# Physics 114 - March 19, 2015

Exam 2 Cometh

Tuesday, March 24 0800 Lower Strong

formula sheets 1 side 8.5 x 11 inch

Q + A Session Prob Mon at 5pm

Exam 2 will cover:

Problem set 3 (last two problems) through problem set 8

Workshops 3 through 7

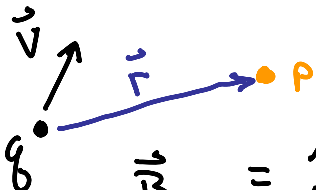
Lectures from Feb. 5 (start of potential) through March 5

Text chapters 23 through 27

- My March 24 office hour is 3-4 instead of 2-3

# Law of Biot-Savart

<Magnetostatics> Steady currents

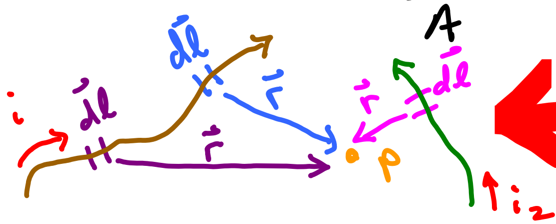



$\vec B$ at point p due to q moving with $\vec v$

$$\vec B_{\substack{\text{at}\\ \text{p due}\\ \text{to q}}} = \frac{\mu_0}{4\pi}\, \frac{q\,\vec v\times\hat r}{r^2}$$

$\mu_0$ = permeability of free space

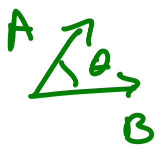
$= 4\pi\times10^{-7}\ \frac{T\cdot M}{A}$



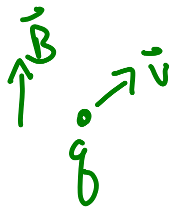

$\vec B$ at some point due to distribution of steady currents

$$\boxed{\vec B_p = \frac{\mu_0}{4\pi}\int_{\substack{\text{Distribution}\\ \text{of}\\ \text{currents}}} \frac{i\,d\vec\ell\times\hat r}{r^2}}$$

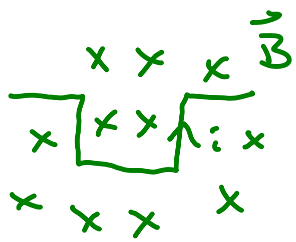
# Right-hand rules



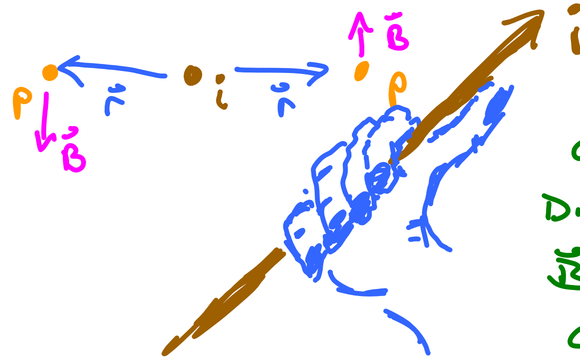

$\vec{A} \times \vec{B}$




$\vec{F} = q\,\vec{v} \times \vec{B}$

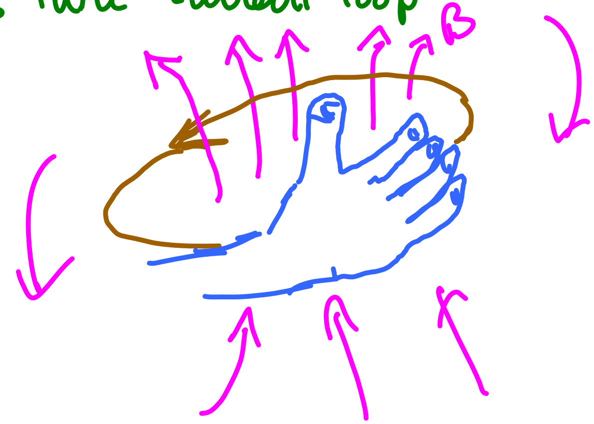



$\vec{F} = i\,\vec{l} \times \vec{B}$

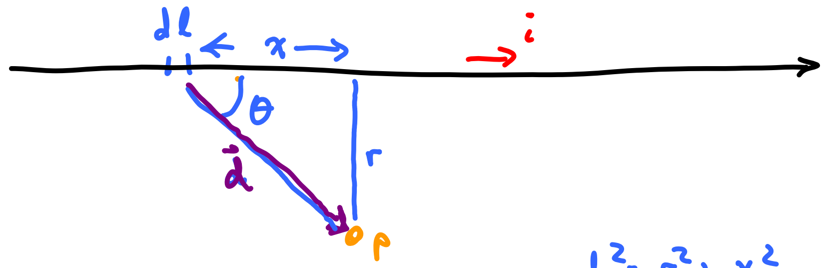



determine Direction of $\vec{B}$ around current i

Determine direction of $\vec{B}$ thru current loop



"simple" example
$\infty$ straight line current




$d^2 = r^2 + x^2$

$$\vec{B}_P = \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \frac{i\, d\vec{l} \times \hat{d}}{d^2}$$

$d\vec{l} \times \hat{d} = |d\vec{l}| \sin\theta \qquad \sin\theta = \frac{r}{d}$

$$\vec{B}_P = \frac{\mu_0}{4\pi} 2i \int_0^{\infty} \frac{\sin\theta\, dx}{r^2 + x^2}$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

$$\vec{B}_P = \frac{\mu_0 2ir}{4\pi} \int_0^{\infty} \frac{dx}{(r^2+x^2)^{3/2}}$$

doable but a pain

## Electrostatics

Gauss' Law

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

closed surface

## Magnetostatics

Ampere's Law

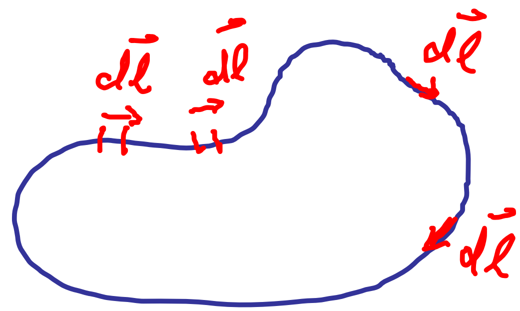
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$

closed curve

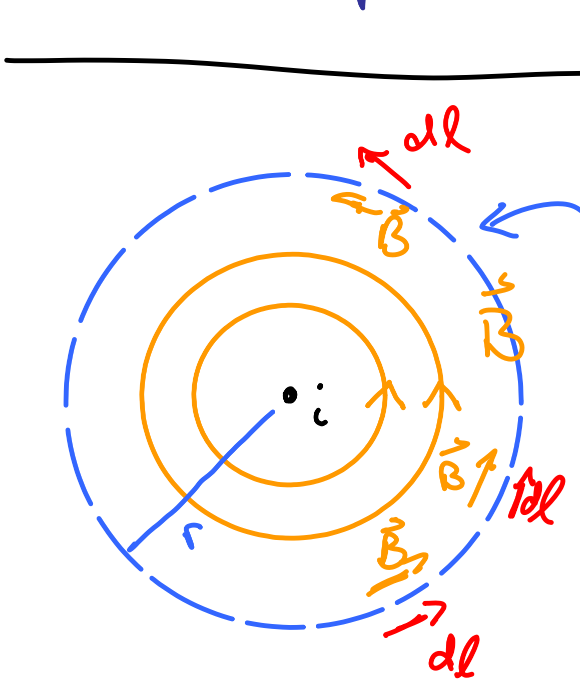
General . . .

Always True

Very useful under certain conditions of symmetry.



Back to earlier example that was painful with Biot-Savart




$\infty$ line current

Amperian loop

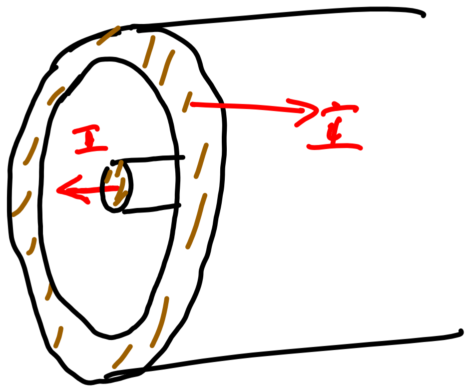
$$\int_{loop} \vec{B} \cdot \vec{d\ell} = \mu_0 I_{enclosed}$$

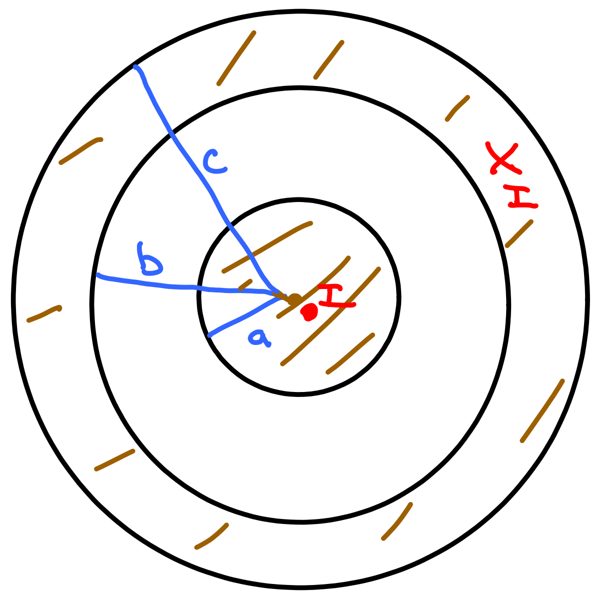
$$\int_{loop} |\vec{B}|\, d\ell = \mu_0 i$$

$$|\vec{B}| \int_{loop} d\ell = \mu_0 i$$

$$|\vec{B}|\, 2\pi r = \mu_0 i \qquad |\vec{B}| = \frac{\mu_0 i}{2\pi r}$$

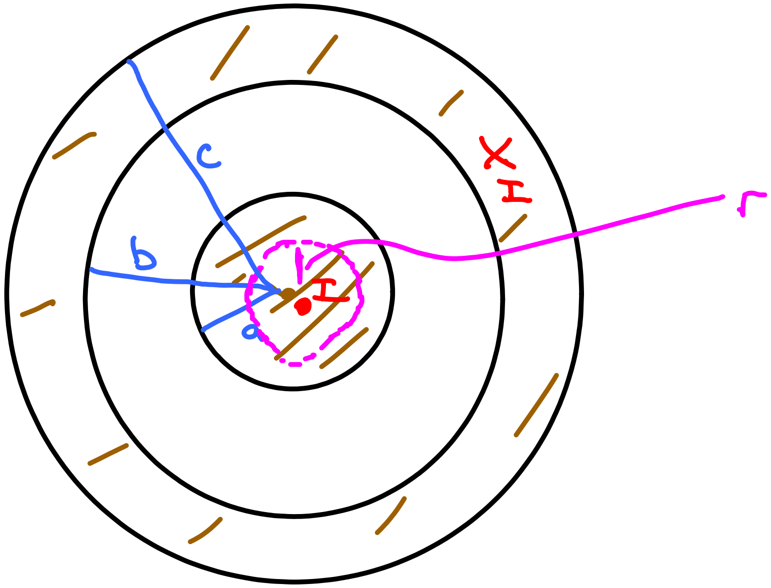
Coaxial cable (long)



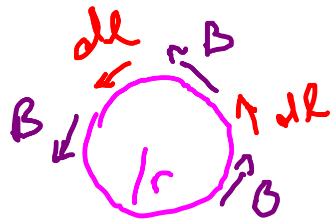

Assume I is uniform across both inner + outer conductors.

Find $\vec{B}$ in all space



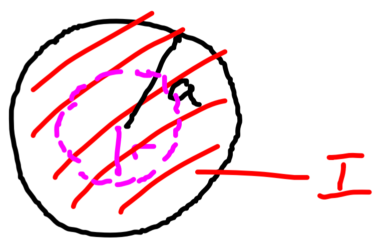

Find $\vec{B}$ for $r<a$

$$\oint_{\text{curve}} \vec{B}\cdot\vec{dl} = \mu_0 i_{enclosed}$$



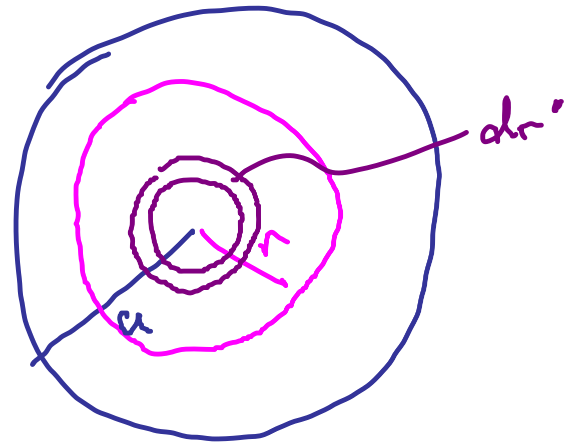

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl = B\oint dl$$

$$B2\pi r$$




$\vec{j}(r) \equiv$ current density

$j(r) = \text{const wrt } r = \dfrac{I}{\pi a^2}$




$i_{encl} = \int j \, dA = \int_0^r j \, 2\pi r' dr'$

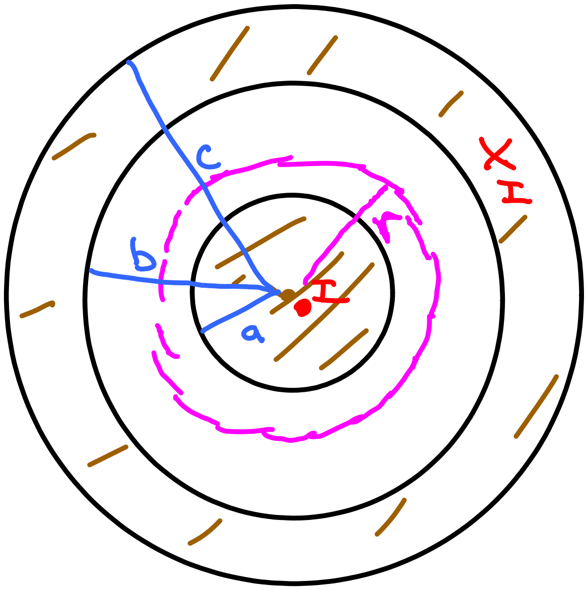
(area of the Amp. loop)

$= j \dfrac{2\pi r^2}{2} = j \pi r^2$

$B \, 2\pi r = \mu_0 j \pi r^2 = \mu_0 \dfrac{I}{\pi a^2}$

$\vec{B}_{at\ r<a} = \dfrac{\mu_0 I}{\pi a^2} \dfrac{\pi r^2}{2\pi r} = \dfrac{\mu_0 I r}{2 a^2 \pi}$

in direction counter-clockwise referring to 1st sketch

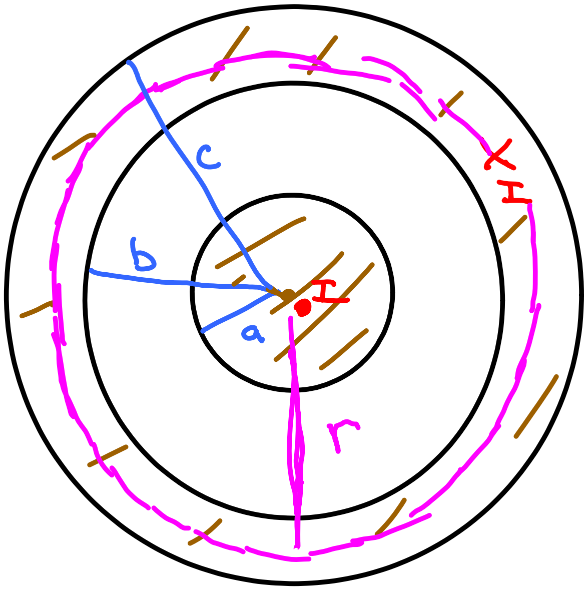



$b > r > a$

$$\oint \vec{B} \cdot d\vec{l} = i_{encl}\,\mu_0$$

$$B\,2\pi r = I\mu_0$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$ counterclockwise



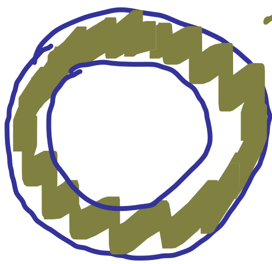

$c > r > b$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{encl}$$

$$B 2\pi r = \mu_0 i_{encl}$$

$$i_{encl} = I - j(\pi r^2 - \pi b^2) = I - \frac{I}{(\pi c^2 - \pi b^2)}(\pi r^2 - \pi b^2)$$

$$= I\left[1 - (\pi r^2 - \pi b^2)/(\pi c^2 - \pi b^2)\right]$$

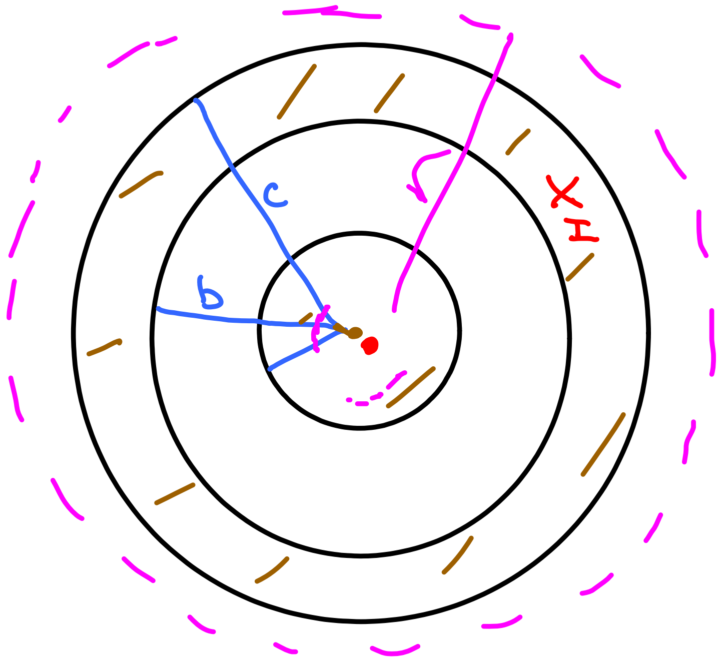


what is current density ?

$$j = \frac{I}{\pi c^2 - \pi b^2}$$

$$\vec{B}_{c>r>b} = \frac{\mu_0}{2\pi r} I \left[ 1 - \frac{(\pi r^2 - \pi b^2)}{(\pi c^2 - \pi b^2)} \right]$$

counterclockwise




$r > c$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \, i_{encl}$$

$$B \, 2\pi r = \mu_0 (0)$$

$$\vec{B} = 0$$