Physics 123 - January 23, 2013

Last Time

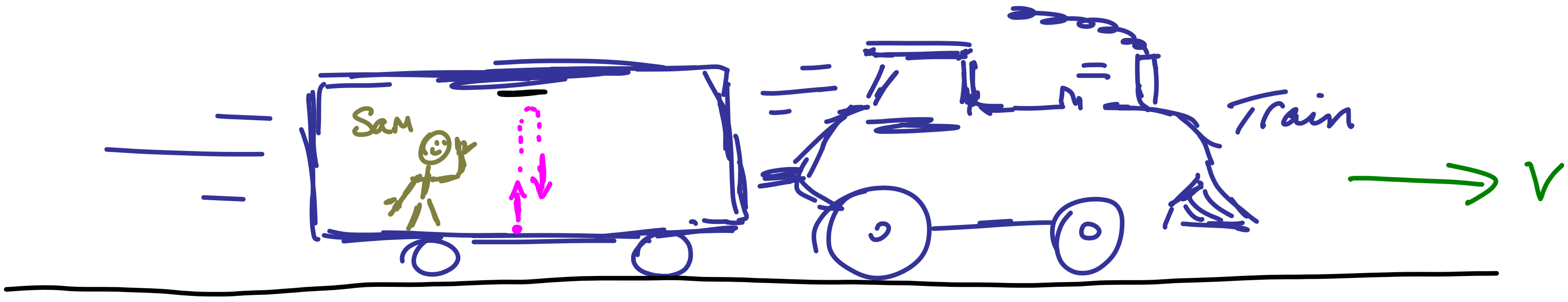
Relativity — 2 postulates

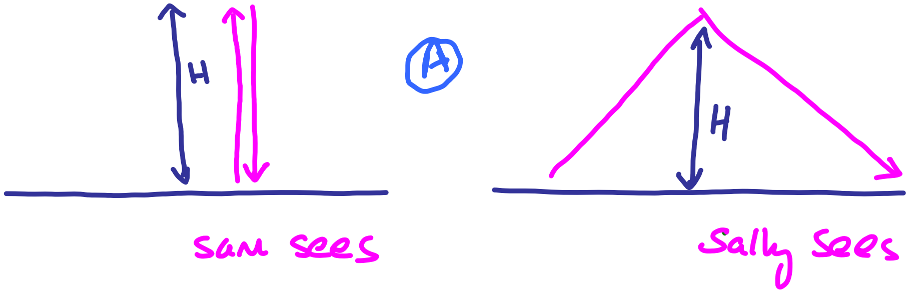
1) The speed of light is the same (c) in all inertial reference frames

Move at constant speed

constraint constitutes the special in special relativity

2) physics is the same in all reference systems





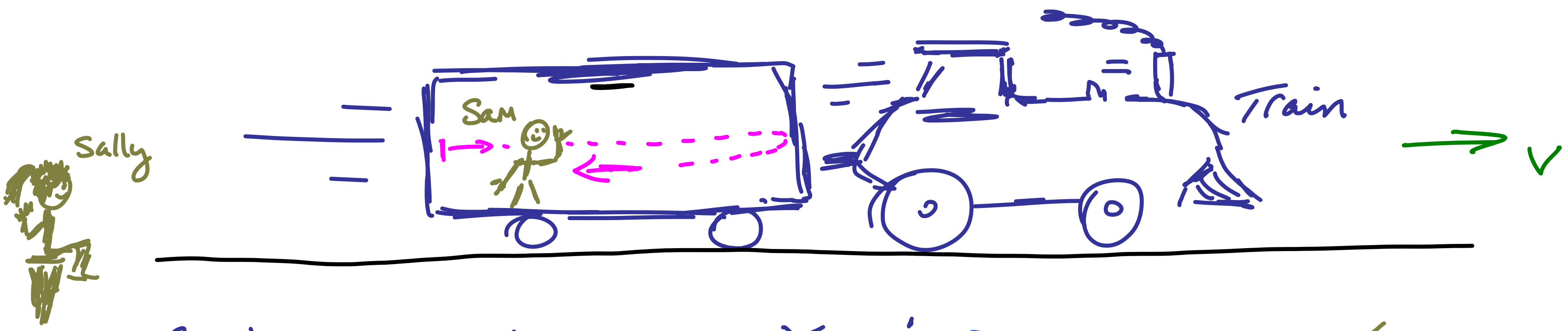
Time dilation

Time is shortest in Proper Frame

Don't get hung up on the prime

$$\Delta t = \gamma \underbrace{\Delta t'}_{\text{proper frame}}$$ Ⓑ

$$T_{Sally} = \underbrace{\boxed{\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}}_{\equiv \gamma\ ,\ >1} T_{Sam}$$ Ⓒ



Similarly, can show $\underbrace{\Delta x}_{\text{proper Frame}} = \gamma \Delta x'$ Ⓐ

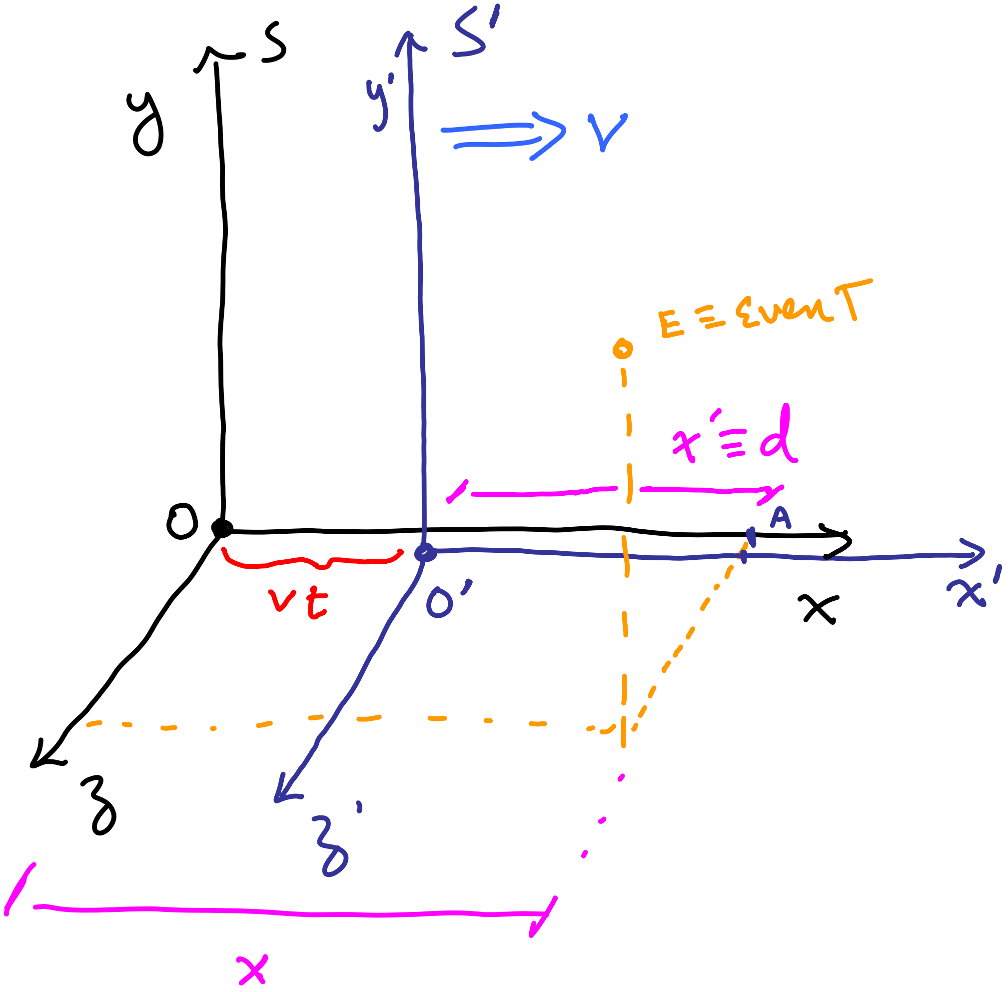
$\Delta x_{Sam} = \gamma \Delta x_{Sally}$

Length contraction

along direction of relative motion

Measured length is longest in the proper frame

Nature of space + time more complex than human bias/experience would lead us to believe.



at $t=0, t'=0 \rightarrow O=O'$

two systems overlap

classical physics

(Galilean Transformations)

$$d = x' = x - vt$$

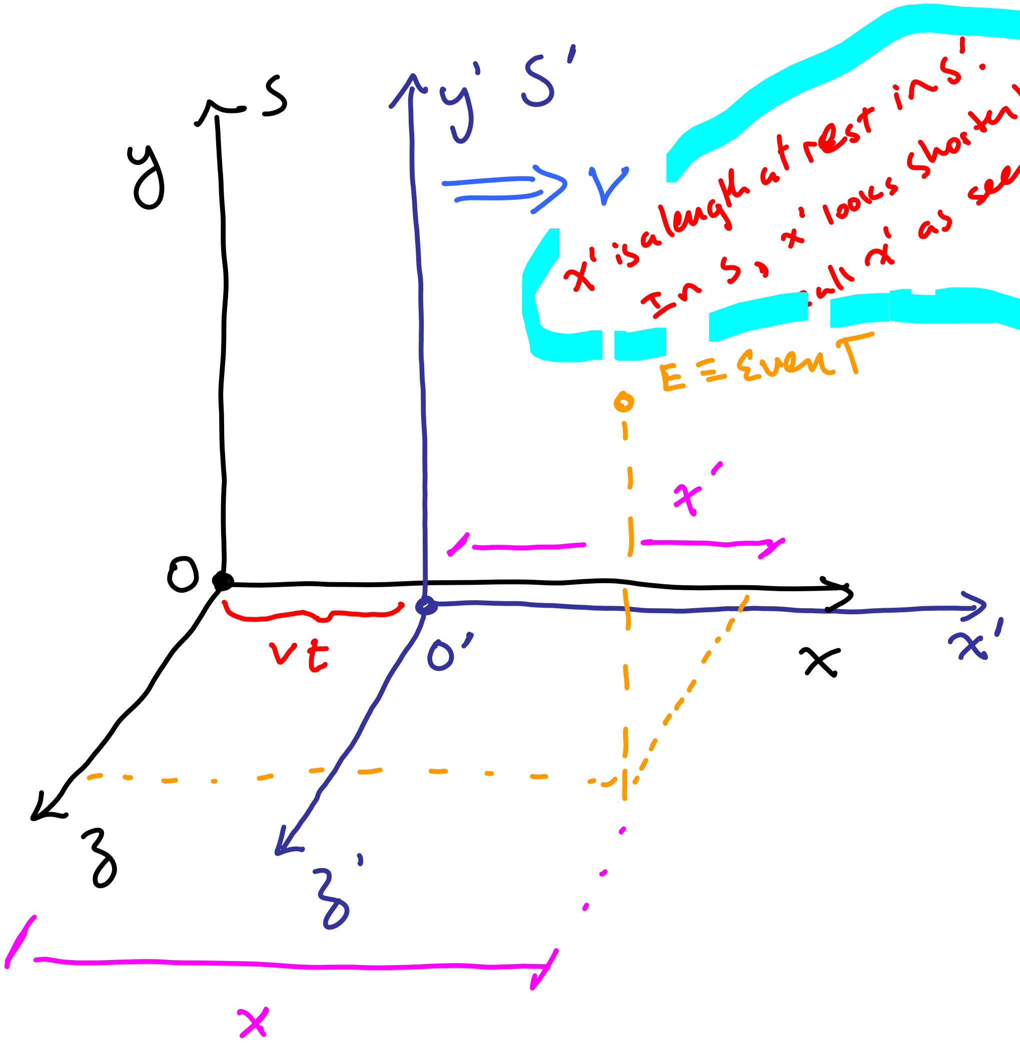
$$y' = y$$

$$z' = z$$

$$t' = t$$

Relativistic Transformations

Transform coords from S To S'



x' is a length at rest in S'.
In S, x' looks shorter by 1/γ
all x' as seen in S → d
$d = x'/\gamma$

Ⓐ $d = \dfrac{x'}{\gamma}$

$x' = d\gamma$

$d = x - vt$

$\dfrac{x'}{\gamma} = x - vt$

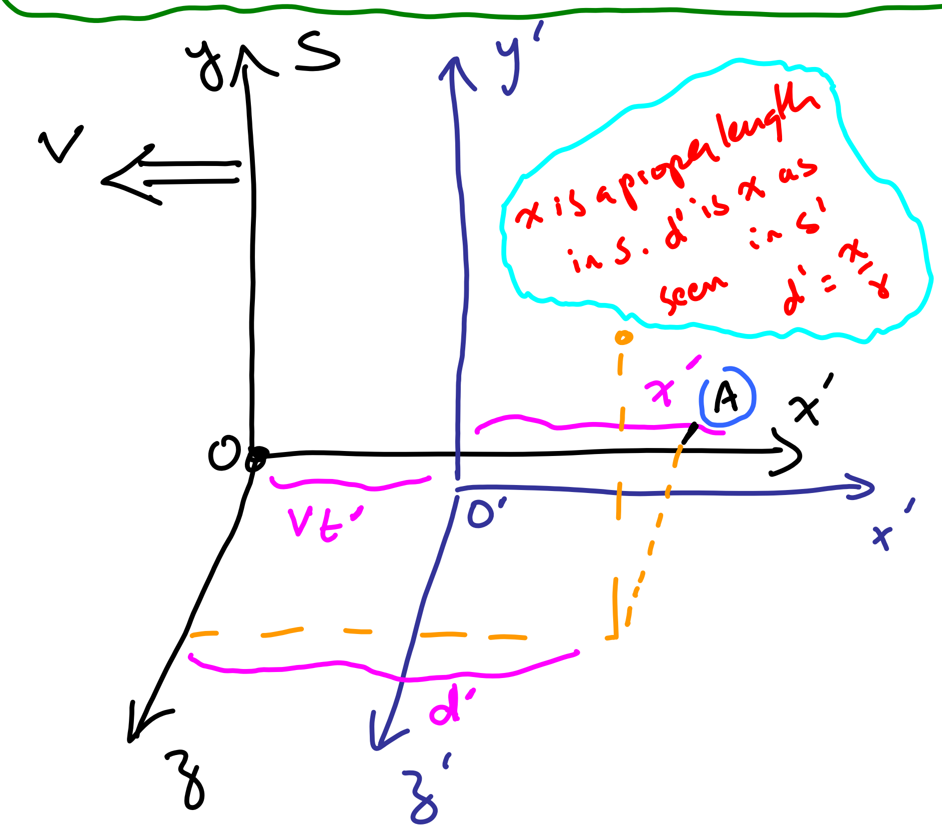
Ⓑ $x' = \gamma(x - vt)$

$y' = y$

$z' = z$

we'll get to t soon

let's transform coordinates the other direction from s' to s



x is distance from O to A in S

$$d' = \frac{x}{\gamma}$$

$$x' = d' - vt'$$

$$x' = \frac{x}{\gamma} - vt'$$

Ⓑ $$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

t ⋯ yet to come

Ⓐ $x=\gamma(x'+vt')$ $x'=\gamma(x-vt)$

sub in

Ⓑ $x=\gamma[\gamma(x-vt)+vt']$

Ⓔ $x=\gamma^2x-\gamma^2vt+\gamma vt'$

Ⓕ $(1-\gamma^2)x=\gamma v(t'-\gamma t)$

$-\gamma^2\beta^2x=\gamma v(t'-\gamma t)$

$-\gamma^2\beta^2x=\gamma vt'-\gamma^2vt$

$t'=\frac{1}{\gamma v}\left[\gamma^2vt-\gamma^2\beta^2x\right]=\gamma t-\frac{\gamma\beta^2}{v}x=\gamma\left[t-\frac{v}{c^2}x\right]$

Ⓒ

$\beta\equiv\frac{v}{c}$

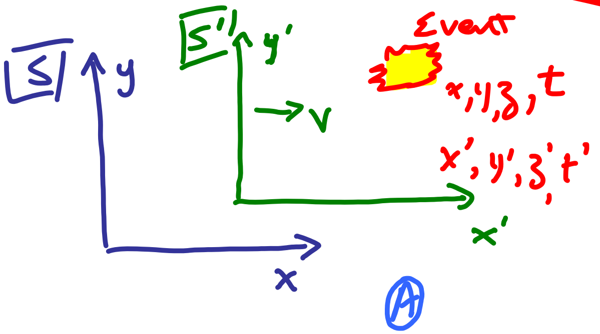
$1-\gamma^2=-\gamma^2\beta^2$

$t'=\gamma\left(t-\frac{v}{c^2}x\right)$

Ⓓ

Similarly .... bit of algebra → $t = \gamma\left(t' + \frac{v}{c^2}x'\right)$ ⑧

Lorentz Transformations



$S \to S'$

Ⓑ

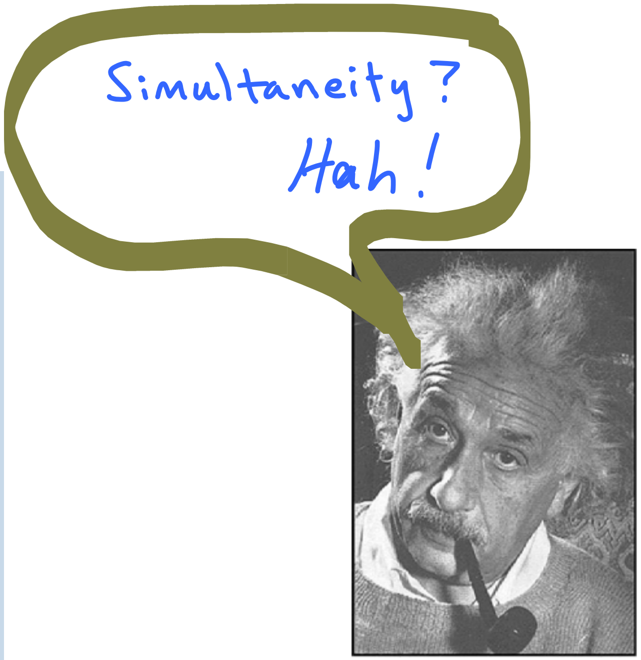
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

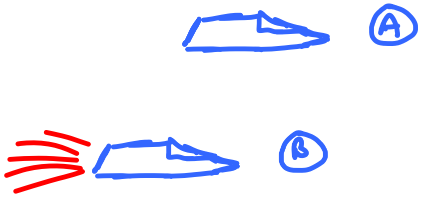
$S' \to S$

Ⓒ

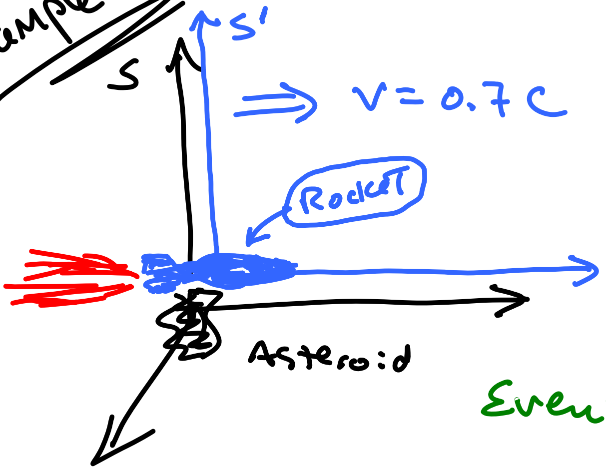
$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$


ON
QC
NL
NB
NS
PE
WA
MT
ND
MN
OR
ID
SD
WI
MI
WY
NE
IA
IL
IN
OH
PA
NY
VT
NH
ME
MA
CT
RI
NJ
DE
MD
NV
UT
CO
KS
MO
KY
WV
VA
CA
AZ
NM
OK
AR
TN
NC
SC
MS
AL
GA
TX
LA
FL
United States
Gulf of California
Gulf of Mexico
Mexico
Cuba
Puerto Rico
©2013 Google
Map data ©2013 Google, INEGI


Simultaneity?
Hah!


Ⓐ
Ⓑ

Example Prob



$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 1.4$$

Event 1: $t = 0$, $t' = 0$ Rocket passed Asteroid

Event 2: Laser flashes at $\underbrace{x = 3\,\text{km}\ ,\ t = 5\,\mu s}_{\text{is } S}$

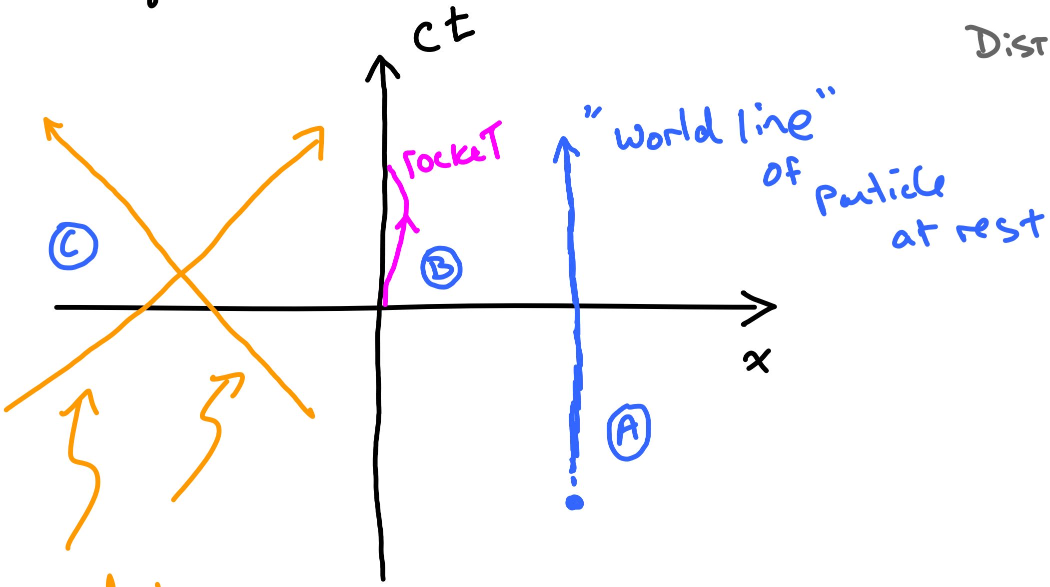
Time ordering changed!

What does event 2 look like in S'?

Ⓐ $$x_2' = \gamma(x_2 - v t_2) = 1.4\left[3 - (.7)(3\times10^5)\,5\times10^{-6}\right] = 2.73\,\text{km}$$

Ⓑ $$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = 1.4\left[5\times10^{-6} - \frac{(.7)(3)}{3\times10^5}\right] = -2.8\,\mu s$$

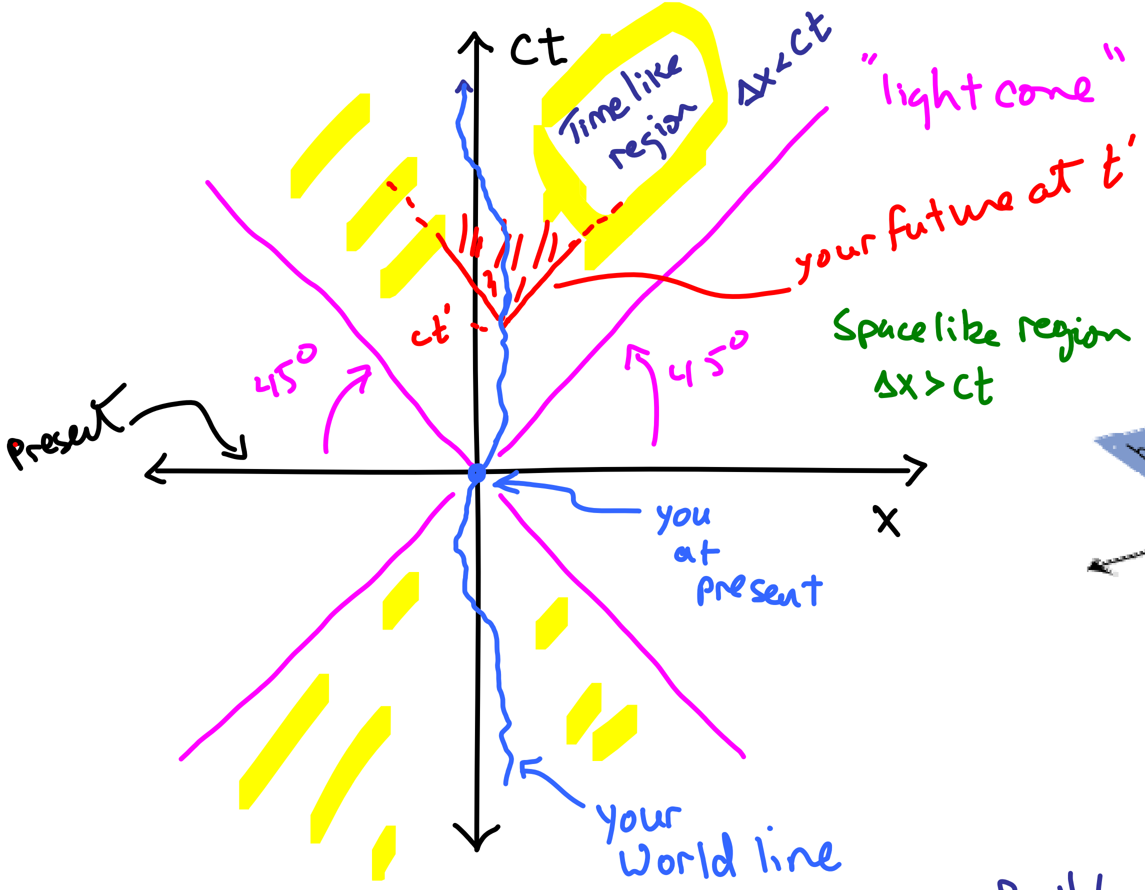
Space-time diagrams

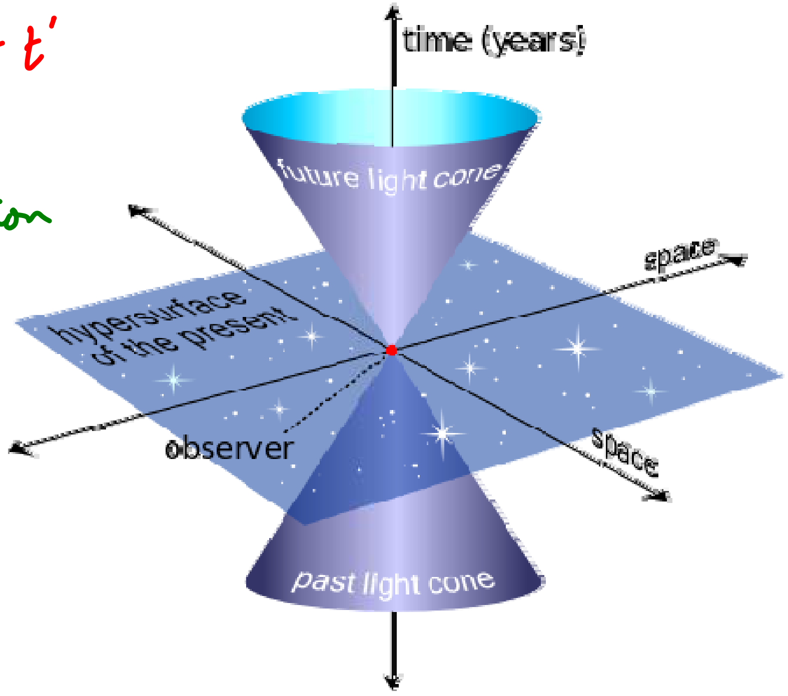


World lines of photons (particles of light) moving in ± x direction

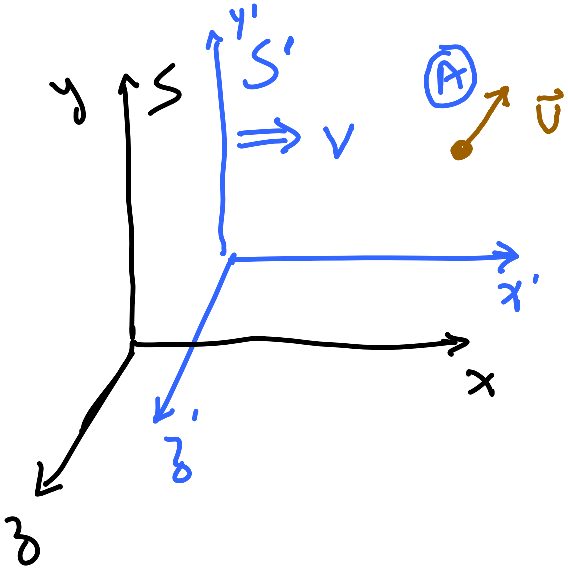
Move at 45°

Distance light travels in time t = ct





Possible causal connection w/ timelike region
No possible causal connection w/ Spacelike region



Velocity Transformations

Ⓐ $\vec{U} \equiv 3$ vector velocity in S

in S

$$U_x = \frac{dx}{dt}$$ Ⓑ

$$U_y = \frac{dy}{dt}$$

$$U_z = dz/dt$$

Ⓓ

$$\boxed{U'_x = \frac{U_x - V}{1 - \frac{V}{c^2} U_x}}$$

Ⓔ

Ⓒ in S'

$$U'_x = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma(dt - \frac{v}{c^2}dx)} = \frac{\gamma\left(\overbrace{\frac{dx}{dt}}^{U_x} - v\right)}{\gamma\left(1 - \frac{v}{c^2}\underbrace{\frac{dx}{dt}}_{U_x}\right)}$$

$$U_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{\overset{U_y}{\boxed{\frac{dy}{dt}}}}{\gamma\left(1 - \frac{v}{c^2}\underset{U_x}{\boxed{\frac{dx}{dt}}}\right)}$$

Ⓐ

Ⓑ

$$\boxed{U_y' = \frac{U_y}{\gamma\left(1 - U_x\frac{v}{c^2}\right)}}$$

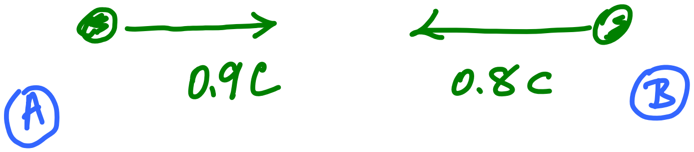
similarly

Ⓒ

$$\boxed{U_z' = \frac{U_z}{\gamma\left(1 - U_x\frac{v}{c^2}\right)}}$$

Example problem

Consider two subatomic particles



What is their relative velocity?

Stop + ponder this for a few moments

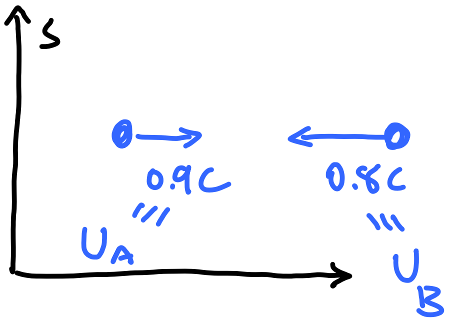
The Answer is **NOT** that they approach each other at 1.7 c

really ?

WTF ?

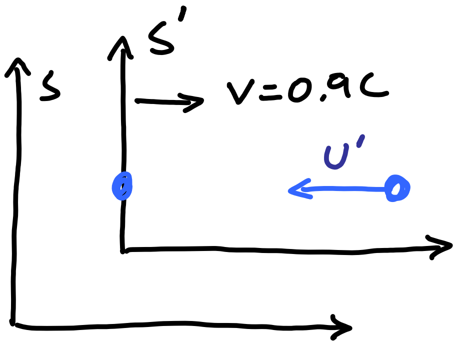
Solution

I



Situation set up
in Problem

II



look at problem from
rest frame of
particle A ⟶ s' system

Ask what is speed of particle B in s'
⟶ That is the Answer to the problem!

III

$$u' = \frac{u_B - v}{1 - \frac{v}{c^2}u_B} = \frac{-0.8c - 0.9c}{1 - \frac{(0.9c)(-0.8c)}{c^2}} = \frac{-1.7c}{1.72} = -0.988c$$

Tis Flu season –

you have Stomach bug – get sick every 20 Minutes
Must go to hospital on Next planet over (5×10^{7} km away)
How fast does the spaceship Need to travel in order that you Not get sick on the spaceship?

what velocity is relevant? NOT dx/dt really

rather

$$\frac{dx}{d\boxed{\tau}}$$

This is known as a Proper velocity. Transforms like spacetime NOT complicated like dx/dt

This is your propertime – Time in your reference frame.

Proper velocity $\eta_x = \frac{dx}{d\tau}$ Ⓐ transforms like x

We already know $dt = \gamma \, d\tau$ Ⓑ (time dilation)

$$\eta_x = \frac{dx}{d\tau} = \gamma \frac{dx}{dt} = \gamma v_x$$ Ⓒ

recall that $\underbrace{\vec{r} = (x,y,z),\ t}$ get mixed up in a Lorentz transformation

→ 4 components → Spacetime 4-vector (x,y,z,t)

can define relativistic Momentum $\equiv m\vec{\eta} = (m\eta_x, m\eta_y, m\eta_z)$

what is 4th component?