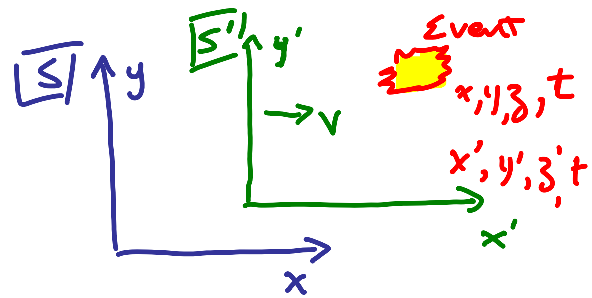
Physics 123 - January 28, 2013

- Workshops begin this week
- P.S. 2 updated due Thurs (Fri)
- Owe you P.S. 1 solns
- Will post relativity chapter from
  Griffiths - Intro to Electrodynamics
  on BlackBoard

Last Time

Lorentz Transformations



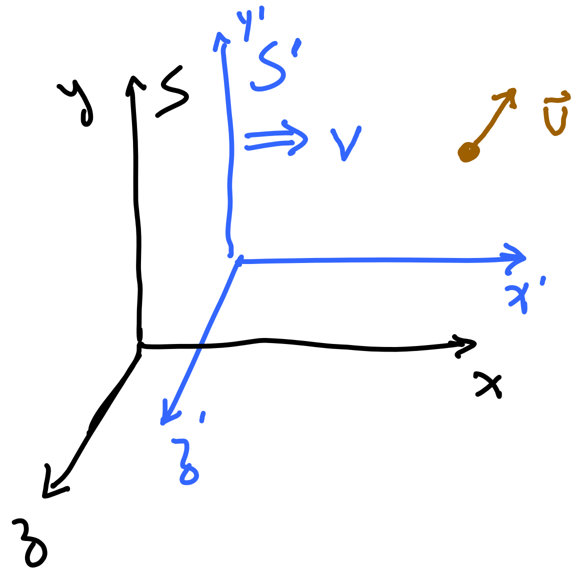
$S \to S'$

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$S' \to S$

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Velocity Transformations



$\vec{U} \equiv$ 3 vector velocity in S

in S

$$U_x = \frac{dx}{dt}$$

$$U_y = \frac{dy}{dt}$$

$$U_z = dz/dt$$

$$U_x' = \frac{U_x - V}{1 - \frac{V}{c^2} U_x}$$

$$U_y' = \frac{U_y}{\gamma\left(1 - U_x \frac{V}{c^2}\right)}$$

$$U_z' = \frac{U_z}{\gamma\left(1 - U_x \frac{V}{c^2}\right)}$$

We looked briefly at proper velocity

$$\frac{dx}{dt} \rightarrow \frac{dx}{d\tau}$$

will come back to this . . .

Matrix multiplication

$A_{\mu\nu}$ = Element in row μ, column ν

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} (A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) \\ (A_{21}B_{11} + A_{22}B_{21}) & (A_{21}B_{12} + A_{22}B_{22}) \end{pmatrix}$$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} \bigcirc & \bigcirc \\ \bigcirc & \end{pmatrix}$$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & \\ & \end{pmatrix}$$

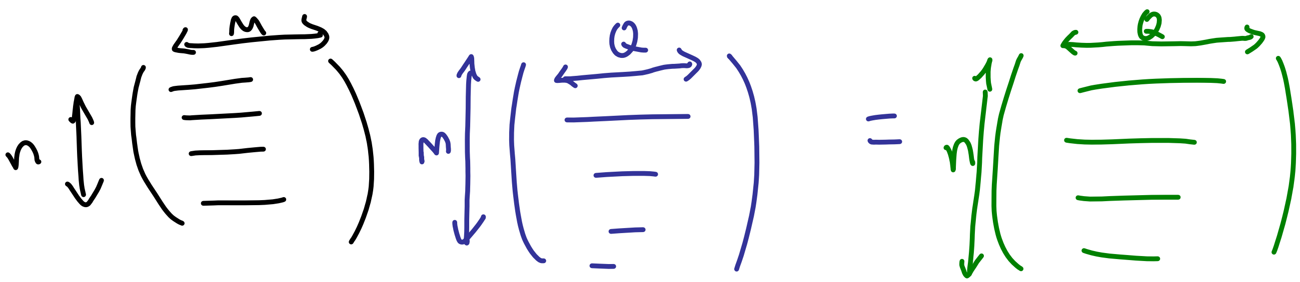
$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} & A_{11}B_{12} + A_{12}B_{22} \\ & \end{pmatrix}$$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} & \\ A_{21}B_{11} + A_{22}B_{21} & \end{pmatrix}$$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} & \\ & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix}$$

example

$$\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 6+9 & -6+12 \\ 4+6 & -4+8 \end{pmatrix} = \begin{pmatrix} 15 & 6 \\ 10 & 4 \end{pmatrix}$$



$n \times m$ $\quad$ $m \times Q$ $\quad$ $n \times Q$

can generalize

columns of matrix 1 must = # rows of matrix 2

$$\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & & & \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1Q} \\ B_{21} & B_{22} & \cdots & B_{2Q} \\ \vdots & & & \\ B_{m1} & B_{m2} & \cdots & B_{mQ} \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} + \cdots + A_{1m}B_{m1} & \cdots \\ A_{21}B_{11} + A_{22}B_{21} + \cdots + A_{2m}B_{m1} & \cdots \\ \vdots & \\ A_{n1}B_{11} + A_{n2}B_{21} + \cdots + A_{nm}B_{m1} & \cdots \end{pmatrix}$$

Vector dot product

$$\underset{1\times 3}{\begin{pmatrix} A_x & A_y & A_z \end{pmatrix}} \underset{3\times 1}{\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}} = \underset{1\times 1}{A_x B_x + A_y B_y + A_z B_z}$$

Relativity and 4-vectors

$x_0 \equiv ct \quad x_1 \equiv x \quad x_2 \equiv y \quad x_3 \equiv z \qquad$ 4-vector x_μ

$$\begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{Lorentz transformation Matrix}} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

usually called $L_{\mu\nu}$

Choices made:
- relative motion along x
- 0 component is the 4th one

Form of $L_{\mu\nu}$ depends on this ... called the Metric look at 1st component

$$\sum_{\nu=1}^{4} L^{1\nu} x_\nu$$

$$x_0' = \gamma x_0 - \gamma\beta x_1$$

$$ct' = \gamma ct - \gamma\beta x$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$\beta = \frac{v}{c}$$

$$\frac{\beta}{c} = \frac{v}{c^2}$$

2ND component

$$\sum_{\nu=1}^{4} L^{2\nu} x_\nu$$

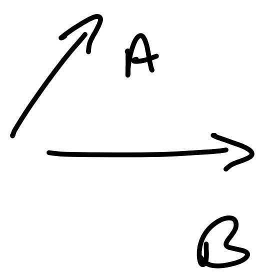
$$x_1' = -\gamma\beta x_0 + \gamma x_1$$

$$x' = -\gamma\beta ct + \gamma x = \gamma(x - vt)$$

recall 3-vector dot product

$$\begin{pmatrix} A_x & A_y & A_z \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = A_x B_x + A_y B_y + A_z B_z$$

scalar → invariant under transformations like rotations or translations



$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

For 4 vectors we have a similar beast ... this is invariant under Lorentz transformations

$$\underbrace{a \cdot b}_{\text{dot product of two 4-vectors } a,\, b} = -a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3$$

Note the negative sign here

often written $a^{\mu} b_{\mu}$ ... μ runs from 0 to 3 for us

$a_\mu \quad , \quad b_\mu$

$$\Delta x_\mu = a_\mu - b_\mu \doteq \begin{pmatrix} a_0 - b_0 \\ a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{pmatrix}$$

$$(\Delta x)_\mu (\Delta x)^\mu = -(a_0-b_0)^2 + (a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2$$

invariant interval $= -c^2\Delta t^2 + d^2$

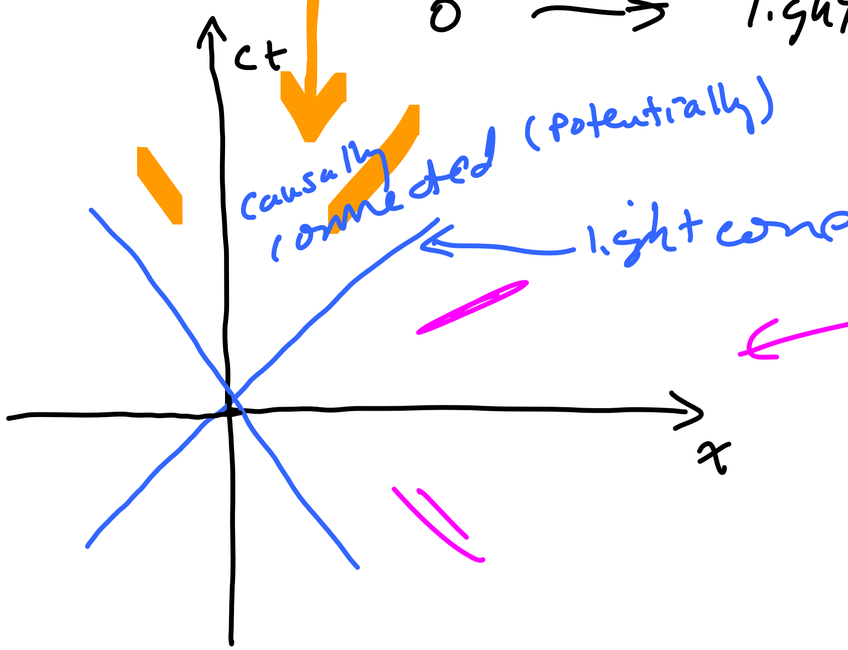
Lorentz invariant

inv. interval is negative → timelike

positive → spacelike

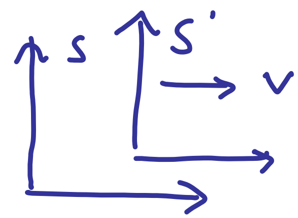
0 → "light like"

Spacetime diagram



No causal connection possible

Momentum Conservation



$$m_A u_A + m_B u_B = m_C u_C + m_D u_D$$

is this true in S' as well? ... i.e. Does momentum conservation work as we move from 1 ref. frame to another?

$$m_A u_A' + m_B u_B' \overset{?}{=} m_C u_C' + m_D u_D'$$

Similar terms

$$m_A\left(\frac{u_A - v}{1 - \frac{v}{c^2}u_A}\right) + m_B\left(\frac{u_B - v}{1 - \frac{v}{c^2}u_B}\right) \overset{?}{=} m_C \cdots + m_D \cdots$$

Very messy... does not work

Define $\underline{\underline{\text{proper}}}$ $\underline{\underline{\text{velocity}}}$ $\underline{\underline{\text{4-vector}}}$

$$\eta_0 = c\,\frac{dt}{d\tau} = c\gamma$$

$$\eta_1 = \frac{dx}{d\tau} = \gamma v_x$$

$$\eta_2 = \frac{dy}{d\tau} = \gamma v_y$$

$$\eta_3 = \frac{dz}{d\tau} = \gamma v_z$$

$$\begin{pmatrix} \eta_0' \\ \eta_1' \\ \eta_2' \\ \eta_3' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

$$\eta_1' = -\gamma\beta\,\eta_0 + \gamma\,\eta_1$$

Try momentum conservation using proper velocity instead of regular velocity

$$m_A \eta_{A_1} + m_B \eta_{B_1} = m_C \eta_{C_1} + m_D \eta_{D_1}$$ ⟵ statement of p cons. in S

What happens in S' frame?

$$m_A \eta'_{A_1} + m_B \eta'_{B_1} \stackrel{?}{=} m_C \eta'_{C_1} + m_D \eta'_{D_1}$$

$$\eta_1' = -\gamma\beta\eta_0 + \gamma\eta_1$$

(from transformations on last page)

$$m_A(-\gamma\beta\eta_{A_0} + \gamma\eta_{A_1}) + m_B(-\gamma\beta\eta_{B_0} + \gamma\eta_{B_1})$$

$$\stackrel{?}{=} m_C(-\gamma\beta\eta_{C_0} + \gamma\eta_{C_1}) + m_D(-\gamma\beta\eta_{D_0} + \gamma\eta_{D_1})$$

regroup terms

$$m_A \gamma u_{A_1} + m_B \gamma u_{B_1} = m_C \gamma u_{C_1} + m_D \gamma u_{D_1}$$

$$- m_A \gamma \beta M_{A_0} - M_B \gamma \beta M_{B_0} \qquad\qquad - M_C \gamma \beta M_{C_0} - M_D \gamma \beta M_{D_0}$$

know this are equal because true in frame S

So P cons. in S' works if orange terms are equal.

$$- m_A \gamma \beta M_{A_0} - M_B \gamma \beta M_{B_0} = - M_C \gamma \beta M_{C_0} - M_D \gamma \beta M_{D_0}$$

$$m_A M_{A_0} + m_B M_{B_0} = m_C M_{C_0} + m_D M_{D_0}$$

$$m_A c\gamma + m_B c\gamma = m_C c\gamma + m_D c\gamma$$

$m_A c^2 \gamma + \cdots$ ——

Define $\gamma m c^2 \equiv$ Relativistic Energy

Have relativistic momentum - Energy conservation

$$\left.\begin{aligned} P_0 &= m\eta_0 = m\gamma c = \frac{m\gamma c^2}{c} = E/c \\ P_1 &= m\,\eta_1 \\ P_2 &= m\,\eta_2 \\ P_3 &= m\,\eta_3 \end{aligned}\right]$$

Momentum-energy 4-vector