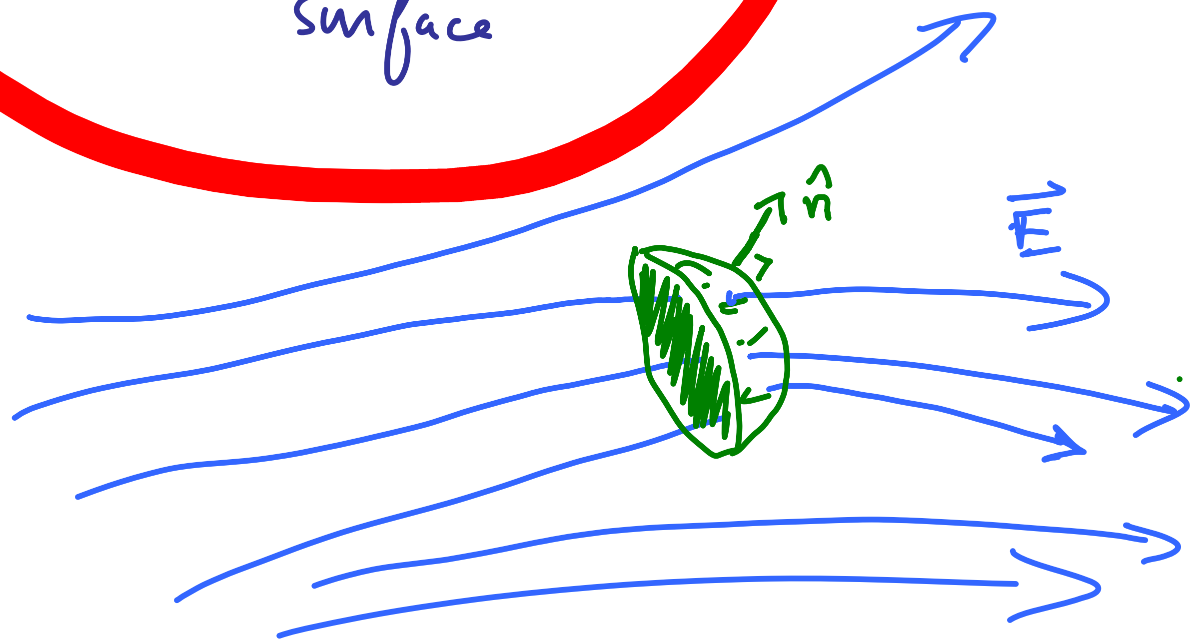
Physics 114 - February 3, 2015

Last Time

Electric Flux

$$\Phi_{surf} = \int_{surface} \vec{E}\cdot \overbrace{d\vec{A}}^{\hat{n}\,dA}$$



Gauss' Law

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$ ②

True in general

integral over volume inside gaussian surface (NOT necessarily all Charge)

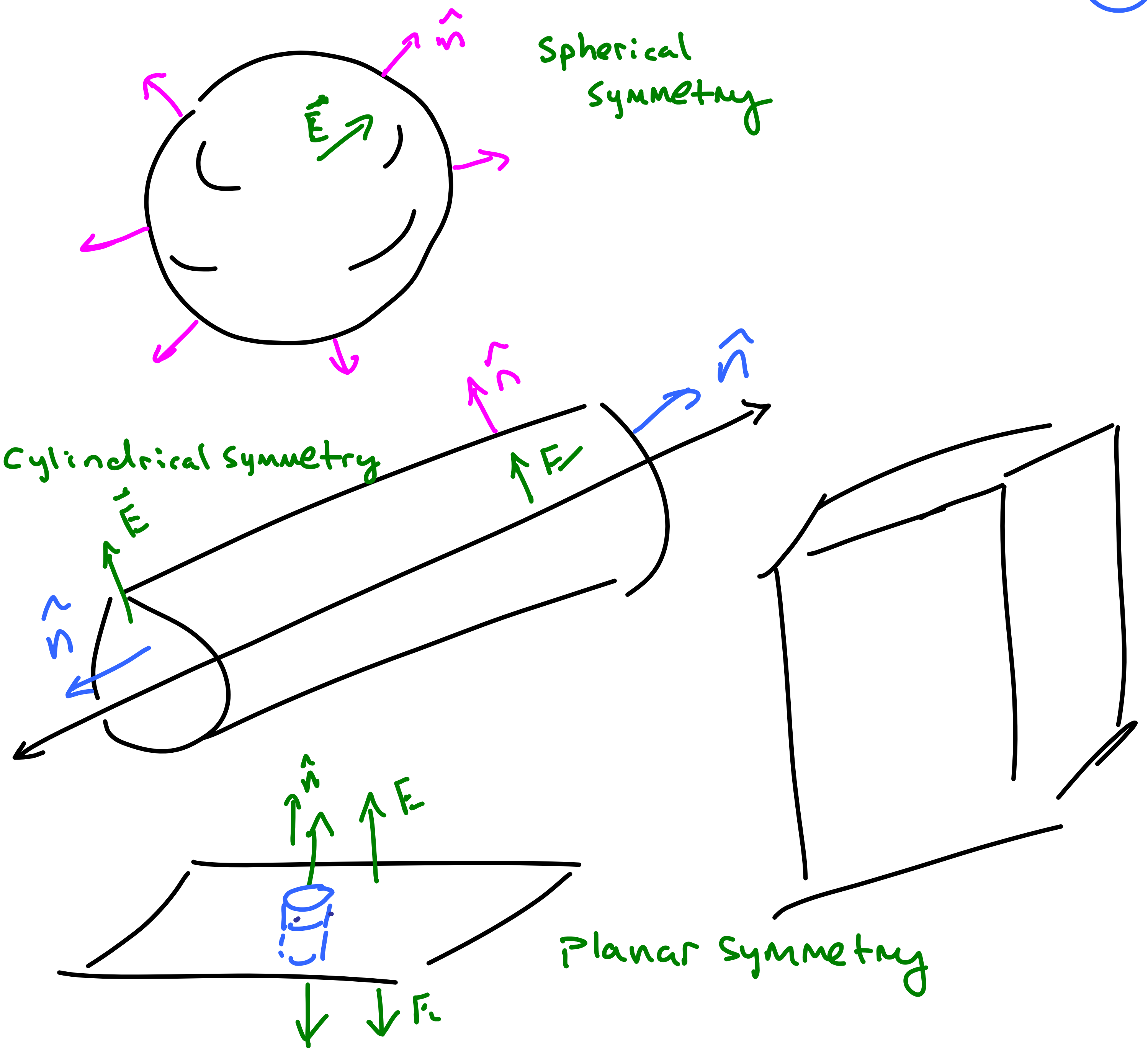
$$\oint \underbrace{\vec{E}\cdot d\vec{A}} = \frac{Q_{encl}}{\epsilon_0}$$

$$\int_V \rho \, dv$$

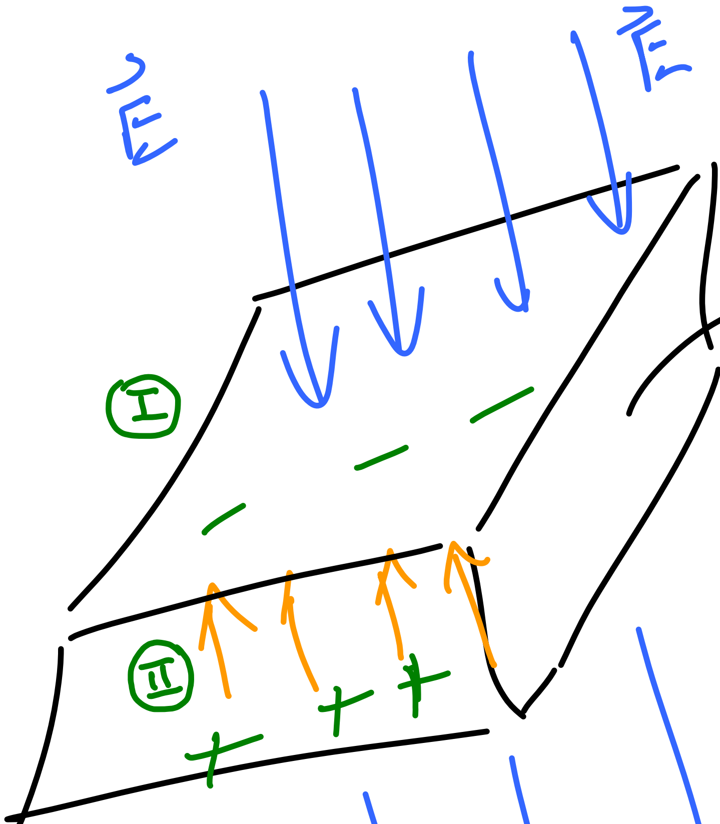
integral over Gaussian surface

Easy if $\vec{E} \perp d\vec{A}$ or $\vec{E} \parallel d\vec{A}$

Easy if $|\vec{E}|$ is constant on surface – can pull out of integral


Spherical symmetry
$\hat{n}$
$\vec{E}$
Cylindrical symmetry
$\hat{n}$
$\vec{E}$
$\hat{n}$
$\hat{n}$
E
$\hat{n}$
E
Planar symmetry
E

4

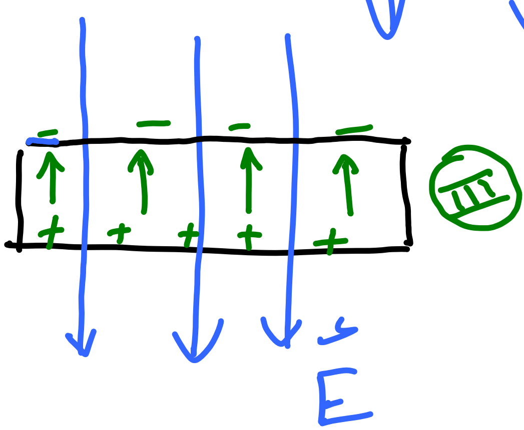


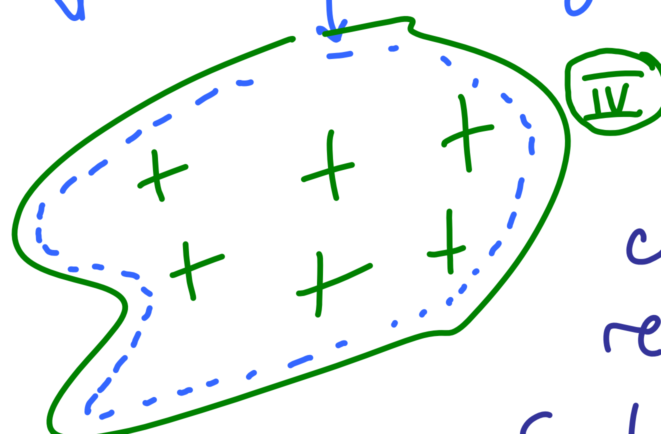
$$\vec{E}_{\text{inside conductor}} = 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$= 0$

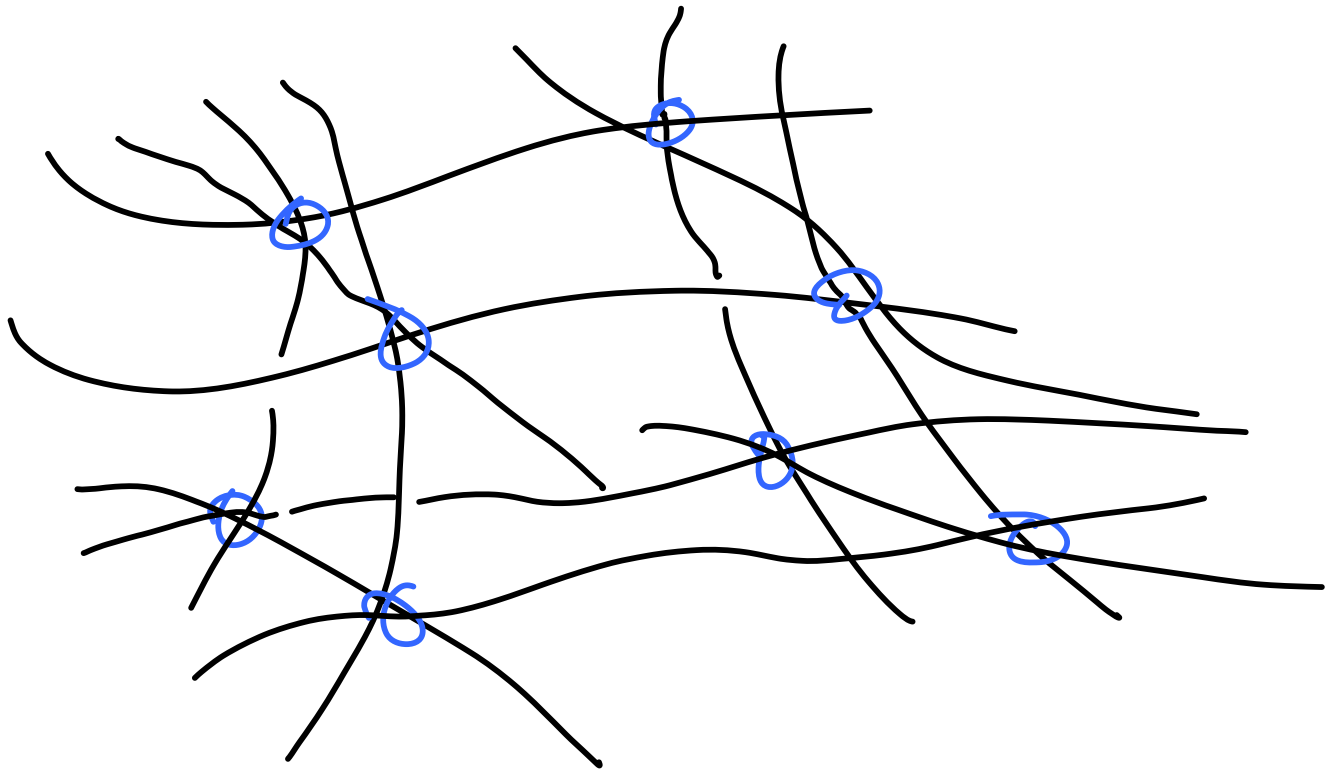
Imaginary Gaussian surface





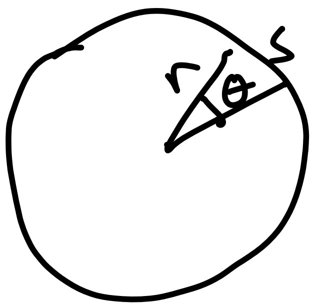
charge resides on surface of conductor

Curvilinear Coordinate System



Recall

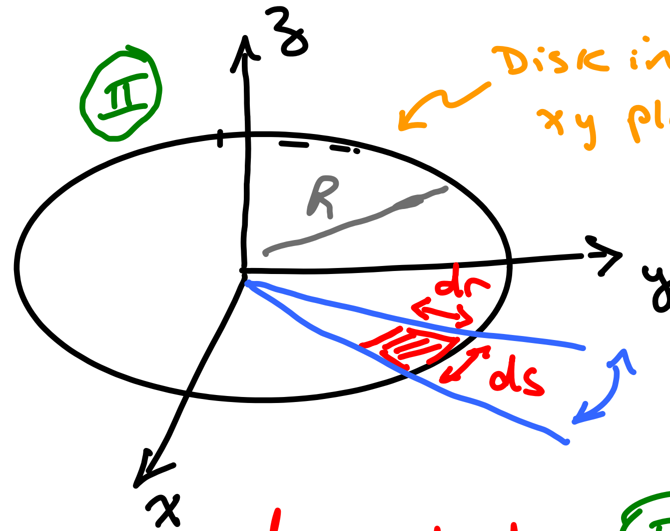
I



$$S = r\theta$$

Find area of circular disk

II



Disk in xy plane

$$da = dr\,ds \quad \text{III}$$
$$= r\,d\theta\,dr \quad \text{IV}$$

$$ds = r\,d\theta$$

R integral

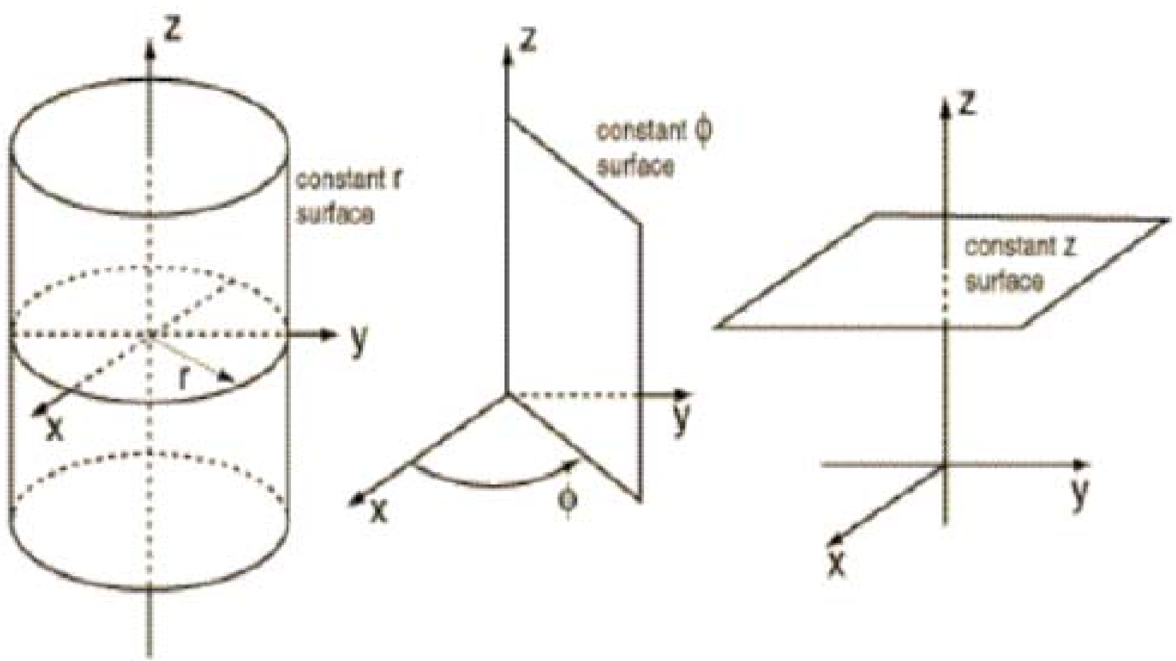
θ integral

$$\text{Area} = \int da = \int_0^R \int_0^{2\pi} r\,d\theta\,dr \quad \text{V}$$

$$\text{VI} \quad = \int_0^R r\,dr \int_0^{2\pi} d\theta = \int_0^R r\,dr\,(2\pi)$$

$$\text{VII} \quad = \frac{R^2}{2} 2\pi = \pi R^2$$

cylindrical coordinates



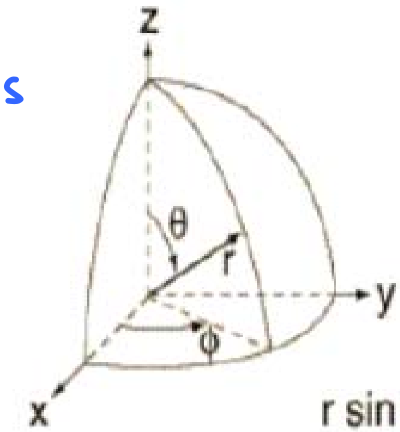
Some figures in this section from:

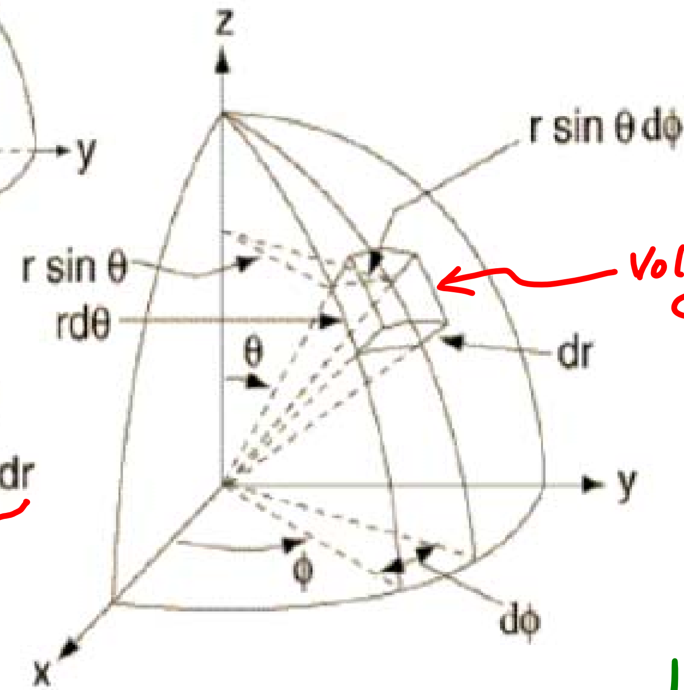
http://hyperphysics.phy-astr.gsu.edu/hbase/sphc.html

Also Griffiths, Intro to Electromagnetism

Spherical polar coordinates

Variables Defined





Volume element dV

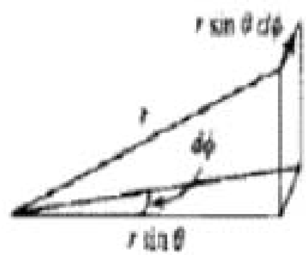
Differential Volume element

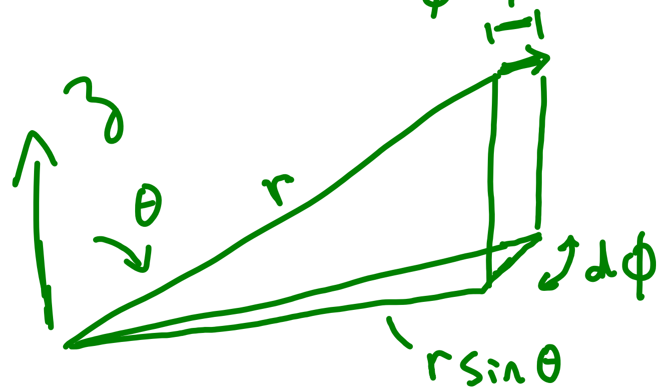
$dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$

This is important
Spend some time
studying this
and make
sure you
understand it.

$dl_\phi = r\sin\theta\, d\phi$

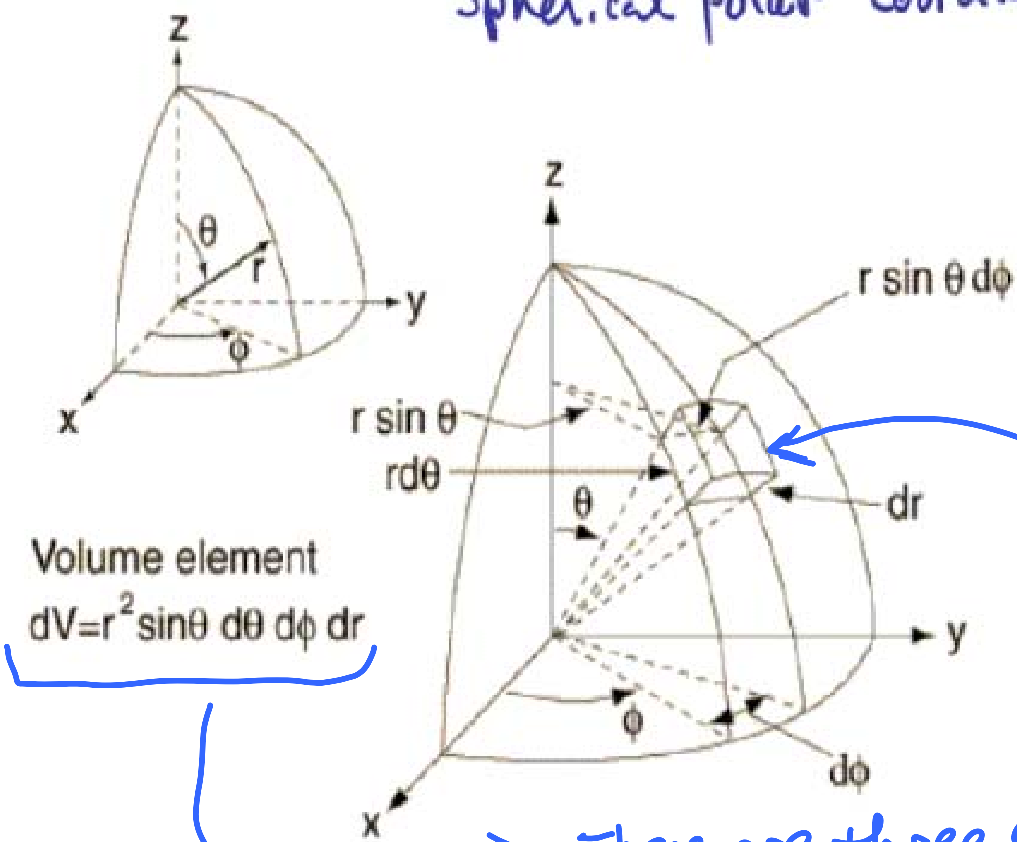
$dl_\phi = r \sin\theta \, d\phi.$





Spherical polar coordinates

(8a)

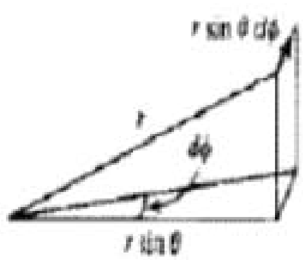


if this were a cartesian coordinate problem

$dv = dx\,dy\,dz$

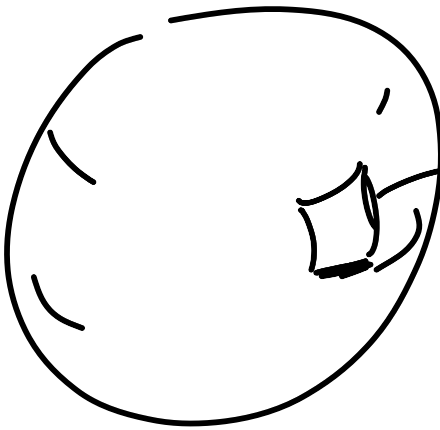
There are three elements that make up dv

$dl_\phi = r \sin\theta \, d\phi.$



in r	dr
in θ	$r\,d\theta$
in ϕ	$r\sin\theta\,d\phi$

Find Area of Sphere



da

θ integral

ϕ integral

I $\quad \text{Area} = \int da = \int_0^{\pi} \int_0^{2\pi} \underbrace{r^2}_{\text{CONSTANT}} \sin\theta \, d\theta \, d\phi$

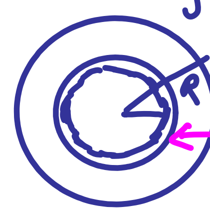
II $\quad = r^2 \, 2\pi \underbrace{\int_0^{\pi} \sin\theta \, d\theta}_{2} = 4\pi r^2$

[Can Choose shape of dv to simplify problem]

$da = dr \, r \, d\theta$

$A = \int_0^R \int_0^{2\pi} r \, d\theta \, dr$

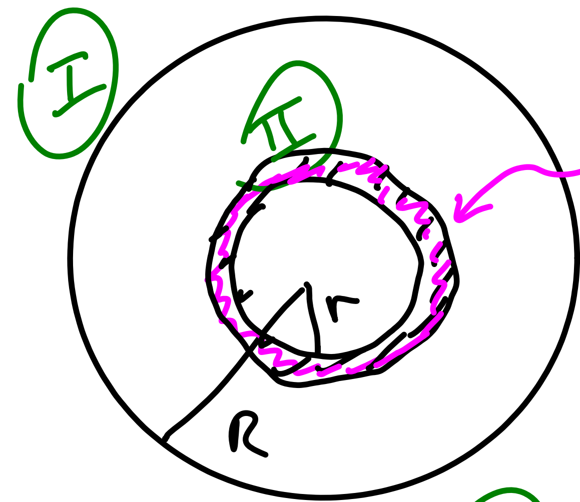
or integrate over θ by constuction



$da = 2\pi r \, dr$

area of circle

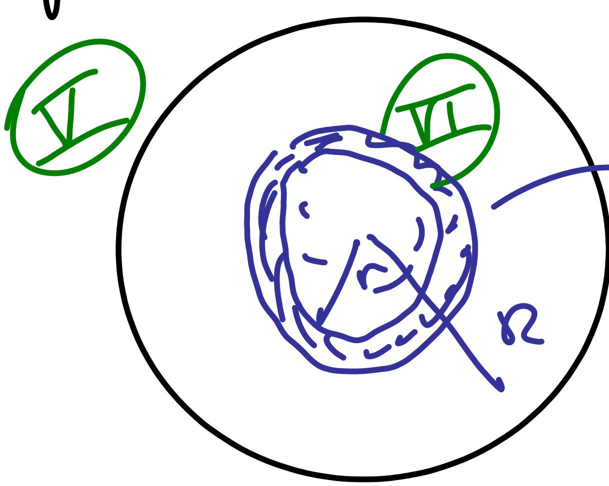
- solved as a 1-dimensional integral



III $da = 2\pi r\,dr$

IV $\int_0^R 2\pi r\,dr = \pi r^2$

Vol of Sphere



Shell

$dv = 4\pi r^2 dr$

$\text{Vol sph} = \int_0^R dv = \int_0^R 4\pi r^2 dr = \frac{4}{3}\pi R^3$

$$\rho = k r^3 \quad r < R$$
$$= 0 \quad r \geq R$$

units of k must make things work

III $\rho \to C/m^3$ so

$k \to C/m^6$

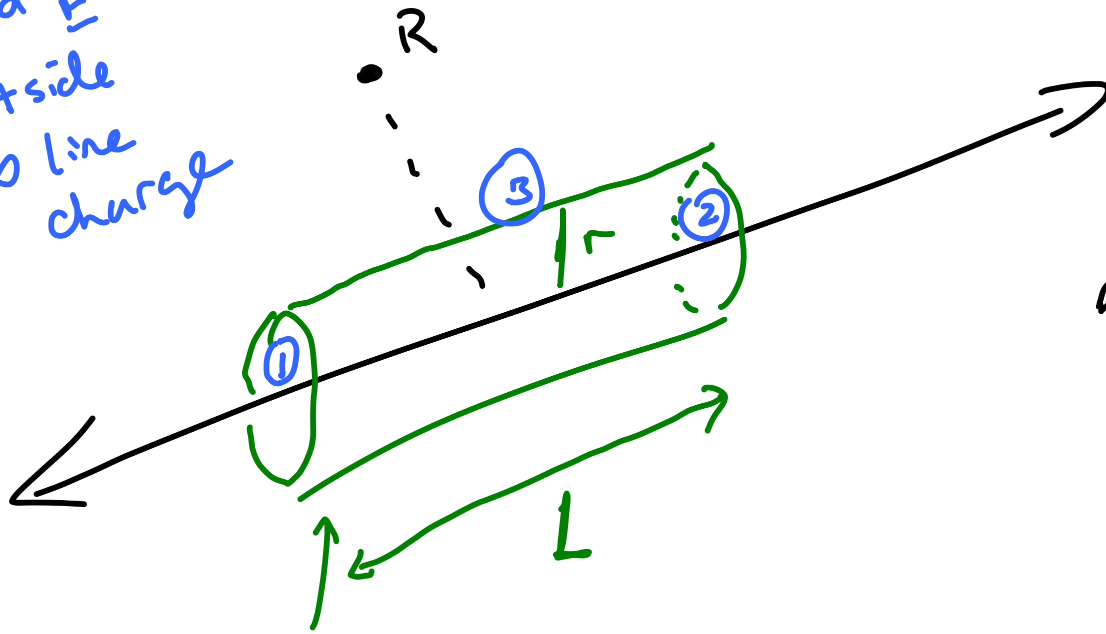
Find TOTAl charge

I $$Q_{TOT} = \int \rho \, dv = \int_0^R \rho \underbrace{4\pi r^2 dr}_{dv}$$

II $$Q_{TOT} = \int_0^R k r^3 4\pi r^2 dr = k 4\pi \int_0^R r^5 dr$$

$$= k 4\pi \frac{R^6}{6}$$

Find $\vec{E}$ outside ∞ line charge



$+\lambda$ chg/length

∞ line charge

Find electric field at R

cylindrically symmetric gaussian surface

$\vec{E}$ radially out (symmetry)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$\vec{E} \cdot d\vec{A} = 0$ for 1 + 2 because $d\vec{a} \perp \vec{E}$

For "endcaps"

$\vec{E}\cdot\vec{dA} = |\vec{E}|_r\, dA$ for surf 3

13

$$\oint \vec{E}\cdot\vec{dA} = |\vec{E}| \int_{\text{surf } 3} dA = \frac{Q_{encl}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$2\pi r L$

surface area of pipe part of cylinder + caps

$$|\vec{E}|\, 2\pi r L = \frac{\lambda L}{\epsilon_0}$$

$$\boxed{|\vec{E}| = \frac{\lambda}{2\pi r \epsilon_0}}$$

$\vec{E}$ is radial out by symmetry

(Assuming λ is positive)