P114 - Spring 2015 - PS 1 solutions

21- 1 From Coloumb's law we know that the magnitude of the force between two charges is given by

$$F = k\frac{q_1 q_2}{r^2}$$

Over here one wants to find the force of attraction between the nucleus having a charge of $26e$ and an electron of charge e in the inner most shell at a distance of $r = 1.5 \times 10^{-12} m$. $e = -1.602 \times 10^{-26} C$. Hence the magnitude of the force (a positive quantity) turns out to be

$$\begin{aligned} F &= 8.988 \times 10^9 Nm^2/C^2 \times \frac{1.602\times10^{-26}C\times26\times1.602\times10^{-26}C}{(1.5\times10^{-12}m)^2} \\ &= 2.7 \times 10^{-3} N \end{aligned}$$

21- 2 One knows that charge is quantised and the amount of charge carried by one electron is $e = -1.602 \times 10^{-26} C$. Hence the number of electrons needed to hold a charge of $-38\mu C$ is

$$\frac{-38 \times 10^{-6} C}{-1.602 \times 10^{-26} C} = 2.37 \times 10^{14}$$

21- 6 From Coulomb's law we know that the force of attraction is proportional to $\frac{1}{r^2}$. If the charges are the same then the force times the distance squared should be a constant. In this case the distance is changed by a factor of $\frac{1}{8}$. $F_1 = 3.2 \times 10^{-2} N$ and $r_2 = r_1/8$.

$$\begin{aligned} F_1 r_1^2 &= F_2 r_2^2 \\ F_2 &= F_1 \frac{r_1^2}{r_2^2} \\ F_2 &= 64 F_1 \\ &= 2N \end{aligned}$$

21- 10 The electrostastatic force of attraction is given by the Coulomb's law

$$F_e = k\frac{q_e q_p}{r^2}$$

where as the gravitational force of attraction is given by

$$F_g = G\frac{m_e m_p}{r^2}$$

Hence the ratio of the forces turns out to be

$$\begin{aligned} \frac{F_e}{F_g} &= \frac{8.988\times10^9 Nm^2/C^2\times(1.602\times10^{-26}C)^2}{6.67\times10^{-11}Nm^2/kg^2\times9.11\times10^{-31}kg\times1.67\times10^{-27}kg} \\ &= 2.3\times10^{39} \end{aligned}$$

21- 12 One again uses the Coulomb's law to find out the force and its direction this time. While doing so one must use the distance between the charges for whom the force of attraction or repulsion is being found. The sign of the charges helps us to find the direction along which they are acting. The forces on the various charges are as follows

$$\begin{aligned} \vec{F}_{+75} &= -k\frac{75\mu C\times48\mu C}{(0.35m)^2}\hat{i} + k\frac{75\mu C\times85\mu C}{(0.70m)^2}\hat{i} \\ &= -147.2N\hat{i} \\ \vec{F}_{+48} &= k\frac{75\mu C\times48\mu C}{(0.35m)^2}\hat{i} + k\frac{48\mu C\times85\mu C}{(0.35m)^2}\hat{i} \\ &= 563.5N\hat{i} \\ \vec{F}_{-85} &= -k\frac{75\mu C\times85\mu C}{(0.70m)^2}\hat{i} - k\frac{85\mu C\times48\mu C}{(0.35m)^2}\hat{i} \\ &= -416.3N\hat{i} \end{aligned}$$

21- 13 The forces between the charges act as per the Coulomb's law. The direction along which they will act depends on the charges under consideration. For a positive charge interacting with another positive charge the force of repulsion will act away from the charge where as if the positive charge interacts with a negative charge the force of attraction will act towards the positive charge. For this problem we first find out the magnitude of the forces acting between the particles. In order to find out the direction of the net force we will use decompose the forces into X and Y components. The fact that the charges lie on an equilateral triangle with equal sides and each angle being 120° will be used as well. The magnitude of the forces acting are

$$
\begin{aligned}
F_{12} = F_{21} &= k\frac{|q_1 q_2|}{r^2} = 8.988 \times 10^9 Nm^2/C^2 \times \frac{7\mu C \times 8\mu C}{(1.20m)^2} \\
&= 0.3495N \\
F_{13} = F_{31} &= k\frac{|q_1 q_3|}{r^2} = 8.988 \times 10^9 Nm^2/C^2 \times \frac{7\mu C \times 6\mu C}{(1.20m)^2} \\
&= 0.2622N \\
F_{23} = F_{32} &= k\frac{|q_2 q_3|}{r^2} = 8.988 \times 10^9 Nm^2/C^2 \times \frac{8\mu C \times 6\mu C}{(1.20m)^2} \\
&= 0.2996N \\
F_{1x} = F_{12x} + F_{13x} &= (-0.3495N)cos60^\circ + (0.2622N)cos60^\circ = -4.365 \times 10^{-2} N \\
F_{1y} = F_{12y} + F_{13y} &= (-0.3495N)sin60^\circ - (0.2622N)sin60^\circ = -5.297 \times 10^{-1} N \\
F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} &= 0.53N \\
\theta_1 = tan^{-1}\frac{F_{1y}}{F_{1x}} &= 265^\circ \\
F_{2x} = F_{21x} + F_{23x} &= (0.3495N)cos60^\circ - (0.2996N) = -1.249 \times 10^{-1} N \\
F_{2y} = F_{21y} + F_{23y} &= (0.3495N)sin60^\circ + 0 = -3.027 \times 10^{-1} N \\
F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} &= 0.33N \\
\theta_2 = tan^{-1}\frac{F_{2y}}{F_{2x}} &= 112^\circ \\
F_{3x} = F_{31x} + F_{32x} &= (-0.2662N)cos60^\circ + (0.2996N) = 1.685 \times 10^{-1} N \\
F_{3y} = F_{31y} + F_{32y} &= (0.3495N)sin60^\circ + 0 = 2.271 \times 10^{-1} N \\
F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} &= 0.26N \\
\theta_3 = tan^{-1}\frac{F_{3y}}{F_{3x}} &= 53^\circ
\end{aligned}
$$

21- 18 The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges. Consideration of the various possible cases leads to the conclusion that the third charge must be postive and placed between the other two charges. Let the new charge be Q placed at a distance x from the charge $-Q_0$. $(0 < x < l)$

The force on $-Q_0$ due to charge Q acting towards it is given by $k\frac{QQ_0}{x^2}$.

The force on $-Q_0$ due to charge $-4Q_0$ acting away from it is given by $k\frac{4Q_0^2}{l^2}$.

Since $-Q_0$ is in equlibrium the two forces are equal to each other.

The force on $-4Q_0$ due to charge Q acting towards it is given by $k\frac{4QQ_0}{(l-x)^2}$.

The force on $-4Q_0$ due to charge $-Q_0$ acting away from it is given by $k\frac{4Q_0^2}{l^2}$.

Since $-4Q_0$ is in equlibrium the two forces are equal to each other.

From the above relations we say that

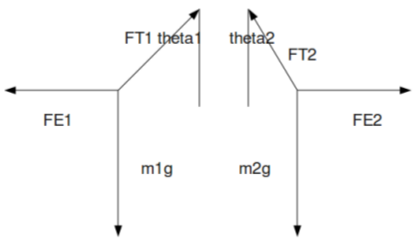
$$\begin{aligned}
k\tfrac{QQ_0}{x^2} &= k\tfrac{4QQ_0}{(l-x)^2} \\
(l-x)^2 &= 4x^2 \\
(l-x) &= 2x \\
x &= \tfrac{1}{3}l
\end{aligned}$$

(ignoring the negative root since x is positive)

Putting this value of x back in

$$\begin{aligned}
k\tfrac{QQ_0}{x^2} &= k\tfrac{4Q_0^2}{l^2} \\
Q &= 4Q_0\tfrac{x^2}{l^2} \\
Q &= \tfrac{4}{9}Q_0
\end{aligned}$$

21- 20 If all the angles to the vertical are considered to be small ($sin\theta \sim tan\theta \sim \theta$) then the spheres have horizontal displacements only. The electric forces will be horizontal. The free body diagram has been drawn below. The right is taken to be the positive horizontal direction and up as the positive vertical direction. Since the bodies are in equilibrium the net forces on the bodies should be zero.



(a)

$$\Sigma F_{1x} = F_{T1}sin\theta_1 - F_{E1} = 0, F_{E1} = F_{T1}sin\theta_1$$

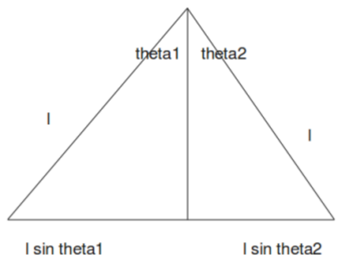
$$\Sigma F_{1y} = F_{T1}cos\theta_1 - m_1g = 0, F_{T1} = \frac{m_1g}{cos\theta 1}$$

$$F_{E1} = \frac{m_1g}{cos\theta_1}sin\theta_1 = m_1gtan\theta_1 = m_1g\theta_1$$

Similarly we can show that $F_{E2} = m_2g\theta_2$. Since the electric forces are balance each other using Newton's third law we can say that $F_{E1} = F_{E2}$ or $\frac{\theta_1}{\theta_2} = \frac{m_2}{m_1} = 1$.

(b) The same calculation can be done as above and we get that $F_{E1} = F_{E2}$ or $\frac{\theta_1}{\theta_2} = \frac{m_2}{m_1} = 2$.

(c) From the diagram below we see that the distance between the two bodies is given by $d = l(\theta_1 + \theta_2)$. For the first case



$$\begin{aligned} d = 2l\theta_1, F_{E1} &= k\frac{Q\times 2Q}{d^2} \\ d &= (\frac{4lkQ^2}{mg})^{\frac{1}{3}} \end{aligned}$$

since $\theta_1 = \theta_2$.

For the second case

$$\begin{aligned} d = \frac{3}{2}l\theta_1, F_{E1} &= k\frac{Q\times 2Q}{d^2} \\ d &= (\frac{3lkQ^2}{mg})^{\frac{1}{3}} \end{aligned}$$

since $\theta_1 = \theta_2/2$.