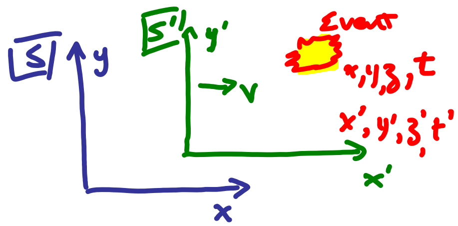
Physics 123 – January 30, 2013

Last Time

Lorentz Transformations



$S \to S'$

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$S' \to S$

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

can be written as

$$x^{\nu} = L^{\mu\nu} x_{\mu}$$

4-vector index

Lorentz matrix index

repeated indices imply summation

"Einstein Summation notation"

you've seen it ...

Will see much more if you keep taking physics

That said ... we won't use it ...

Define Space time 4-vector

$x_0 = ct \quad x_1 = x \quad x_2 = y \quad x_3 = z$

$$\begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{Lorentz Transformation matrix}} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \rightarrow \left\{ \begin{aligned} x_0' &= \gamma x_0 - \beta\gamma x_1 \\ x_1' &= -\beta\gamma x_0 + \gamma x_1 \\ x_2' &= x_2 \\ x_3' &= x_3 \end{aligned} \right.$$

Dot product between 4-vectors is $\underbrace{\text{Lorentz invariant}}$

$\boxed{\text{Meaning What ?}}$

$$a^\mu b_\mu \quad \text{or} \quad a \cdot b = \underbrace{-a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3}_{\text{Lorentz invariant}}$$

where a and b are 4-vectors

Suppose we have events a and b described by 4-vectors a and b

$$\Delta x_\mu = a - b \doteq \begin{pmatrix} a_0 - b_0 \\ a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{pmatrix}$$

displacement 4 vector

$$\Delta x \cdot \Delta x = -c^2(\Delta t)^2 + \underbrace{\Delta x^2 + \Delta y^2 + \Delta z^2}_{d^2}$$

(spatial distance bet. events)^2

invariant interval

Proper velocity 4-vector

$$\eta_0 = c\frac{dt}{d\tau} = c\gamma$$

$$\eta_1 = \frac{dx}{d\tau} = \gamma v_x$$

$$\eta_2 = \frac{dy}{d\tau} = \gamma v_y$$

$$\eta_3 = \frac{dz}{d\tau} = \gamma v_z$$

or v_x, u_y, u_z

Momentum + Energy conservation hold across Lorentz transformations if they are Defined through the

Energy-Momentum 4-vector

$$p_0 = m\eta_0 = m\gamma c = \frac{m\gamma c^2}{c} = E/c$$

$$p_1 = m\,\eta_1$$

$$p_2 = m\,\eta_2$$

$$p_3 = m\,\eta_3$$

Where

$E \equiv m\gamma c^2$

$\equiv$ Relativistic Energy

$$\underbrace{p^\mu p_\mu = P\cdot P}_{} = -\frac{E^2}{c^2} + \frac{m^2u^2}{1-\frac{v^2}{c^2}}$$

$$= -\frac{m^2c^4}{c^2\left(1-\frac{v^2}{c^2}\right)} + \frac{m^2u^2}{\left(1-\frac{v^2}{c^2}\right)} = \frac{m^2u^2c^2 - m^2c^4}{\left(1-v^2/c^2\right)} = \frac{m^2c^2(u^2-c^2)}{c^2-u^2}$$

$$\frac{(m\gamma c^2)^2}{c^2}$$

$$= -m^2c^2$$

invariant Mass

Lorentz invariant

$$P\cdot P = -m^2c^2 = -\frac{E^2}{c^2} + \underbrace{m^2u^2\gamma^2}_{p^2}$$

$$E^2 = m^2c^4 + p^2c^2$$

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{E}{c^2} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{1}{c^2}$$

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Taylor Series

$$(1-x)^{-1/2} = 1 - \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3 + \cdots$$

$$x \downarrow$$

$$\left(1-\frac{v^2}{c^2}\right) = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\left(\frac{v^2}{c^2}\right)^2 - \cdots$$

$$E = \underbrace{mc^2}_{\text{rest Energy}} + \underbrace{\frac{1}{2}mv^2}_{\text{kinetic Energy}} + \underbrace{\frac{3}{8}m\frac{v^4}{c^4} - \cdots}_{\text{higher order terms}}$$

small unless v is close to c
we will ignore these

mass of Proton is 1.67×10^{-27} Kg $\sim$ 1 amu

Moves at $0.7c$

$$E = (1.4)\,mc^2 = 1.4\,(1.67 \times 10^{-27}\,\text{kg})(3 \times 10^8\,\text{m/s})^2$$

$$= 2.1 \times 10^{-10}\,\text{Kg}\,\frac{\text{m}^2}{\text{s}^2} \rightsquigarrow \text{J}$$

Joules are not a convenient unit for ATOMIC + Subatomic particles

1 eV $\equiv$ electron-Volt $\equiv$ Energy gained by one electron moving thru potential diff of 1 volT

Energy = mc^2
$\uparrow$ eV

unit of Momentum $\sim$ eV/c

unit of Mass $\sim$ eV/c^2

$$E^2 = P^2c^2 + m^2c^4$$

$M_{electron} = 0.511 \ MeV/c^2$

$M_{proton} = 938 \ MeV/c^2$

$1000 \ MeV = GeV$

1000 MeV proton passing by ...

Total energy

$M_p = \underbrace{1.67 \times 10^{-27} kg} = 1.5 \times 10^{-10} \frac{J}{c^2} \frac{1 eV}{1.6 \times 10^{-19} J} = 938 \ \frac{MeV}{c^2}$

$E_{TOT} = \underbrace{E_0}_{mc^2} + KE = 938 \frac{MeV}{c^2} c^2 + 1000 = 1938 \ MeV$

Momentum of that proton

$$E^2 = p^2c^2 + E_0^2$$

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{1938^2 - 938^2}}{c} = 1696\ \frac{\text{MeV}}{c}$$

velocity in lab?

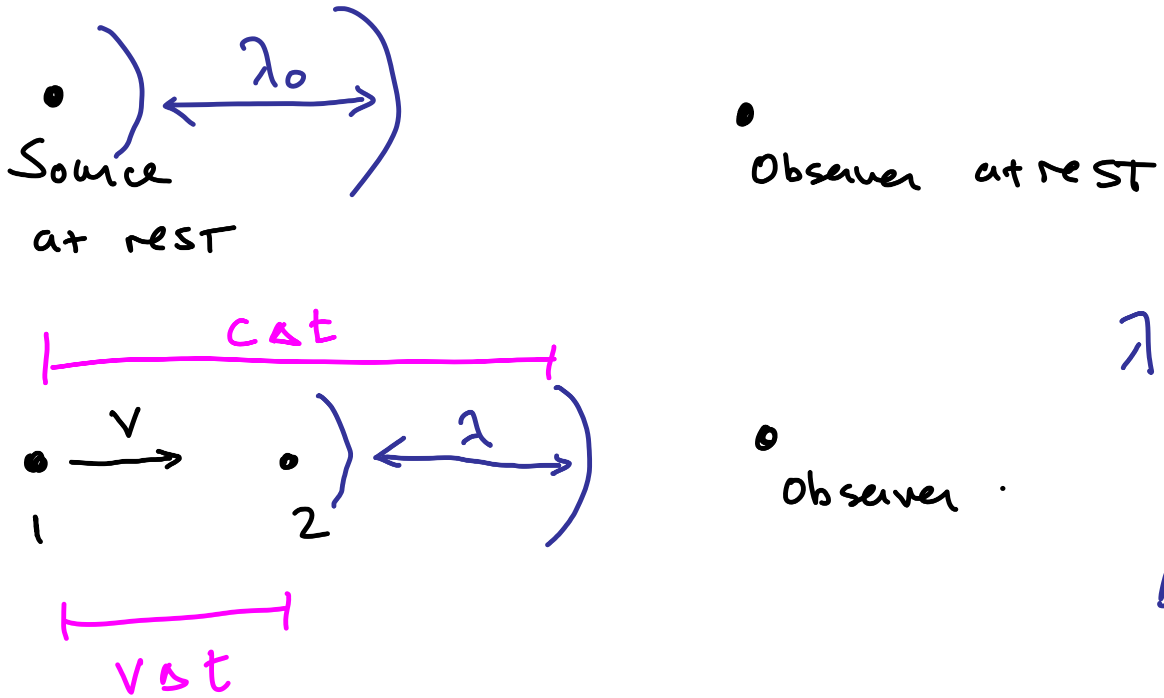
$$E = \gamma mc^2 = \qquad mc^2 = \sqrt{1 - \left(\frac{v}{c}\right)^2}\ E$$

$$v = c\sqrt{1 - \left(\frac{E_0}{E}\right)^2}$$

$$v = c\sqrt{1 - \left(\frac{938}{1938}\right)^2} = 0.87\,c$$

Doppler shift of light

Follows Giancoli's discussion



$$\lambda = c\Delta t - v\Delta t$$

Δt is O frame

Δt_0 is S frame

proper frame

$$\Delta t = \Delta t_0 \gamma$$

ν = frequency

$v = \lambda\nu$

$$\lambda = (c-v)\gamma\Delta t_0 = \frac{c-v}{\sqrt{1-v^2/c^2}}\Delta t^0 = \frac{c-v}{\sqrt{c^2-v^2}}\underbrace{\Delta t_0 c}_{\lambda_0}$$

$$\lambda_{\text{observer}} = \underset{\uparrow \text{ Source}}{\lambda_0}\sqrt{\frac{c-v}{c+v}} \qquad \nu = \nu_0\sqrt{\frac{c+v}{c-v}}$$

Source + Observer Moving toward each other

Moving Away from each other

$$\lambda_{obs} = \lambda_0\sqrt{\frac{c+v}{c-v}} \qquad \nu = \nu_0\sqrt{\frac{c-v}{c+v}}$$

Vesto Slipher (1875-1969) Lowell Observatory discovers a strange thing in 1912 …
Most nearby galaxies are moving away from us
Made use of the Doppler shift in atomic spectra

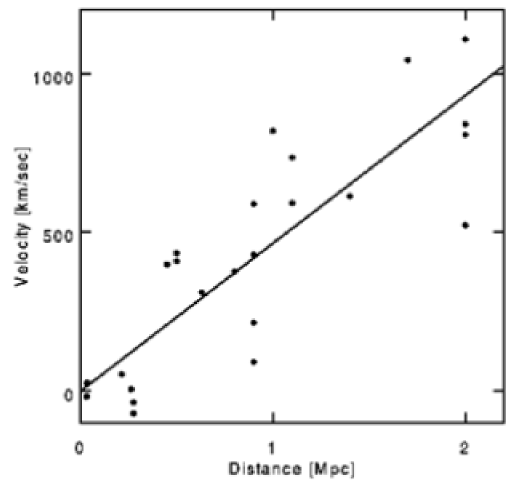
Edwin Hubble (1889-1953) and Milton Humason (1891-1972) at Mount Wilson Observatory combine Hubble's distance measurements (Cephied variable stars) with Slipher's reshift information and discover …

Galaxies that are further away are moving away from us faster

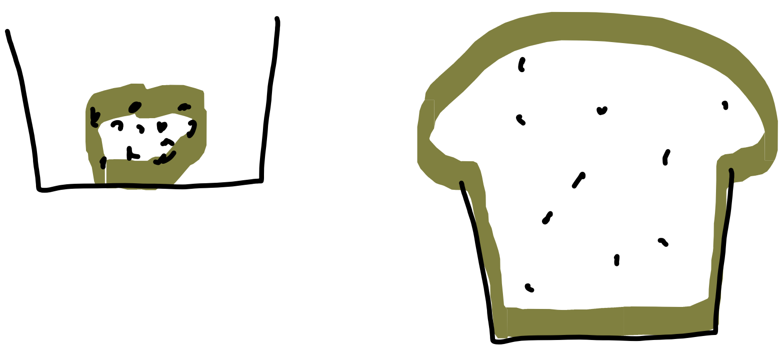
Hubble's Law V=Hd

Hubble

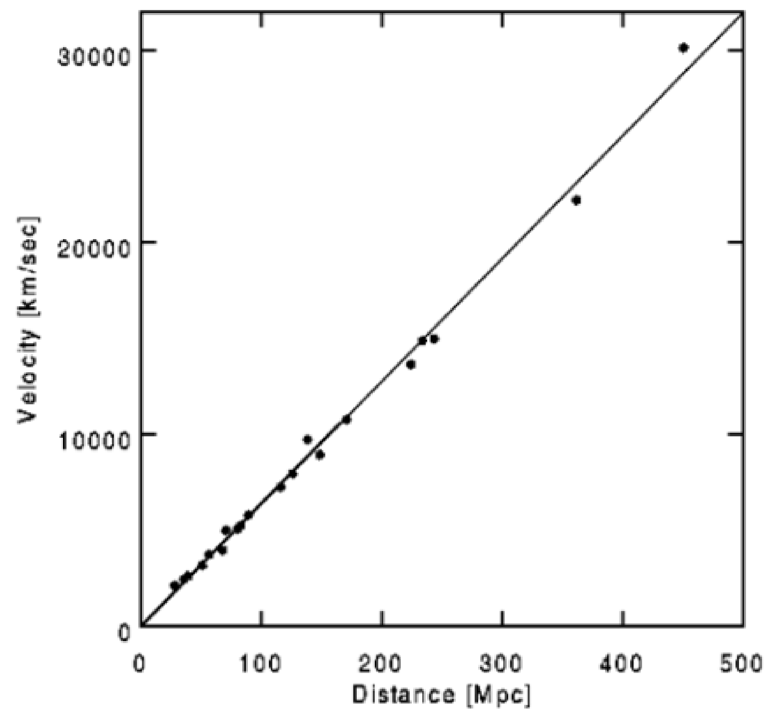
Humason



Welcome to the Expanding Universe!



Type Ia SNe from Riess, Press and Kirshner (1996)



Light travels from NYC to San Francisco in 1/100 second …. and it travels 1 Mpc in 3 million years