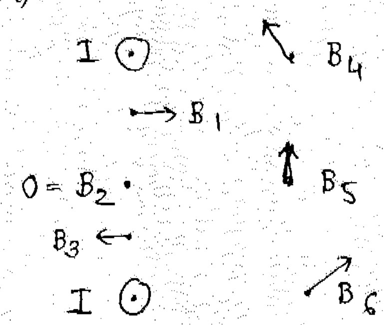
P114 SOLUTION SET 9

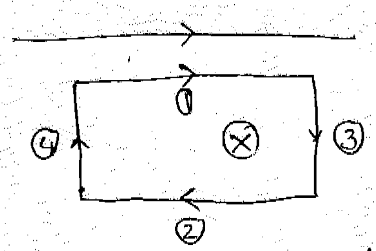
2.8.3. Since the current on the wires is along the same direction, the force between them is attractive.

$$F_2 = \frac{\mu_0}{2\pi}\,\frac{I_1 I_2}{d}\,\ell_2 = \frac{(4\pi\times10^{-7}\ \mathrm{T/m\,A})}{2\pi}\,\frac{(35\,\mathrm{A})^2}{(0.04\,\mathrm{m})}\,(25\,\mathrm{m}) = 0.15\ \mathrm{N}$$

28.13. We use the right hand thumb rule to find direction of magnetic. We also know that its strength decreases inversely as the square of the distance. Hence at 2, the fields add up to zero.



2.8.18.



The magnetic field created due to the wire points into the paper. We divide the loop into four parts and find the forces on each.

Wire 3, 4 are at equal distance from the wire carrying current. However the force due to magnetic field points to the left on wire 4 and points to the right for wire 3. The forces are same in magnitude. Hence net force on wire 3 and 4 is total.

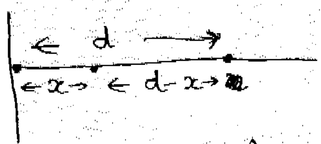
Force on wire 1 and wire 2 are opposite in direction. (upward for 1 and downward for 2). Hence net force on the loop is –

$$F_{net} = F_1 - F_2 = \frac{\mu_0}{2\pi}\,\frac{I_1 I_2}{d_1}\,\ell_1 - \frac{\mu_0}{2\pi}\,\frac{I_1 I_2}{d_2}\,\ell_2 = \frac{\mu_0}{2\pi}\,I_1 I_2\, l\left(\frac{1}{d_1} - \frac{1}{d_2}\right)$$

$$= \frac{(4\pi\times10^{-7}\ \mathrm{T\,m/A})}{2\pi}\,(3.5\,\mathrm{A})^2\,(0.1\,\mathrm{m})\left(\frac{1}{0.03\,\mathrm{m}} - \frac{1}{0.08\,\mathrm{m}}\right)$$

$$= 5.1\times10^{-6}\ \mathrm{N}$$ upwards towards the wire

28.19. The magnetic field due to wire on left will be along +ve Y axis while field due to right one will be along -ve Y axis.



$$\vec{B}_{net} = \frac{\mu_0 I}{2\pi x}\hat{j} - \frac{\mu_0 I}{2\pi(d-x)}\hat{j} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right)\hat{j} = \frac{\mu_0 I}{2\pi}\frac{(d-2x)}{x(d-x)}\hat{j}$$

28.26. The magnetic field inside a solenoid is given by-

$$B = \frac{\mu_0 N I}{l} \qquad N = \frac{Bl}{\mu_0 I} = \frac{(0.3T)(0.32\,m)}{(4\pi\times10^{-7}\,Tm/A)(4.5A)} = 1.7\times10^{4} \text{ turns}$$

28.35. The current in the straight part of the wires flows out radially, there is no magnetic field produced by them in the center. The upper wire creates a magnetic field ~~along~~ ~~outward~~ inward while lower produces a field outward.

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{upper}\frac{ds}{R^2}\hat{k} + \frac{\mu_0 I_2}{4\pi}\int_{lower}\frac{ds}{R^2}(-\hat{k}) = \frac{\mu_0 \pi R}{4\pi R^2}(I_1 - I_2)\hat{k}$$

$$= \frac{\mu_0}{4R}(0.35 - 0.65)I = -\frac{0.3\mu_0 I}{4R} = -\frac{3\mu_0 I}{40R}$$

28.32. The current flowing through the two cylinders is same that is I_0. The current densities can be used to find c_1, c_2 since I_0 is same.

$$I_0 = \int_0^{R_1} c_1 R \underbrace{2\pi R}_{\text{perimeter of a circle}} \underbrace{dR}_{\text{small length element along radius}} = 2\pi c_1 \int_0^{R_1} R^2 dR = 2\pi c_1 \frac{R_1^3}{3} \qquad \therefore c_1 = \frac{3I_0}{2\pi R_1^3}$$

$$-I_0 = \int_{R_2}^{R_3} 2\pi R\,dR\,C_2 R = +2\pi C_2 \int_{R_2}^{R_3} R^2 dR = \frac{2\pi C_2}{3}(R_3^2 - R_2^2)$$

$$\therefore C_2 = \frac{3I_0}{2\pi(R_2^2 - R_3^2)}$$

a) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl.} = \mu_0 \int_0^R (c_1 R')\,2\pi R'\,dR' = \mu_0 2\pi c_1 \int_0^R R'^2 dR'$

$$|\vec{B}|\cdot 2\pi R = 2\pi \mu_0 c_1 \frac{R^3}{3}$$

$$\therefore |\vec{B}| = \frac{\mu_0 c_1 R^2}{3} = \frac{\mu_0 R^2}{3}\cdot\frac{3I_0}{2\pi R_1^3} = \frac{\mu_0 I_0}{2\pi}\frac{R^2}{R_1^3}$$

b) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl.} = \mu_0 I_0$

$$|\vec{B}|\cdot 2\pi R = \mu_0 I_0 \qquad \therefore |\vec{B}| = \frac{\mu_0 I_0}{2\pi R}$$

c) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl.}$

$$= \mu_0 \left[I_0 + \int_{R_2}^{R} (C_2 R') 2\pi R' dR' \right]$$

$$= \mu_0 I_0 - 2\pi C_2 \mu_0 \int_{R_2}^{R} R'^2 dR'$$

(-ve sign comes due to opposite direction of current)

$$= \mu_0 I_0 - \frac{2\pi}{3} C_2 \mu_0 (R^3 - R_2^3)$$

$$= \mu_0 I_0 + \frac{2\pi}{3} \frac{\mu_0 3 I_0}{2\pi (R_2^3 - R_3^3)} (R^3 - R_2^3)$$

$$= \mu_0 I_0 + \mu_0 I_0 \frac{(R^3 - R_2^3)}{(R_2^3 - R_3^3)}$$

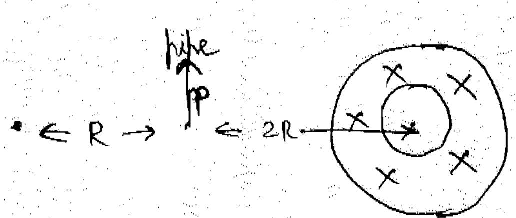
$$= \mu_0 I_0 \left(1 + \frac{R^3 - R_2^3}{R_2^3 - R_3^3} \right)$$

$$= \mu_0 I_0 \left(\frac{R_2^3 - R_3^3 + R^3 - R_2^3}{R_2^3 - R_3^3} \right)$$

$$= \mu_0 I_0 \frac{(R^3 - R_3^3)}{(R_2^3 - R_3^3)} \qquad |\vec{B}| = \frac{\mu_0 I_0}{2\pi R} \frac{(R^3 - R_3^3)}{(R_2^3 - R_3^3)}$$

d) Outside the wire the net current enclosed is zero. $\therefore |\vec{B}| = 0$.
For all the above calculations the magnetic field is radially outward.

Prob 8



a) We find the magnetic field due to the pipe at P using

$\int \vec{B}\cdot d\vec{s} = \mu_0 I_{enclosed}$

$|\vec{B}| \, 2\pi r = \mu_0 I$

$\therefore |\vec{B}| = \frac{\mu_0 I}{2\pi r}$ Hence $r = 2R$

$\therefore \vec{B} = \frac{\mu_0 I}{4\pi R} \hat{r}$ The magnetic field is ~~radially outward~~ pointing upward.

Hence the current in the wire should also be into the paper to produce a magnetic field in downward direction at P.

Let the current be I'

Magnetic field due to wire $|\vec{B}| = \frac{\mu_0 I'}{2\pi R}$

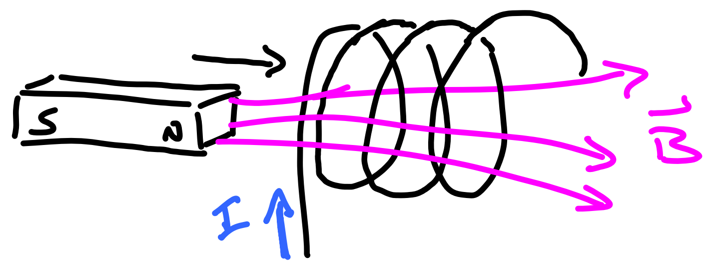
$$\frac{\mu_0 I'}{2\pi R} = \frac{\mu_0 I}{4\pi R} \qquad \therefore I' = \frac{I}{2}$$

b) The magnetic field in the center of the pipe is zero as the inside of the pipe does not enclose any current. So the magnetic field inside the ~~wi~~ pipe will be only due to the wire which is 3R away.

B = mu_0*I/2*pi*3R in the up direction.

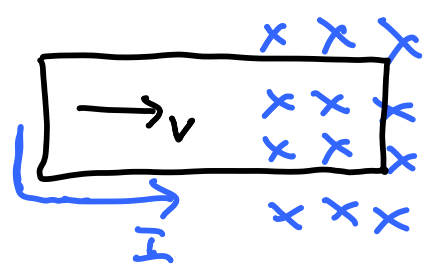
Note Title 3/30/2010

29-2



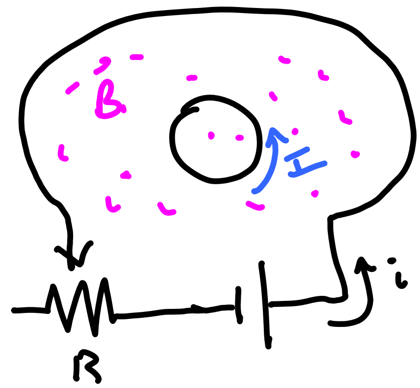
magnet being inserted
B field strength increasing
I induced in direction to decrease the increasing field ... goes in direction shown to produce $\vec{B}$ thru loop from Right to left.

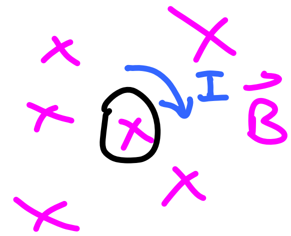
29-3



loop being pushed into field means B flux into board increasing.
Induce current counter-clockwise to "fight" this increase.

29-8



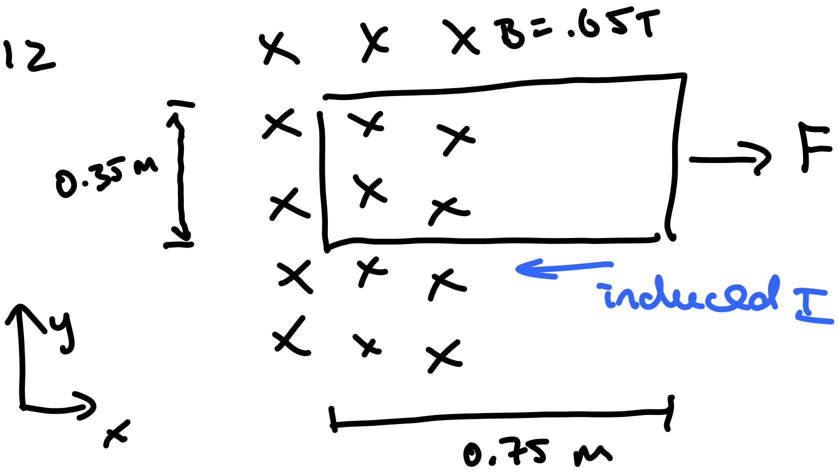


R slowly increased, v unchanged → i slowly decreasing

B coming out but flux decreasing
I in smaller loop counter-clockwise to fight the decreasing field.

Outside big loop $\vec{B}$ going in but decreasing so induced I is clockwise to fight the decrease.

29-12



R = 0.28 Ω

v = 3.4 m/s

$$|\mathcal{E}| = \frac{d\Phi_m}{dt} = \frac{d}{dt} B(xy) = By\frac{dx}{dt} = (.65)(.35)\,3.4$$

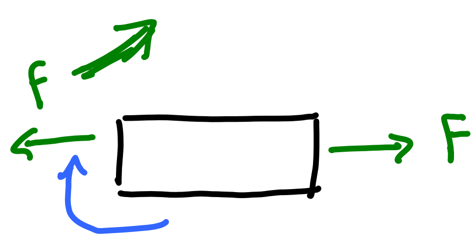
$$|\mathcal{E}| = 0.77 \text{ volts}$$

$|\mathcal{E}| = IR$ $\qquad .77 = I \; .28$

$$I = 2.75 \text{ Amps}$$

Force (to pull loop out at constant v) = $I\overset{.35}{L}B$

$$F = (2.75)(.35)(0.65) = 0.63 \text{ N}$$



29.18. a) The emf is given by -

$$E = -N\frac{d\phi}{dt} = -75\,\frac{d}{dt}\left(8.8t - 0.51t^3\right)\times10^{-2}\ \mathrm{T\,m^2}$$

$$= (-6.6 + 1.1457\,t^2)\ \mathrm{V}$$

b) $E(t=1) = -6.6 + 1.1457\times(1)^2 \simeq -5.5\,\mathrm{V}$

$E(t=4) = -6.6 + 1.1457\times4^2 \simeq 12\,\mathrm{V}$

29.19. The energy given out is the power in the circuit times the amount of time it dissipates energy.

$\mathcal{E} = \text{emf} = -\frac{\Delta\phi}{\Delta t}$ $\qquad P = \frac{\mathcal{E}^2}{R}$

$$E = P\Delta t$$

$$= \frac{\mathcal{E}^2}{R}\Delta t = \left(\frac{\Delta\phi}{\Delta t}\right)^2\frac{\Delta t}{R}$$

$$= \frac{(\Delta\phi)^2}{R\,\Delta t}$$

$\Delta\phi = \text{Area}\times\Delta B = \pi\,(0.125\,\mathrm{m})^2\,(0.4\,\mathrm{T})$

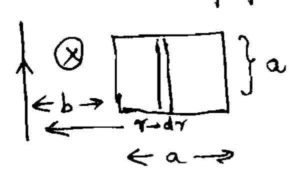
$$= \frac{\pi^2\times(0.125\,\mathrm{m})^4\,(0.4\,\mathrm{T})^2}{(150\,\Omega)\,(0.12\,\mathrm{s})} = 2.1\times10^{-5}\ \mathrm{J}$$

29.22. The magnetic field inside a solenoid is given by $B = \mu_0 n I$.
The induced emf is given by $E = -\frac{d\Phi}{dt} = -A_1\frac{dB_{solenoid}}{dt}$

$$= -A_1\mu_0 n\frac{dI}{dt} = -A_0\mu_0 n\frac{d}{dt}(I_0\cos\omega t)$$

$$= A_0\mu_0 n\,\omega I_0\sin\omega t$$

29.25. a) The magnetic field produced by the wire is into the paper and given by-



$$\vec{B} = \frac{\mu_0}{4\pi}\times\frac{2I}{r}$$

Flux produced $= \int\vec{B}\cdot d\vec{A}$

$$= \int\frac{\mu_0}{2\pi}\frac{I}{r}\,a\,dr = \frac{\mu_0 I a}{2\pi}\int_b^{a+b}\frac{dr}{r}$$

$$= \frac{\mu_0 I a}{2\pi}\ln\left(\frac{b+a}{b}\right)$$

b) The speed of the loop is given by $V = \frac{db}{dt}$, the rate at which dist. between wire and loop changes.

$$E = -\frac{d\phi}{dt} = -\frac{\mu_0 I a}{2\pi}\frac{d}{dt}\left[\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 I a}{2\pi}\,\frac{1}{\left(1+\frac{a}{b}\right)}\times\left(-\frac{a}{b^2}\right)\times\frac{db}{dt}$$

$$= \frac{\mu_0 I a}{2\pi} \times \frac{a}{\frac{(b+a)}{b}} \times \frac{1}{b^2} \times V$$

$$= \frac{\mu_0 I a^2 V}{2\pi b (b+a)}$$

c) The produced magnetic field is into the paper and decreasing. The induced current will be clockwise to create a downward magnetic field.

d) Power dissipated is given by -

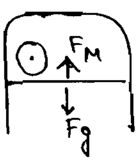
$$P = \frac{E^2}{R}$$

$$F = \frac{P}{V} = \frac{E^2}{RV} = \frac{\mu_0^2 I^2 a^4 V^2}{4\pi^2 b^2 (a+b)^2 V R} = \frac{\mu_0^2 I^2 a^4 V}{4\pi b^2 (a+b)^2 R}$$

29.28. $E = \ell \vec{B} \times \vec{V} = \ell B V \sin 90° = B \ell V$

$$V = \frac{E}{B\ell} = \frac{0.12\,V}{(0.9\,T)(0.132\,m)} = 1\ m/sec$$

29.31.



The rod attains the terminal velocity when the magnetic and gravitational force are equal to each other.

$$F_M = \frac{B^2 \ell^2 V}{R} = mg$$

$$V = \frac{mgR}{B^2 \ell^2} = \frac{(3.6 \times 10^{-3}\,Kg)(9.8\,m/s^2)(0.0013\,\Omega)}{(0.06\,T)^2\,(0.18\,m)^2} = 0.39\,m/s$$