P114 - SOLUTION SET 3

22.6. From Gauss's law we know that the flux from a closed surface depends on net charge enclosed by it.

$$\Phi_1 = \frac{(Q - 3Q)}{\epsilon_0} = \frac{-2Q}{\epsilon_0} \qquad \Phi_3 = \frac{(2Q - 3Q)}{\epsilon_0} = -\frac{Q}{\epsilon_0}$$

$$\Phi_2 = \frac{(Q + 2Q - 3Q)}{\epsilon_0} = 0 \qquad \Phi_4 = 0$$

$$\Phi_5 = \frac{2Q}{\epsilon_0}$$

22.17. Due to the spherical symmetry of the problem, the electric field can be found by using Gauss's law and finding the charge enclosed in a sphere. The charge enclosed is found by multiplying the constant charge density with the appropriate volume.

$$\int \vec{E} \cdot d\vec{A} = |\vec{E}|\, 4\pi r^2 = \frac{Q_{encl.}}{\epsilon_0} \qquad \therefore |\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q_{encl.}}{r^2}$$

a) There is negative charge enclosed in this region.

$$Q_{encl.} = \rho_{(-)} \frac{4}{3}\pi r^3$$

$$\therefore |\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{\rho_{(-)} \frac{4}{3}\pi r^3}{r^2} = \frac{\rho_{(-)} r}{3\epsilon_0} = \frac{(-5\ C/m^3)\, r}{3(8.85\times10^{-12} C^2/N m^2)}$$

$$= -1.9 \times 10^{11}\ (N/C\,m)\, r$$

b) For this region we have both negative and positive charge enclosed.

$$Q_{encl.} = \rho_{(-)} \frac{4}{3}\pi r_1^3 + \rho_{(+)} \frac{4}{3}\pi (r^3 - r_1^3)$$

(The second term considers the volume between r_1 and r having +ve charge)

$$= \frac{4\pi}{3}(\rho_{(-)} - \rho_{(+)}) r_1^3 + \frac{4\pi}{3}\rho_{(+)} r^3$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q_{encl.}}{r^2}$$

$$= \frac{1}{3\epsilon_0}\left[\frac{r_1^3}{r^2}(\rho_{(-)} - \rho_{(+)}) + \rho_{(+)} \frac{r^3}{r^2} \right]$$

$$= \left[\frac{(0.06)^3 m^3 (-5 - 8)\ C/m^3}{r^2} + (8.0\ C/m^3)\, r \right] \frac{1}{3\times(8.85\times10^{-12})\ C/Nm^2}$$

$$= \frac{-1.1\times10^{8}\ (Nm^2/C)}{r^2} + 3\times10^{11}\ (N/C\,m)\, r$$

c) In this region all the charge is enclosed.

$$Q_{encl.} = \rho_{(-)} \frac{4}{3}\pi r_1^3 + \rho_{(+)} \frac{4}{3}\pi (r_2^3 - r_1^3)$$

$$= \frac{4\pi}{3} r_1^3 (\rho_{(-)} - \rho_{(+)}) + \frac{4\pi}{3} r_2^3 \rho_{(+)}$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q_{encl}}{r^2}$$

$$= \frac{1}{3\epsilon_0} \left[(\rho_{(-)} - \rho_{(+)}) \frac{r_1^3}{r^2} + \rho_{(+)} \frac{r_2^3}{r^2} \right]$$

$$= \frac{1}{3 \times (8.85 \times 10^{-12} c^2/Nm^2)\, r^2} \left[(-5-8)\, c/m^3 (0.06 m)^3 + 8\, c/m^3 (0.12 m)^3 \right]$$

$$= \frac{4.1 \times 10^8 \ N m^2/c}{r^2}$$

22.21. a) consider a spherical gaussian surface at a radius of 3.00 cm. This encloses all of the charge.

$$|\vec{E}| = \frac{Q}{4\pi r^2 \epsilon_0} = (8.988 \times 10^9 N m^2/c^2) \frac{5.5 \times 10^{-6} c}{(3 \times 10^{-2})^2 m^2} = 5.49 \times 10^7 N/c$$

The field points radially outward.

b) A radius of 6 cm is inside the conducting material and so the field must be zero. Electric field inside a conductor is always zero. The $+5.5 \mu c$ charge in the center induces a charge of $-5.5 \mu c$ on the surface of conductor at radius of 4.5 cm and a charge of $5.5 \mu c$ on the outermost surface of conductor.

c) By considering a ~~sperі~~ spherical gaussian surface at a radius of 30 cm, we enclose all the charge in it.

$$|\vec{E}| = \frac{1}{4\pi r^2} \frac{Q}{\epsilon_0}$$

$$= (8.988 \times 10^9 N m^2/c^2) \frac{5.50 \times 10^{-6} c}{(30 \times 10^{-2} m)^2}$$

$$= 5.49 \times 10^5 N/c$$

The electric field points outward.

22.29. a) We use Gauss's law by choosing spherical gaussian surfaces and finding charged enclosed in it.

For region $0 < r < r_1$, there is no charge enclosed.

$|\vec{E}| = 0$

b) For this region the charge is found by multiplying the charge density with the volume of the shell. Let the charge density be ρ.

$\rho \times \frac{4}{3}\pi\left(r_0^3 - r_1^3\right) = Q$ $\quad \therefore \rho = \frac{3Q}{4\pi(r_0^3 - r_1^3)}$

Charge enclosed by a gaussian surface of radius $r_0 < r$.

$Q_{encl.} = \rho V_{encl.} = \rho \frac{4\pi}{3}(r^3 - r_1^3)$

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\frac{4\pi}{3}\rho(r^3 - r_1^3)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}\frac{(r^3 - r_1^3)}{r^2} = \frac{Q(r^3 - r_1^3)}{4\pi\epsilon_0 r^2(r_0^3 - r_1^3)}$$

(using ρ from previous expression)

c) For region $r > r_0$, the enclosed charge is Q in all.

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2}$$

22.34. For this problem, we consider a cylindrical gaussian surface.

$$\int \vec{E}\cdot d\vec{A} = |\vec{E}|\int|d\vec{A}| = |\vec{E}|\, 2\pi r l = \frac{Q}{\epsilon_0}$$

l, R ~~being~~ being the length and radius of cylinder under consideration.

a) charge enclosed by cylinder = Volume of cylinder × Charge density

$= \pi R_0^2 l \times \rho_E$

$$\therefore |\vec{E}| = \frac{Q}{2\pi\epsilon_0 R l} = \frac{\pi R_0^2 l \rho_E}{2\pi\epsilon_0 R l} = \frac{R_0^2 \rho_E}{2\epsilon_0 R}$$

The electric field points outward.

b) Charge enclosed by cylinder now $= \pi R^2 l \times \rho_E$

$$|\vec{E}| = \frac{Q}{2\pi\epsilon_0 R l} = \frac{\pi R^2 l \rho_E}{2\pi\epsilon_0 R l} = \frac{\rho_E R}{2\epsilon_0}$$

The electric field points outward again radially.

22.35. In this problem as well, we use a cylindrical gaussian surface.

a) For $0 < R < R_1$, there is no charge enclosed and electric field is zero.

b) For $R_1 < R < R_2$, a charge $+Q$ is enclosed.

$$|\vec{E}| = \frac{Q}{2\pi\epsilon_0 R l}, \text{ radially outward}$$

c) For $R > R_2$, the net charge enclosed in $Q - Q = 0$. This makes the electric field zero in this region.

d) The force on an electron placed between the cylinder points in the direction opposite to the electric field, so the force is inward radially. The electric force produces the centripetal acceleration for the electron to move in circular orbit.

$$F = eE = \frac{eQ}{2\pi\epsilon_0 R l} = \frac{mV^2}{R} \quad (\text{centripetal } \cancel{\text{acc}} \text{ force})$$

$$V^2 = \frac{eQ}{2\pi m \epsilon_0 l}$$

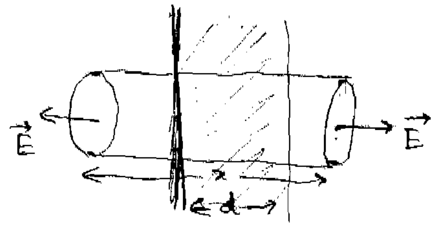
$$\text{Kinetic energy} = \frac{1}{2} mV^2 = \frac{eQ}{4\pi\epsilon_0 l} \quad (\text{independent of } R)$$

22.60. The field due to the plane is $E = \frac{\sigma}{2\epsilon_0}$. Because the slab is very large, and we assume that we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in its field being perpendicular to the slab, with a constant magnitude for a constant distance from its center.

a) To determine the field to the left of the plane, we choose a cylindrical gaussian surface of length $x > d$ and cross section A. Place it so that the plane is centered inside the cylinder. There will be no flux through the curved wall of the cylinder. From the symmetry, the electric field is parallel to surface area vectors on both ends. We know that the field is same on both ends, of a plane and similarly for a slab.

$$\oint \vec{E}\cdot d\vec{A} = \int_{\text{ends}} \vec{E}\cdot d\vec{A} + \int_{\text{side}} \vec{E}\cdot d\vec{A} = \int_{\text{ends}} \vec{E}\cdot d\vec{A} + 0 = \frac{Q_{\text{encl.}}}{\epsilon_0}$$

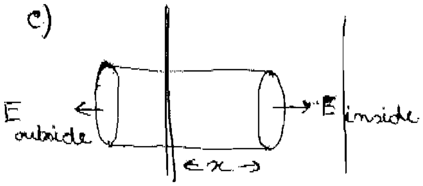
$$2|\vec{E}|A = \frac{\sigma A + \rho_E dA}{\epsilon_0} \qquad \therefore |\vec{E}|_{\text{left}} = \frac{\rho_E d + \sigma}{2\epsilon_0}$$



b) As above, the field is symmetric.

$$|\vec{E}|_{right} = \frac{\sigma + \rho_E d}{2\epsilon_0}$$

c)



We choose a cylindrical gaussian surface of cross section A and a distance x into the slab.
Due to symmetry, the field again is parallel to the surface area vector on both ends. There is no flux along curved walls.

$$\oint \vec{E}\cdot d\vec{A} = \int_{left} \vec{E}\cdot d\vec{A} + \int_{right} \vec{E}\cdot d\vec{A} + \int_{side} \vec{E}\cdot d\vec{A} = E_{outside} A + E_{inside} A + 0 = \frac{Q_{encl}}{\epsilon_0}$$

$$Q_{encl.} = \sigma A + \rho_E x A$$

Using previous result,

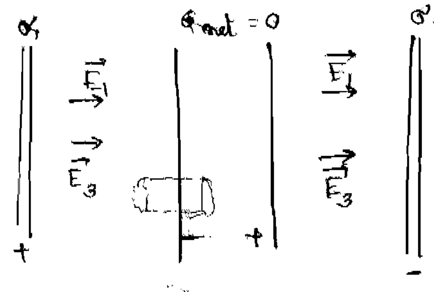
$$\left(\frac{\sigma + \rho_E d}{2\epsilon_0}\right) A + E_{inside} A = \frac{\sigma A + \rho_E x A}{\epsilon_0}$$

$$A\ E_{inside} = \frac{(2\sigma A + 2\rho_E x A - \sigma A - \rho_E d A)}{2\epsilon_0}$$

$$\therefore E_{inside} = \frac{\sigma + \rho_E (2x - d)}{2\epsilon_0}, \quad 0 < x < d$$

22.63. a) In an electrostatic situation, there is no electric field inside a conductor. Thus $E = 0$ in a conductor.

b)



The positive sheet produces an electric field, external to itself, directed away from the plate with a magnitude as $E_1 = \frac{|\sigma_1|}{2\epsilon_0}$. The negative sheet produces an electric field, external to itself, directed towards the plate with a magnitude $E_2 = \frac{|\sigma_2|}{2\epsilon_0}$. Between the left and middle sheet these two fields are parallel and add to each other.

$$E_{left} = E_1 + E_2 = \frac{|\sigma_1| + |\sigma_2|}{2\epsilon_0} = \frac{2(5\times10^{-6}\ c/m^2)}{2(8.85\times10^{-12}\ C^2/Nm^2)} = 5.65\times10^{5}\ N/C \text{ to right}$$

c) The same field exists between the middle and right sheets.

$E_{right} = 5.65 \times 10^5 \text{ N/C}$

d) To find the charge density on the surface of the left of the middle sheet choose a gaussian cylinder of ends with area A. Let one end be inside the conducting sheet where there is no electric field and other end be in the area between left and middle sheet. There is no flux in the right end of cylinder and the curved surfaces.

$$\oint \vec{E}\cdot d\vec{A} = \int_{left} \vec{E}\cdot d\vec{A} + \int_{right} \vec{E}\cdot d\vec{A} + \int_{side} \vec{E}\cdot d\vec{A} = -E_{left}A + 0 + 0 = \frac{Q_{encl}}{\epsilon_0}$$

$$= \sigma_{left}\frac{A}{\epsilon_0}$$

$$\sigma_{left} = -\epsilon_0 E_{left} = -\epsilon_0\left(\frac{|\sigma_1| + |\sigma_2|}{2\epsilon_0}\right)$$

$$= -5 \times 10^{-6} \text{ C/m}^2$$

e) Because the middle plate is a conducting sheet and has no net charge, the charge density on the right side must be opposite to charge density on left.

$$\sigma_{right} = -\sigma_{left} = 5 \times 10^{-6} \text{ C/m}^2$$

23.1. Since energy is conserved, change in potential energy is opposite to that of kinetic energy.

$$\Delta U = q\Delta V = -\Delta K$$

$$\Delta V = -\frac{\Delta K}{q} = \frac{K_{initial} - K_{final}}{q} = \left(\frac{mv^2}{2} - 0\right)\frac{1}{q}$$

$$= \frac{(9.11 \times 10^{-31}\text{ kg})(5 \times 10^5 \text{ m/sec})^2}{2(-1.6 \times 10^{-19}\text{ C})} = -0.71\text{ V}$$

The final potential is lower than the initial to make it stop.

23.8. Assuming that electric field is same, its magnitude is -

$$E = \frac{V}{d} = \frac{110\text{ V}}{4 \times 10^{-3}\text{ m}} = 2.8 \times 10^4 \text{ V/m}$$