PHY 114 SOLUTION SET 4

23.11. The electric field is uniform and goes from higher to lower potential. We also know that $V = -\int_a^b \vec{E}\cdot d\vec{l}$

a) $V_{BA} = 0$ (The electric field is perpendicular to the position vector between A and B)

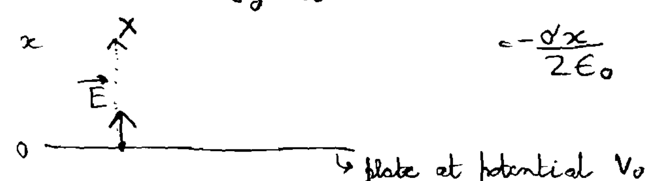
b) $V_{CB} = V_C - V_B = (-4.20\ N/C)\,(4 - (-3)) = -29.4\,V$ (considering the difference in x coordinate of C, B as $\vec{E}$ lies along X axis)

c) $V_{CA} = V_C - V_A = (V_C - V_B) + (V_B - V_A) = -29.4 + 0 = -29.4\,V$

23.12. The electric field produced due to a uniform large plate is $|\vec{E}| = \frac{\sigma}{2\epsilon_0}$

The electric field points away from the surface if the charge on it is positive.

$$V(x) - V(0) = V(x) - V_0 = -\int_0^x \vec{E}\cdot d\vec{l} = -\int_0^x \frac{\sigma}{2\epsilon_0}\,\hat{i}\cdot dx\,\hat{i} = -\frac{\sigma x}{2\epsilon_0}\Big|_0^x = -\frac{\sigma x}{2\epsilon_0}$$



$$\therefore V(x) = V_0 - \frac{\sigma x}{2\epsilon_0}$$

23.14. a) The potential due to a charged ~~surface~~ sphere of radius r_0 on its surface having a charge Q is given by - $V_0 = \frac{Q}{4\pi r_0 \epsilon_0}$

$$\therefore Q = 4\pi\epsilon_0 r_0 V_0$$

$$\text{charge density } (\sigma) = \frac{\text{charge}}{\text{Surface area of sphere}} = \frac{Q}{4\pi r_0^2} = \frac{4\pi\epsilon_0 r_0 V_0}{4\pi r_0^2} = \frac{V_0\epsilon_0}{r_0}$$

$$= \frac{(680\,V)(8.85\times10^{-12}\,C^2/Nm^2)}{(0.16\,m)} = 3.761\times10^{-8}\,C/m^2$$

b) The potential due to charge Q at a distance r from its centre -

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{4\pi\epsilon_0 r_0 V_0}{4\pi\epsilon_0 r} = \frac{r_0 V_0}{r} \qquad \therefore r = \frac{r_0 V_0}{V} = \frac{(0.16\,m)(680\,V)}{(25\,m)}$$

$$= 4.352\,m$$

23.19. a) If we want to find the electric field due to a sphere having charge Q, outside the sphere, we can consider it as a point charge and the electric field would be given as $|\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2}$, $r > r_0$

$$V = -\int_\infty^r \vec{E}\cdot d\vec{r} = -\int_\infty^r \frac{Q}{4\pi\epsilon_0 r^2}\,dr = \frac{Q}{4\pi\epsilon_0 r}\Big|_\infty^r = \frac{Q}{4\pi\epsilon_0 r}$$

b) We now need to find electric field inside the sphere and hence the charge enclosed by a gaussian sphere of radius r. $(r < r_0)$

Charge enclosed $= \dfrac{Q}{\frac{4}{3}\pi r_0^3} \times \dfrac{4}{3}\pi r^3 = \dfrac{Q r^3}{r_0^3}$

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q r^3}{\epsilon_0 r_0^3}$$

$$|\vec{E}| \times 4\pi r^2 = \frac{Q r^3}{\epsilon_0 r_0^3} \qquad \therefore \; |\vec{E}| = \frac{Q r}{4\pi \epsilon_0 r_0^3} \qquad (r < r_0)$$

$$V(r) = V(r_0) - \int_{r_0}^{r} \vec{E} \cdot d\vec{l} \qquad \left(\because V(r_0) = -\int_{\infty}^{r_0} \vec{E} \cdot d\vec{l}\right)$$

$$= \frac{Q}{4\pi \epsilon_0 r_0} - \int_{r_0}^{r} \frac{Q r}{4\pi \epsilon_0 r_0^3} \, dr$$

$$= \frac{Q}{4\pi \epsilon_0 r_0} - \frac{Q}{4\pi \epsilon_0 r_0^3} \left.\frac{r^2}{2}\right|_{r_0}^{r} = \frac{Q}{4\pi \epsilon_0 r_0} - \frac{Q}{8\pi \epsilon_0}\left(\frac{r^2}{r_0^3} - \frac{r_0^2}{r_0^3}\right)$$

$$= \frac{Q}{4\pi \epsilon_0 r_0}\left(1 + \frac{1}{2}\right) - \frac{Q}{8\pi \epsilon_0} \frac{r^2}{r_0^3}$$

$$= \frac{3 Q r_0^2}{8\pi \epsilon_0 r_0^3} - \frac{Q r^2}{8\pi \epsilon_0 r_0^3} = \frac{Q}{8\pi \epsilon_0 r_0^3}\left(3 - \frac{r^2}{r_0^2}\right)$$

c)

$$V(r) = \frac{Q}{8\pi \epsilon_0 r_0^3}\left(3 - \frac{r^2}{r_0^2}\right), \quad r < r_0$$

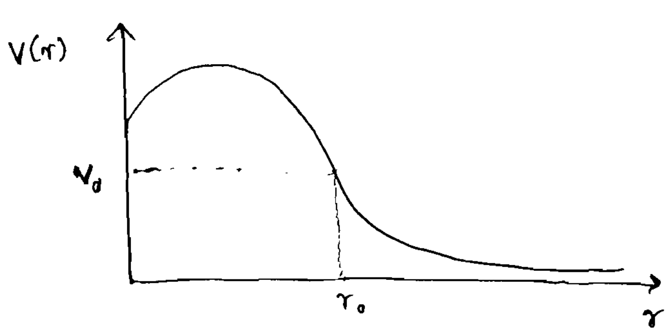
$$= \frac{Q}{4\pi \epsilon_0 r}, \quad r > r_0$$

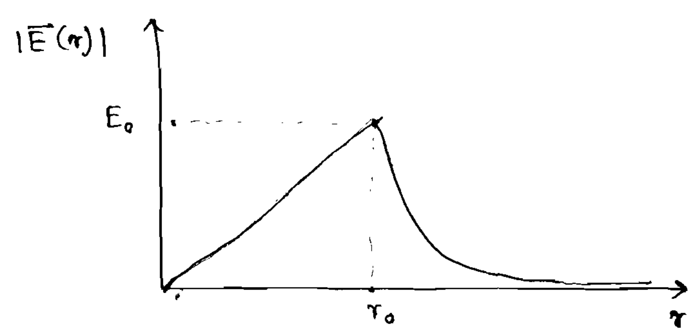
$$= \frac{Q}{4\pi \epsilon_0 r_0}, \quad r = r_0$$

$$|\vec{E}(r)| = \frac{Q r}{4\pi \epsilon_0 r_0^3}, \quad r < r_0$$

$$= \frac{Q}{4\pi \epsilon_0 r^2}, \quad r > r_0$$

$$= \frac{Q}{4\pi \epsilon_0 r_0}, \quad r = r_0$$





23.22. a) To find the electric field we consider the net charge enclosed in each region and then divide by $4\pi r^2$ (surface area of the gaussian sphere) to get the value of electric field.

For $r > r_2$, charge enclosed $= Q + \frac{Q}{2} = \frac{3Q}{2}$.

$$|\vec{E}| = \frac{3Q}{8\pi\epsilon_0 r^2} \quad , \quad r > r_2$$

For $r_1 < r < r_2$, its a conductor and electric field inside it is zero.

For $r < r_1$, charge enclosed is $\frac{Q}{2}$.

$$|\vec{E}| = \frac{Q}{8\pi\epsilon_0 r^2} \quad , \quad r < r_1 .$$

b) For $r > r_2$, we can consider the net charge as a point source and find potential accordingly.

$$V(r) = \frac{3Q}{8\pi\epsilon_0 r} = \frac{\frac{3Q}{2}}{4\pi\epsilon_0 r} \quad , \quad r > r_2 .$$

c) Inside a conductor $(r_2 > r > r_1)$ the potential remains constant along with zero electric field. Thus the potential on the surface is the one inside the conductor as well.

$$V(r) = \frac{3Q}{8\pi\epsilon_0 r_2} \quad , \quad r_1 < r < r_2 .$$

d) For this region, we use -

$$V(r) = -\int_\infty^r \vec{E}\cdot d\vec{r} = -\left[\int_\infty^{r_2} \vec{E}\cdot dr + \int_{r_2}^{r_1} \vec{E}\cdot d\vec{r} + \int_{r_1}^{r} \vec{E}\cdot d\vec{r}\right]$$

$$= -\left[\int_\infty^{r_2} \frac{3Q}{8\pi\epsilon_0 r^2}\, dr + \int_{r_2}^{r_1} 0 \cdot d\vec{r} + \int_{r_1}^{r} \frac{Q}{8\pi\epsilon_0 r^2}\, dr\right]$$

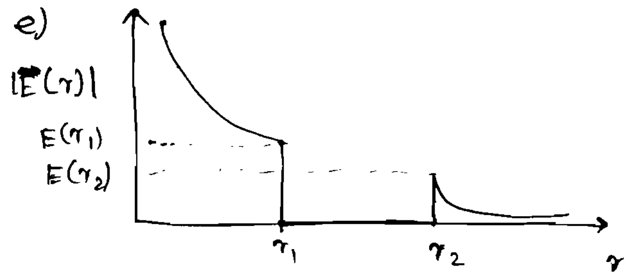
$$= -\left[-\frac{3Q}{8\pi\epsilon_0 r}\Big|_\infty^{r_2} + 0 - \frac{Q}{8\pi\epsilon_0 r}\Big|_{r_1}^{r}\right]$$

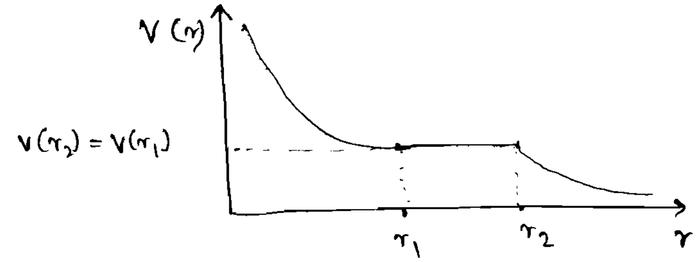
$$= \frac{3Q}{8\pi\epsilon_0 r_2} + \frac{Q}{8\pi\epsilon_0 r} - \frac{Q}{8\pi\epsilon_0 r_1}$$

$$= \frac{Q}{8\pi\epsilon_0}\left(\frac{3}{r_2} - \frac{1}{r_1} + \frac{1}{r}\right) \qquad \because r_2 = 2r_1$$

$$= \frac{Q}{8\pi\epsilon_0}\left(\frac{3}{2r_1} - \frac{1}{r_1} + \frac{1}{r}\right) = \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{2r_1} + \frac{1}{r}\right) \quad , \quad r < r_1$$

e)





23.25.a) The potential due to a point charge at a distance r away is -

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$\therefore Q = \cancel{4\pi\epsilon_0 r V} = \frac{1}{\cancel{8.99 \times 10^9 \, N\, m^2/C^2}} \times$$

$$= (8.99 \times 10^9 \, N\, m^2/C^2) \times \frac{1.6 \times 10^{-19} \, C}{0.50 \times 10^{-10} \, m} = 28.77 \, V$$

b) The potential energy of the electron is the charge of the electron times the potential due to the proton.

$$U = QV = -(1.6 \times 10^{-19} \, C)(28.77 \, V) = -4.6 \times 10^{-18} \, J$$

it is an electron now