Solutions to Problem Set 6

24.55. The removal of the dielectric results in change in the energy of the capacitor and the change in charge.

$$\Delta U = U_f - U_i = \frac{1}{2} C_0 V^2 - \frac{1}{2} K C_0 V^2 = -\frac{1}{2} (K-1) C_0 V^2$$

The work done to maintain a constant voltage is the voltage multiplied by the change in charge ~~due to battery~~.

$$W_{bat} = V(Q_f - Q_i) = V(C_0 V - K C_0 V) = -(K-1) C_0 V^2$$

The work done to remove dielectric is the difference between the change in capacitor's energy and work done by the battery.

$$W = \Delta U - W_{bat} = -\frac{1}{2} (K-1) C_0 V^2 + (K-1) C_0 V^2$$

$$= \frac{1}{2} (K-1) C_0 V^2 = \frac{1}{2} (3.4-1) (8.8 \times 10^{-9} F)(100 V)^2 = 1.1 \times 10^{-4} J$$

24.58. On inserting the mica, the capacitance of the system changes. The voltage is not changed as the same battery is used.

$$\Delta Q = Q_{final} - Q_{initial} = (C_{final} - C_{initial}) V = (K C_{in} - C_{in}) V$$

$$= (K-1) C_{in} V = (7-1)(3.5 \times 10^{-9} F)(32 V) = 6.7 \times 10^{-7} C$$

24.59. The capacitors are in parallel as the potential is same for both. Thus we find net capacitance keeping in mind that the area is changed by a factor of half from the original ~~one~~ ~~being~~ area.

$$C = C_1 + C_2 = K_1 \epsilon_0 \frac{\frac{1}{2}A}{d} + K_2 \epsilon_0 \frac{\frac{1}{2}A}{d}$$

$$= \frac{1}{2} \frac{\epsilon_0 A}{d} (K_1 + K_2)$$

25.1. $I = \frac{\Delta Q}{\Delta t}$

charge of an electron is 1.6×10^{-19} C.

Charge flowing in 1 sec $= 1.3 \times 1 = 1.3$ C.

No. of electrons carrying this charge $= \frac{1.3}{1.6 \times 10^{-19}} = 8.13 \times 10^{18}$ ~~electrons~~

25.6. a) Using Ohm's law - $R = \frac{V}{I} = \frac{240 V}{9.5 A} = 12.63\ \Omega$

b) $\Delta Q = I\,\Delta t = (9.5A) \times 15$ mins $= 9.5A \times 15 \times 60$ secs. $= 8600$ C

25.17. Resistance is proportional to the length of a wire. If the resistance one part is four times that of other their length of the long part should be four times that of short part.

$l = l_{short} + l_{long} = l_{short} + 4\,l_{short} = 5\,l_{short}$

$\therefore\ l_{short} = \frac{1}{5} l = 0.2\,l, \quad l_{long} = l - l_{short} = 0.8\,l$

If $10\,\Omega$ is the resistance in the whole wire, resistance on short half is

$= 0.2 \times 10 = 2\,\Omega.$

Resistance in other part $= (10-2) = 8\,\Omega$ (as they are in series)

25.20. Voltage drop using Ohm's law-

$V = IR.$ $\qquad A = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

where $R = \frac{\rho l}{A}$

$\therefore V = \frac{I \rho l}{A} = \frac{I \rho l}{\frac{\pi d^2}{4}} = \frac{4 I \rho l}{\pi d^2} = \frac{4(12)(1.68 \times 10^{-8})(26)}{\pi (1.628 \times 10^{-3})^2} = 2.5V$

(using resistivity of copper here)

25.25. If the wire is cut the new wire has length half of original. The volume of the wire is constant.

$V_1 = \pi r_1^2 l_1 \qquad V_2 = \pi r_2^2 l_2 \qquad l_2 = \frac{l_1}{2}$

$V_1 = V_2$

$\pi r_1^2 l_1 = \pi r_2^2 l_2 = \pi r_2^2 \frac{l_1}{2}$

$\therefore r_2^2 = 2 r_1^2 \qquad \therefore A_2 = 2 A_1$

$R_1 = \frac{\rho l_1}{A_1} \qquad R_2 = \frac{\rho l_2}{A_2} = \rho\left(\frac{l_1}{2}\right) \times \frac{1}{2A_1} = \frac{\rho l_1}{4 A_1} = \frac{R_1}{4}$

25.38. We know that $P = \frac{V^2}{R}$. Since the resistance of the bulb remains the same. $(R_1 = R_2)$

$\frac{P_1}{P_2} = \frac{V_1^2}{R_1} \times \frac{R_2}{V_2^2} = \frac{V_1^2}{V_2^2} = \left(\frac{120}{240}\right)^2 = \frac{1}{4}$

$P_{USA} = \frac{1}{4} P_{europe}$

25.40. To find out amount of cost we need to find the work done times cost. $W = P t$

$$\text{cost} = (25\,W)(365\,\text{day})\left(\frac{24\,hr}{1\,\text{day}}\right)\left(\frac{\$\,0.095}{K\,W\,hr}\right) \simeq \$\,21$$

25.60. The electric is related to potential as -

$$|\vec{E}| = \frac{\Delta V}{\Delta x} = \frac{70 \times 10^{-3}\,V}{10^{-8}\,m} = 7 \times 10^{6}\,V/m$$