P114 - SOLUTION SET 2

21.24. We know that $\vec{F} = q\vec{E}$

In this case the charge $q = -8.8 \times 10^{-6}$ C

$$\vec{E} = \frac{\vec{F}}{q} = \frac{8.4 \text{ N down}}{-8.8 \times 10^{-6} \text{ C}} = 9.5 \times 10^{5} \text{ N/C up}$$

The direction of electric field is opposite to that of the force since a negative charge is placed.

21.26. We again use the same formula but now for each component of the force we find corresponding electric field.

$$E_x = \frac{F_x}{q} = \frac{3 \times 10^{-3} \text{ N}}{1.25 \times 10^{-6} \text{ C}} = 2400 \text{ N/C}$$

$$E_y = \frac{F_y}{q} = \frac{-3.9 \times 10^{-3} \text{ N}}{1.25 \times 10^{-6} \text{ C}} = -3100 \text{ N/C}$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} = (2400\hat{i} - 3100\hat{j}) \text{ N/C}$$

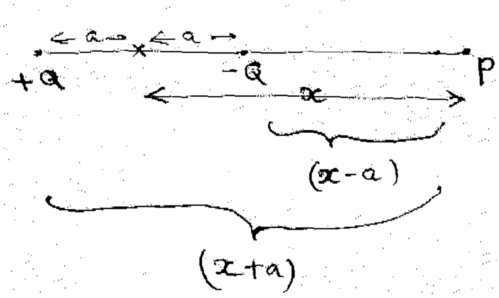
21.27. We use the same formula as above by assuming that the electric force is the only force acting only the electron.

$$\vec{F} = m\vec{a} = q\vec{E}$$

$$\therefore |\vec{a}| = \frac{q|\vec{E}|}{m} = \frac{(1.602 \times 10^{-19} \text{ C})(576 \text{ N/C})}{9.109 \times 10^{-31} \text{ Kg}} = 1.01 \times 10^{14} \text{ m/sec}^2$$

Since the charge is negative the direction of the acceleration is opposite to that of the electric field.

21.35.



The point P is at a distance $(x+a)$ and $(x-a)$ away from the charges $+Q$ and $-Q$ respectively.

The net electric field is given by -

$$E = K\frac{Q}{(x+a)^2} - K\frac{Q}{(x-a)^2} = KQ\left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right)$$

$$= KQ\left(\frac{x^2+a^2-2ax - x^2-a^2-2ax}{(x^2-a^2)^2}\right) \qquad [\because (x+a)(x-a) = x^2-a^2]$$

$$= -\frac{4KQ\,ax}{(x^2-a^2)^2}$$

The negative sign implies that the electric field points towards the left by assuming that direction towards is positive.

21.40. From ~~exp~~ example 21.9 solved in the textbook we know that the electric field at a point at a distance x from the ring is -

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$$

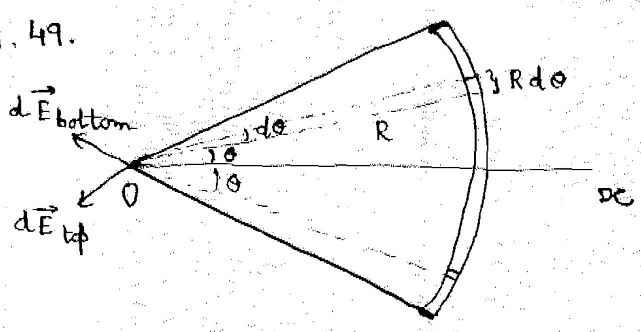
We can use that result in the present problem by shifting the center of the ring once to $x = \frac{l}{2}$ and again to $x = -\frac{l}{2}$. Thus the expression of the electric field changes by choosing appropriate x now. Thus the net electric field now is -

$$\vec{E} = \vec{E}_{right} + \vec{E}_{left}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q\left(x-\frac{l}{2}\right)}{\left[\left(x-\frac{l}{2}\right)^2+R^2\right]^{3/2}}\hat{i} + \frac{1}{4\pi\epsilon_0}\frac{Q\left(x+\frac{l}{2}\right)}{\left[\left(x+\frac{l}{2}\right)^2+R^2\right]^{3/2}}\hat{i}$$

$$= \hat{i}\,\frac{Q}{4\pi\epsilon_0}\left[\frac{\left(x-\frac{l}{2}\right)}{\left(\left(x-\frac{l}{2}\right)^2+R^2\right)^{3/2}} + \frac{\left(x+\frac{l}{2}\right)}{\left(\left(x+\frac{l}{2}\right)^2+R^2\right)^{3/2}}\right]$$

21.49.



We select a small element on the arc at an angle θ with x axis. The length of this element is $R\,d\theta$. Since the charge density is given by λ, the total charge on this length element is

$$dq = \lambda R\,d\theta$$

Electric field created due to this from the top element is -

$$|d\vec{E}_{top}| = \frac{1}{4\pi\epsilon_0}\frac{\lambda R\,d\theta}{R^2}$$

However at pt. O both the top and bottom electric field contributes. The vertical components of $\vec{E}_{top}$, $\vec{E}_{bottom}$ cancel each other symmetrically. So the net electric field in horizontal direction is -

$$dE_{horizontal} = dE_{top,\,horizontal} + dE_{bottom,\,horizontal}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda R\,d\theta}{R^2}\cos\theta + \frac{1}{4\pi\epsilon_0}\frac{\lambda R\,d\theta}{R^2}\cos\theta$$

$$= \frac{2\lambda R\,d\theta\,\cos\theta}{4\pi\epsilon_0 R^2}$$

$$\therefore E_{horizontal} = \int_0^\theta \frac{2\lambda\cos\theta\,d\theta}{4\pi\epsilon_0 R} = \frac{2\lambda}{4\pi\epsilon_0 R}\left[+\sin\theta\right]_0^\theta$$

$$= +\frac{2\lambda}{4\pi\epsilon_0 R}\sin\theta$$

However the electric field points in negative x direction,

$$\vec{E} = -\frac{2\lambda}{4\pi\epsilon_0 R}\sin\theta\,\hat{i}$$

21.56. a) Since the electric field is uniform, it will experience a constant force in the opposite direction to its velocity. It leads to deacceleration.

$$F = ma = qE = -eE \qquad a = -\frac{eE}{m}$$

From equations of motion, we know that

$$V^2 = V_0^2 + 2a\Delta x \qquad (\because V = 0 \text{ that is the electron stops finally})$$

$$\therefore \Delta x = -\frac{V_0^2}{2a} = \frac{V_0^2}{2eE}\times m$$

$$= \frac{(27.5\times10^6)^2\,m^2/sec^2 \times 9.11\times10^{-31}\,kg}{2(1.60\times10^{-19}\,c)\,(11.4\times10^3\,N/c)} = 0.189\,m$$

b) To find the time interval, we use the other equation of motion

$$V = V_0 + at$$

$$\therefore t = \frac{V - V_0}{a}$$

Since the particle comes back to the same pt. it will have been accelerated by same extend to have same starting speed but velocity would pt. in other direction.

$$= -\frac{2V_0}{a}$$

$$= -\frac{2V_0}{-eE}\times m$$

$$= \frac{2mV_0}{eE} = \frac{2(9.11\times10^{-31}\,Kg)(27.5\times10^6\,m/sec)}{(1.6\times10^{-19}\,c)(11.4\times10^3\,N/c)} = 2.75\times10^{-8}\,s$$

21.74. $\overset{+}{C} — \overset{-}{O}$ $\qquad H^+ — N^-$

In this problem there are four pairs of charges among which we need to calculate electric force.

$F_{net} = F_{CH} + F_{CN} + F_{OH} + F_{ON}$ (The distances keep on changing depending on the pair)

$$= \frac{K(0.4e)(0.2e)}{(1\times10^{-9}\,m)^2}\left[-\frac{1}{(0.30)^2} + \frac{1}{(0.40)^2} + \frac{1}{(0.18)^2} - \frac{1}{(0.28)^2}\right]$$

$$= 2.445\times10^{-10}\,N \qquad (K = 8.988\times10^9\,N\,m^2/c^2,\ e = 1.6\times10^{-19}\,c)$$

21.79. The sphere will oscillate sinusoidally about the equilibrium pt. with an amplitude of 5.0 cm. The angular frequency of the sphere is given by $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{126\,N/m}{0.650\,Kg}} = 13.92$ rad/sec

The distance of the sphere from the table is given by -

$r = [0.150 - 0.05\cos(13.92t)]\,m$

Using the distance and charge to give electric field, which points upwards at all times towards negative sphere.

$$E = K\frac{|Q|}{r^2} = \frac{(8.988\times10^9\,N\,m^2/c^2)(3\times10^{-6}\,c)}{[0.15 - 0.05\cos(13.92t)]^2\,m^2} = \frac{2.7\times10^4}{[0.15 - 0.05\cos(13.92t)]^2}$$

$$= \frac{1.08\times10^7}{[3.00 - \cos(13.9t)]^2}\ N/c \text{ upwards}$$

22.1. The electric flux due to an uniform field is given by-

$\Phi = \vec{E}\cdot\vec{A}$ $\qquad$ ($\vec{E}$ - electric field, $\vec{A}$ - area due to the circular region)

a) $\Phi = |\vec{E}||\vec{A}|\cos 0 = (580\,N/c)\,\pi\,(0.13\,m)^2\,(1) = 31\,N\,m^2/c$

b) $\Phi = |\vec{E}||\vec{A}|\cos 45^\circ = (580\,N/c)\,\pi\,(0.13\,m)^2\,\frac{1}{\sqrt{2}} = 22\,N\,m^2/c$

c) $\Phi = |\vec{E}||\vec{A}|\cos 90^\circ = (580\,N/c)\,\pi\,(0.13\,m)^2\,(0) = 0$

22.4. a) We see that the flux outward through the hemispherical surface is the same as the flux inward through the circular base of the hemisphere. On that surface all of the flux is perpendicular to the surface. This means that $\vec{E}$ is parallel to $\vec{A}$.

$$\Phi_E = \vec{E} \cdot \vec{A} = |\vec{E}|\,|\vec{A}| \cos 0 = E \pi r^2 (1) = \pi r^2 E$$

b) If $\vec{E}$ is perpendicular to the axis then the field lines would enter the hemispherical surface and leave it as well. $\Phi_E = 0$

It is more so as $\cos 90^\circ = 0$