Physics 114 - April 2, 2015

Last Time

Integral form of Maxwell's equations

Gauss $\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

No magnetic monopole $\oint_S \vec{B}\cdot d\vec{A} = 0$

Ampere $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \int_S \vec{J}\cdot d\vec{A} + \boxed{\mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E}\cdot d\vec{A}}$

new term - "Maxwell's Displacement current"

Faraday $\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt} \int \vec{B}\cdot d\vec{a}$

Integral Form

(what you've seen except for Maxwell's displacement current term)

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

Vector Calculus Magic

Differential Form

[We will not use in this class]

$$\underbrace{\vec{\nabla}\cdot}_{\text{called the "divergence"}} \vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

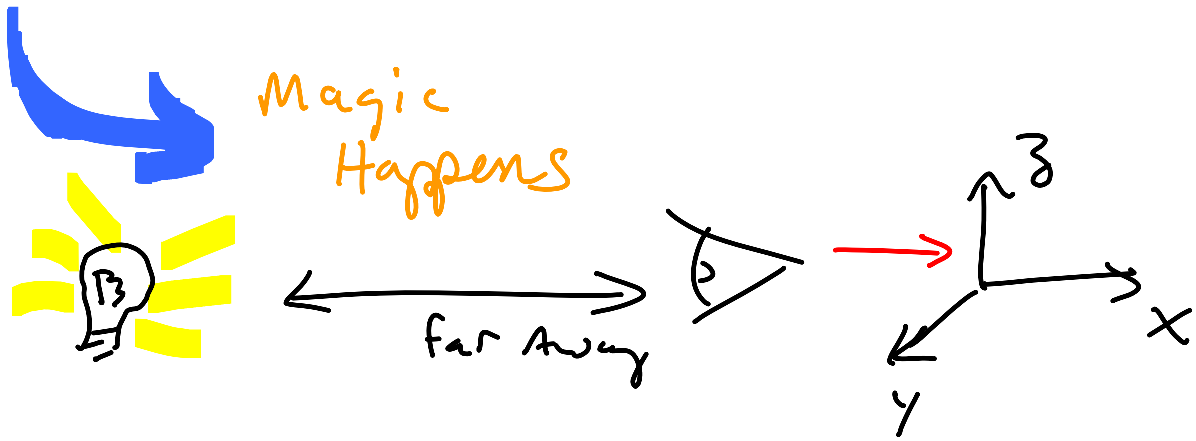
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\underbrace{\vec{\nabla}\times}_{\text{called the "curl"}}\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

vector operator

Just so you can recognize it if you see it



phase (initial condition)

$$E_y(x,t) = E_{oy}\cos(\underbrace{k}_{\frac{2\pi}{\lambda}}x - \underbrace{\omega}_{\frac{2\pi}{T}}t + \phi)$$

$$\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi/T} = \frac{T}{\lambda} = \frac{1}{\text{speed}}$$

Speed of prop

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

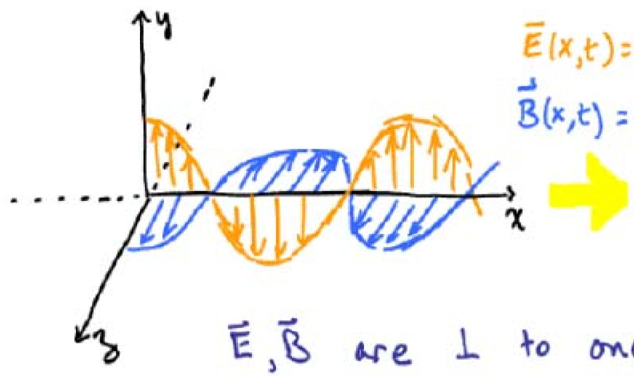
$$B_z(x,t) = \frac{k}{\omega} E_{oy}\cos(kx - \omega t + \phi)$$

$$= \frac{1}{c} E_y$$

very Far Away

Solve Maxwell's eqns

Get Coupled wave eqns for $\vec{E}$, $\vec{B}$



$\vec{E}(x,t) = E_0 \cos(kx - \omega t + \varphi)\,\hat{y}$

$\vec{B}(x,t) = \frac{E_0}{c} \cos(kx - \omega t + \varphi)\,\hat{z}$

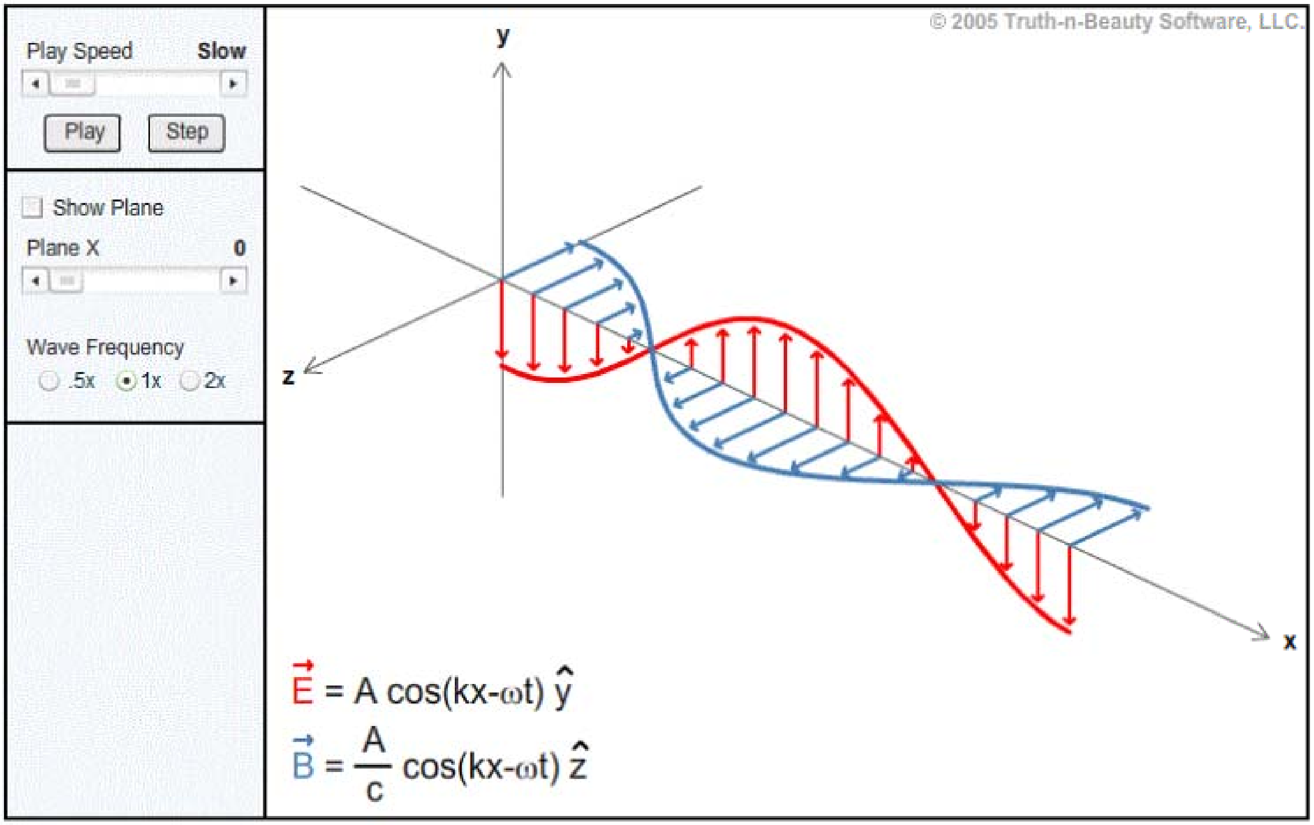
$\vec{E}$, $\vec{B}$ are $\perp$ to one another

Wave propogates in direction of $\vec{E} \times \vec{B}$

$|\vec{B}| = |\vec{E}|/c$

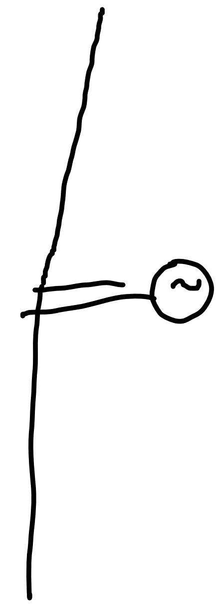
$\vec{E}$, $\vec{B}$ in phase

speed of propagation $\rightarrow c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

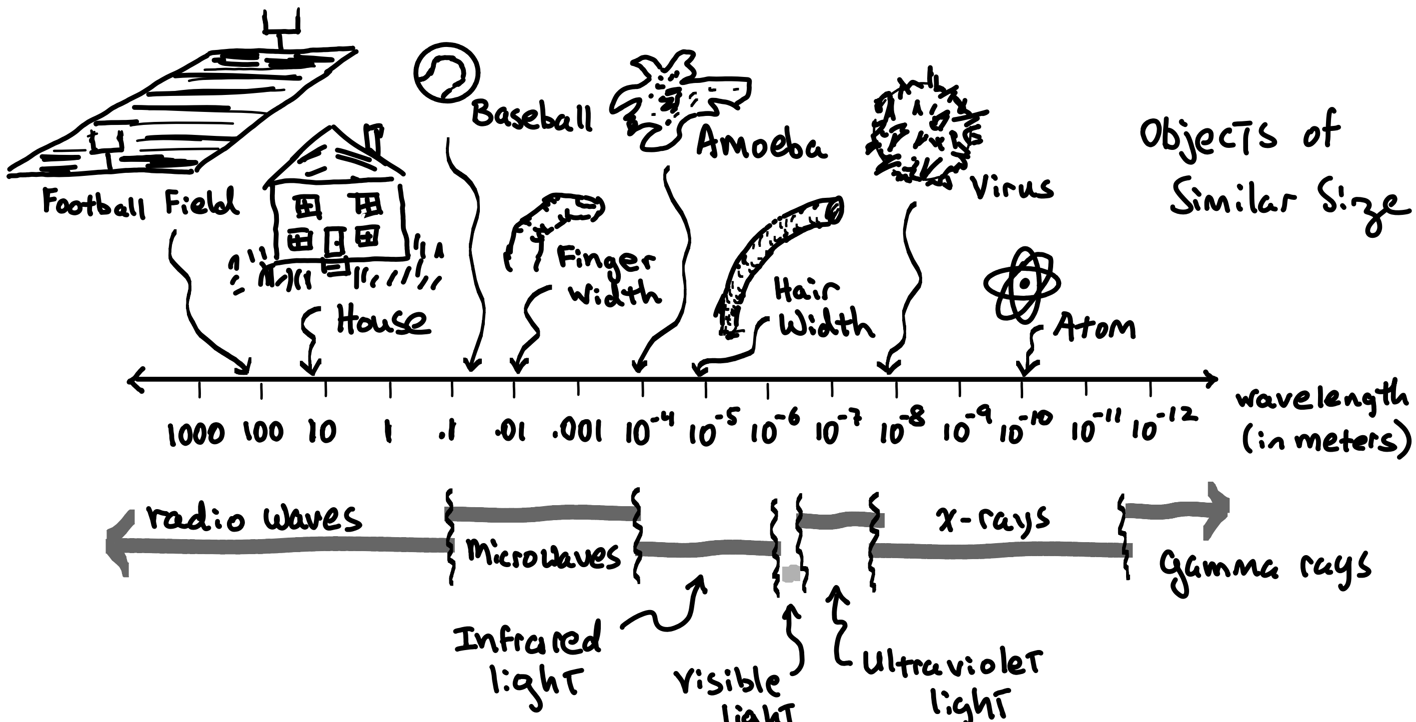


Play Speed
Slow
Play
Step
Show Plane
Plane X
0
Wave Frequency
.5x
1x
2x
© 2005 Truth-n-Beauty Software, LLC.
y
z
x
$\vec{E} = A\cos(kx-\omega t)\,\hat{y}$
$\vec{B} = \frac{A}{c}\cos(kx-\omega t)\,\hat{z}$

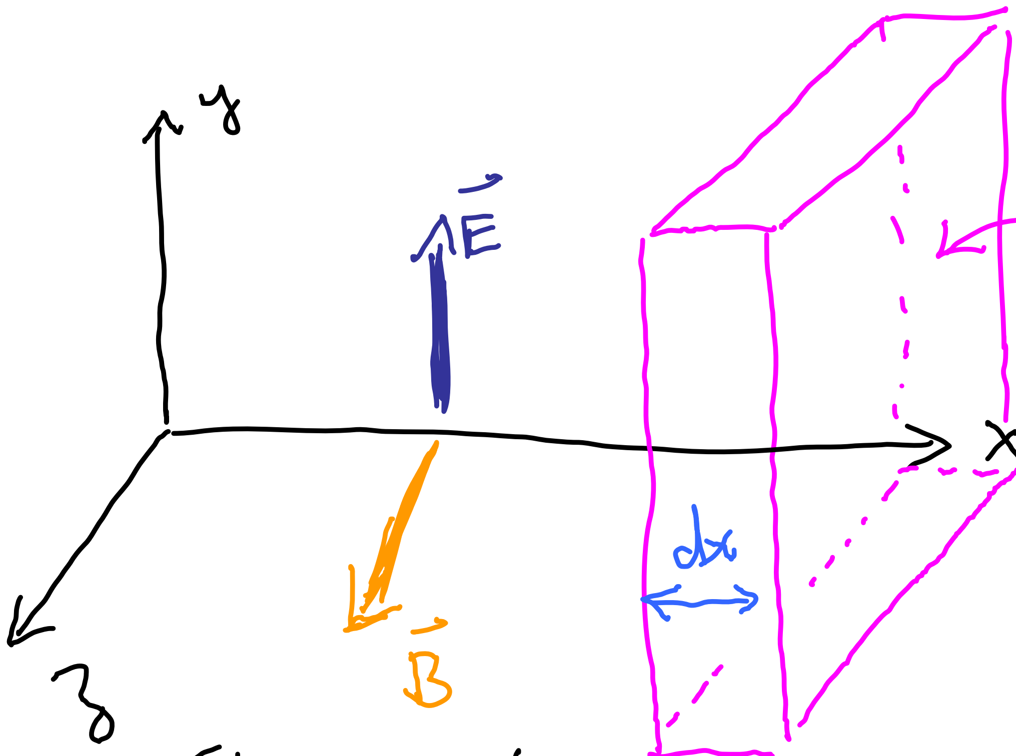
DEMO



The variety of electromagnetic Waves



Energy in EM wave



Electromagnetic plane wave

$$u_E = \frac{\epsilon_0}{2} E^2$$

$$u_B = \frac{1}{2\mu_0} B^2$$

$$dU = (u_E + u_B)\,\text{volume} = (u_E + u_B)\, A\, dx$$

$$du = \left(\frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2\right) A\,dx$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \qquad E = cB \qquad \epsilon_0 = \frac{1}{\mu_0 c^2}$$

$$du = \left[\frac{1}{2\mu_0 c^2} EcB + \frac{1}{2\mu_0} \frac{BE}{c}\right] A\,dx$$

$$\frac{du}{dt} \Big/ \text{Area} = \text{Power thru BOX} = \frac{EB}{\mu_0} \quad \text{WATTS}/m^2$$

Intensity of EM wave

$\vec{S} \equiv$ vector Energy flow $\equiv \dfrac{\vec{E} \times \vec{B}}{\mu_0}$

Poynting vector

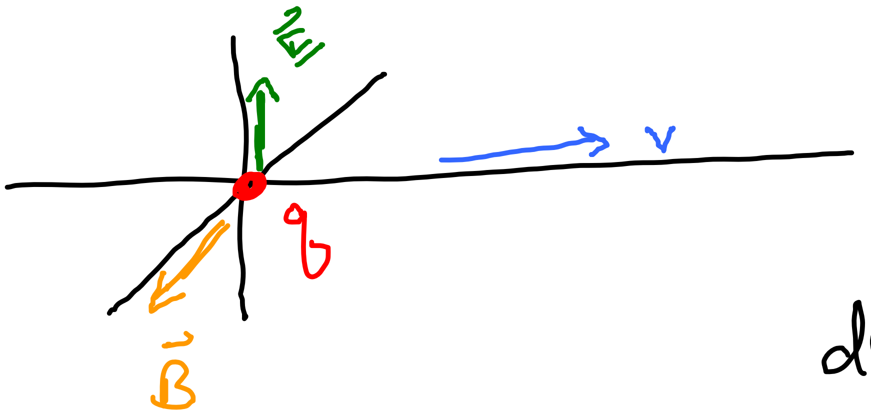
Time Average $\vec{S} \equiv \langle \vec{S} \rangle \equiv \overline{S}$

$E = E_0 \sin \omega t$

$B = \frac{E_0}{c} \sin \omega t$

$$S \sim \frac{1}{\mu_0 c} \underbrace{\sin^2 \omega t}_{\text{Time average} \to \frac{1}{2}}$$

$$\langle S \rangle = \frac{E_0 B_0}{2 \mu_0 c}$$



$$F = \frac{dp}{dt} \sim qvB \sim \overbrace{q\,\underbrace{v}_{\text{dist/time}}\,\frac{E}{c}}^{F} \sim \frac{1}{c}\,\frac{d(\text{work})}{dt}$$

$$dp \sim \frac{d(\text{work})}{c} \sim \frac{\text{Energy}}{c}$$

$$\boxed{p = \frac{U}{c}}$$

Momentum carried by EM wave

$$F = \frac{1}{c}\,\frac{\text{Energy}}{\text{Time}}\,\frac{\text{Area}}{\text{Area}} = \frac{S}{c}\,\text{Area}$$

$$\text{Pressure} = \frac{F}{\text{Area}} = \frac{S}{c} = \text{Radiation Pressure}$$