Physics 123 - February 4, 2013

last time

Relativistic Energy-Momentum 4-vector

$$p_0 = m\eta_0 = m\gamma c = \frac{m\gamma c^2}{c} = E/c$$

$$p_1 = m\eta_1$$

$$p_2 = m\eta_2$$

$$p_3 = m\eta_3$$

Where

$E = m\gamma c^2$

$\equiv$ Relativistic Energy

$$p^\mu p_\mu = -\frac{E^2}{c^2} + m^2 u^2 \gamma^2 = -\frac{E^2}{c^2} + p^2 = \underline{-m^2 c^2}$$

$$\boxed{E^2 = m^2 c^4 + p^2 c^2}$$

$$E = mc^2 \gamma = \frac{mc^2}{\sqrt{1-(v/c)^2}}$$

Taylor expand in powers of v/c

$$\boxed{E = mc^2 + \frac{1}{2} m v^2 + \frac{3}{8} m \frac{v^4}{c} - \ldots}$$

Doppler Shift

in proper frame of source $\longrightarrow \lambda_s, \nu_s$

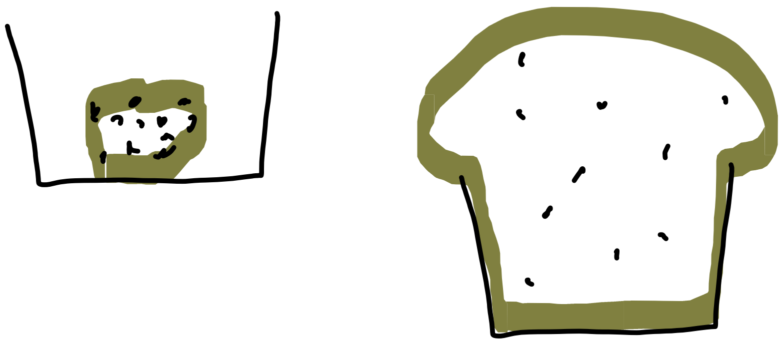
Source + observer Approaching each other

$$\lambda = \lambda_s \sqrt{\frac{c-v}{c+v}} \qquad \nu = \nu_s \sqrt{\frac{c+v}{c-v}}$$

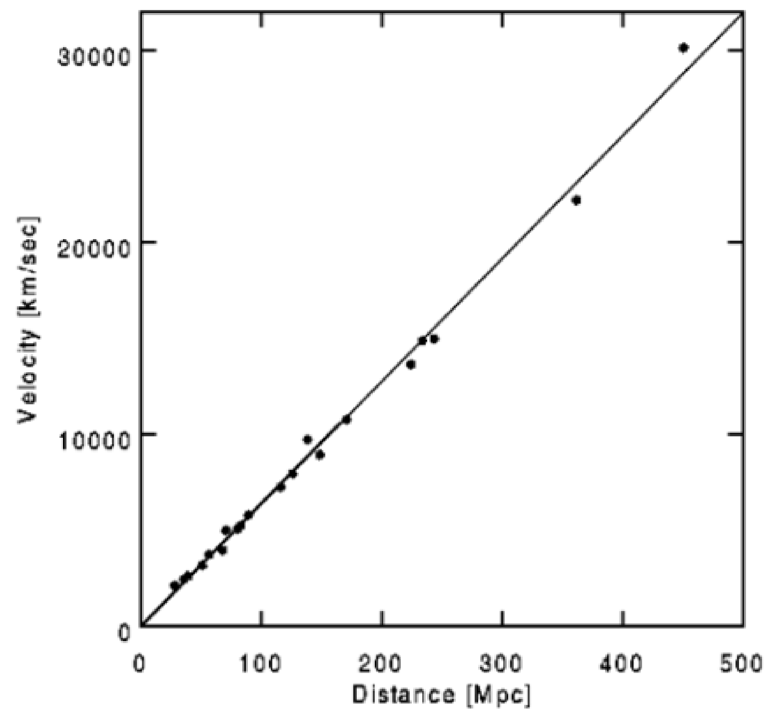
Source + observer Moving away from each other

$$\lambda = \lambda_s \sqrt{\frac{c+v}{c-v}} \qquad \nu = \nu_s \sqrt{\frac{c-v}{c+v}}$$

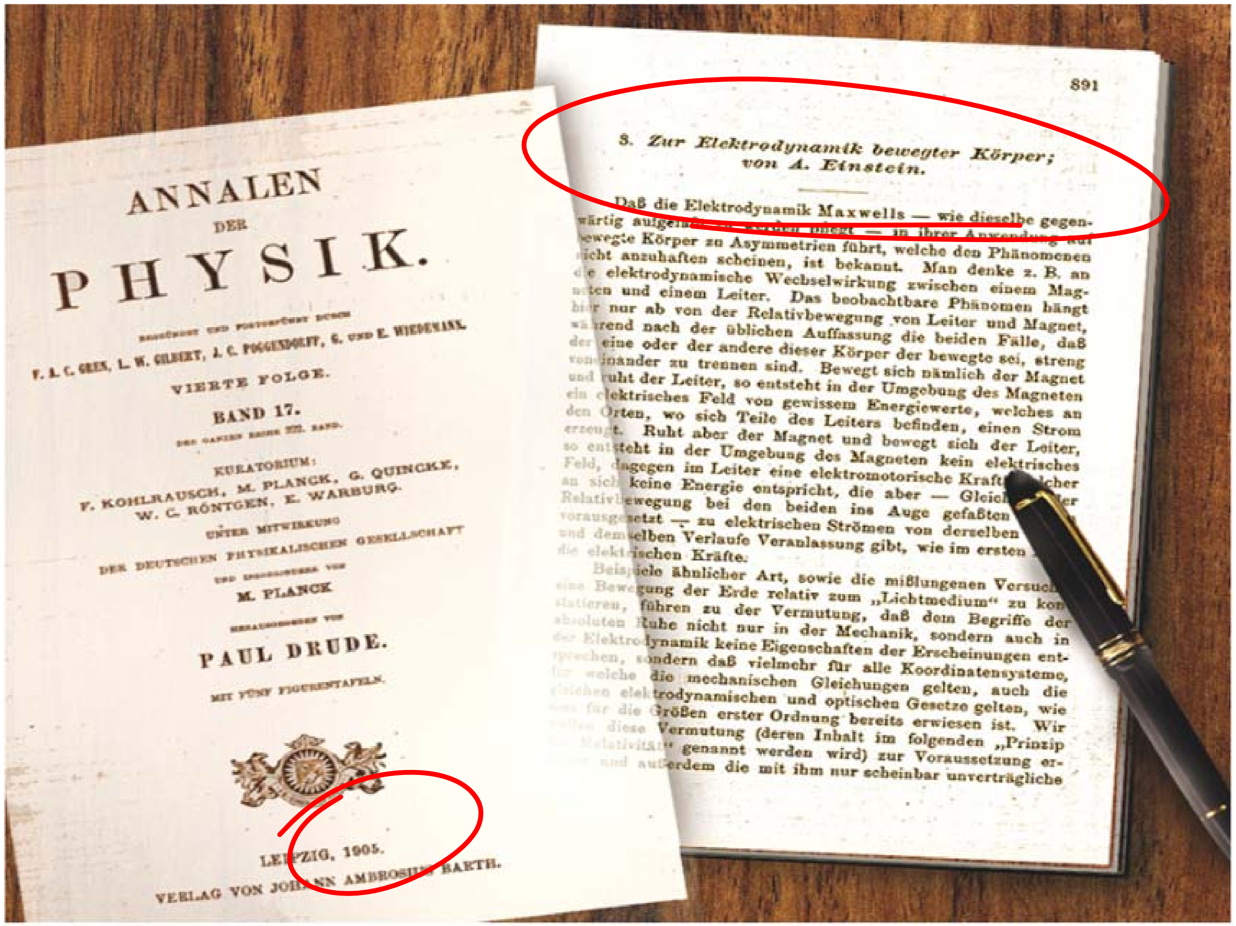
Welcome to the Expanding Universe!



Type Ia SNe from Riess, Press and Kirshner (1996)



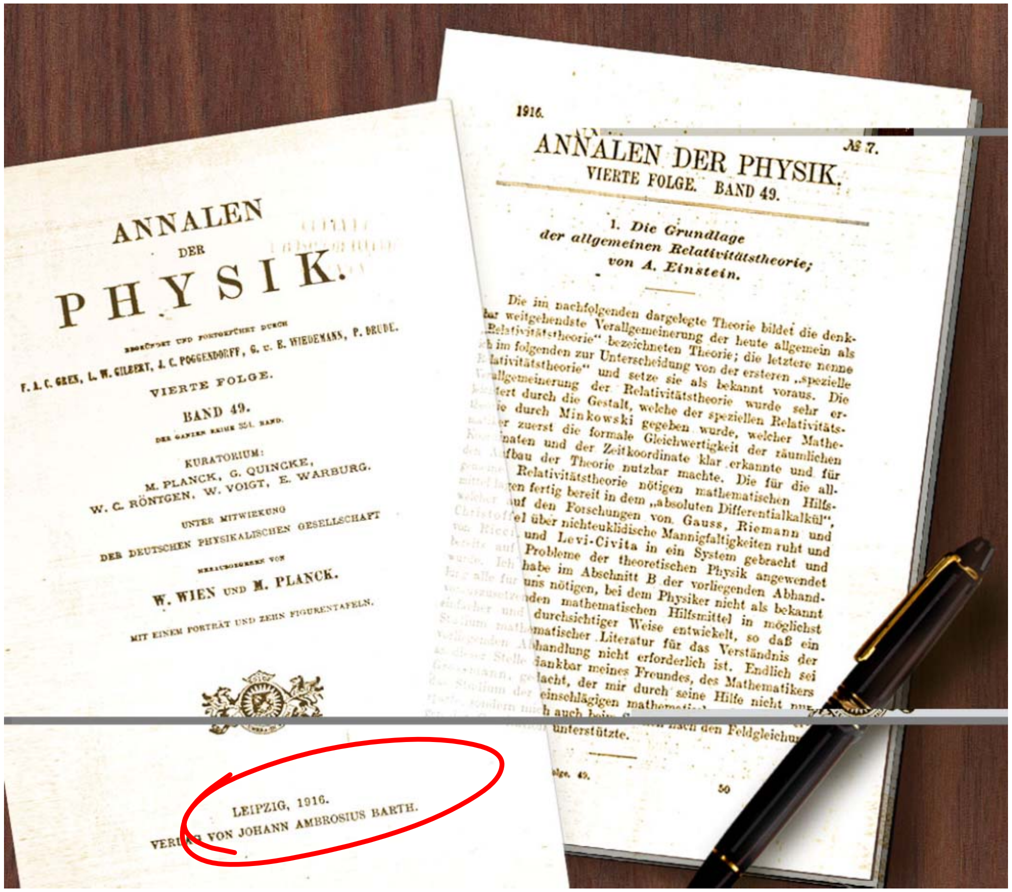
Light travels from NYC to San Francisco in 1/100 second …. and it travels 1 Mpc in 3 million years

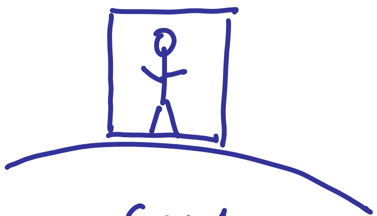


The Special theory of relativity

On the electrodynamics of Moving Bodies

The general theory
of relativity





Earth

Vs

1g

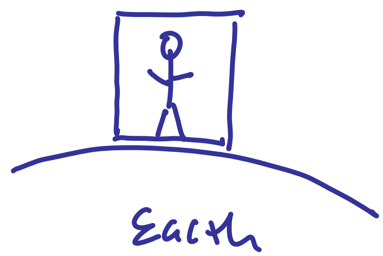
Acceleration

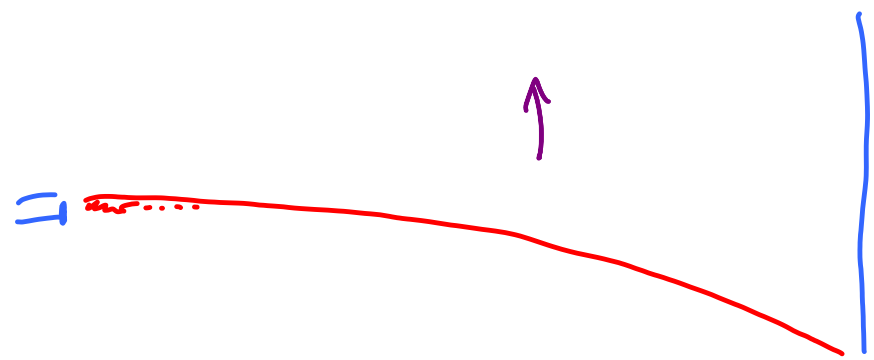
Einstein's Equivalence Principle

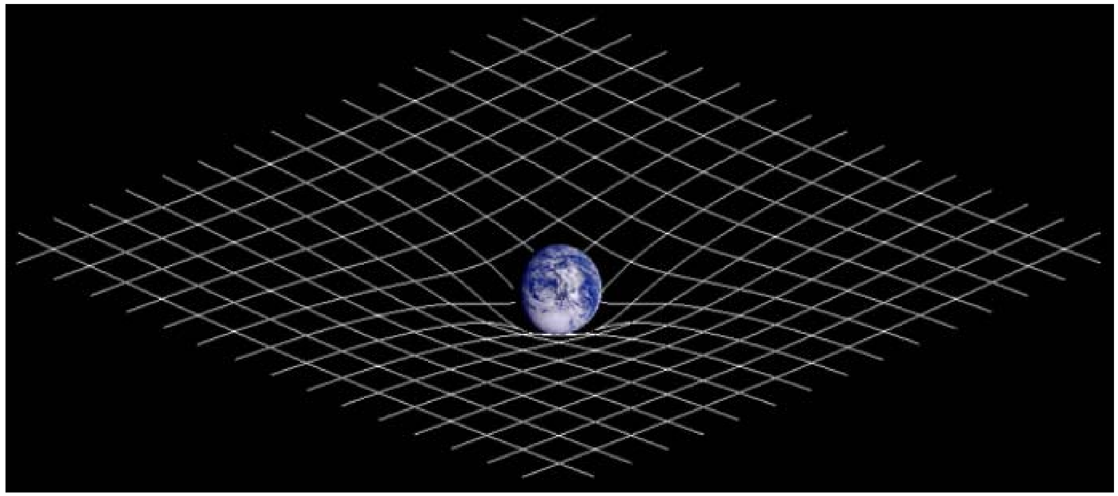
accelerated reference frames

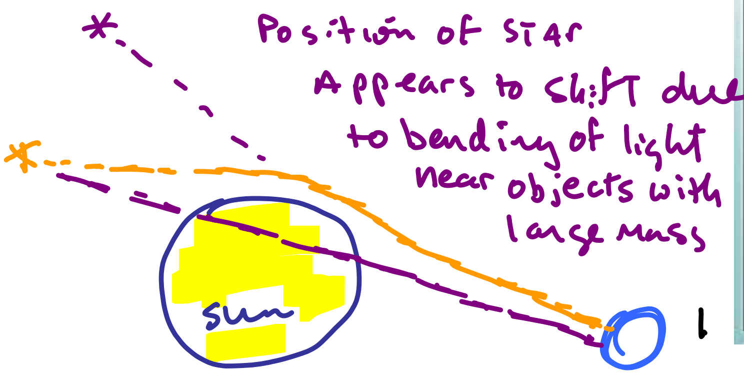
$\equiv$

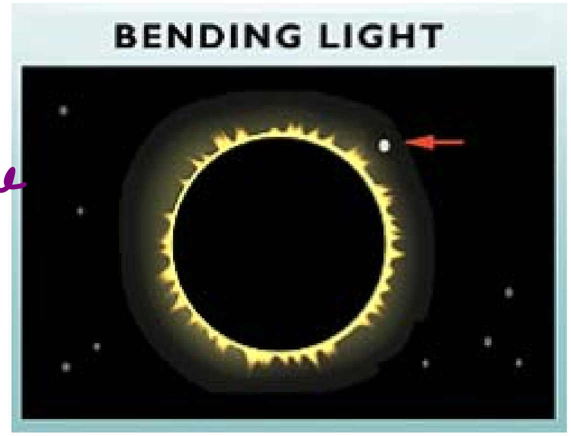
gravitational field

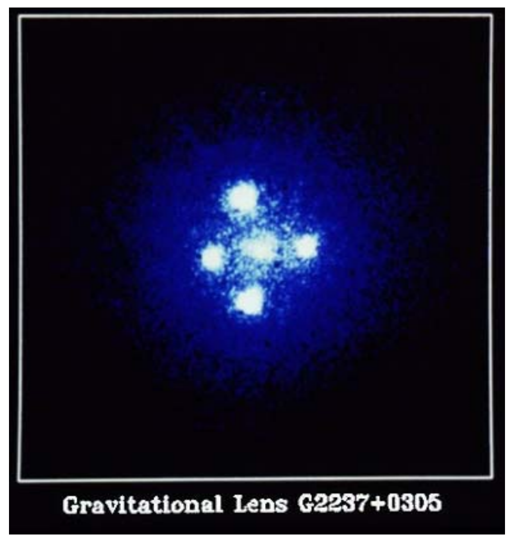

Earth

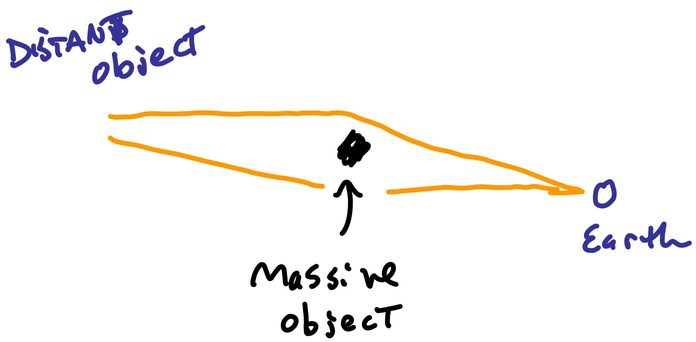




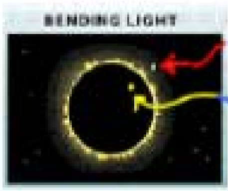

Position of star
Appears to shift due
to bending of light
near objects with
large mass
sun


BENDING LIGHT


Gravitational Lens G2237+0305


DISTANT object
Massive object
Earth

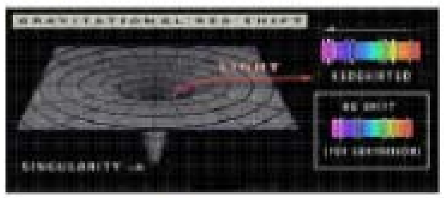
Apparent position



- Bending of light by gravitational field ✓

Actual Position

- Gravitational Redshift of light ✓



✓

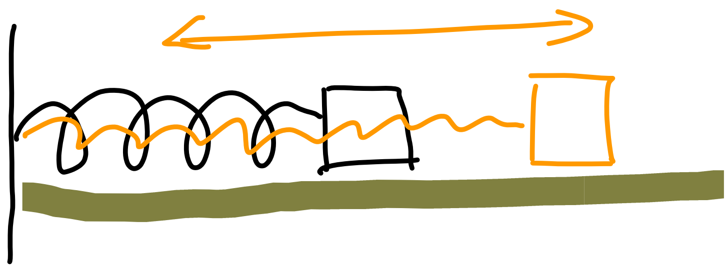
- Perihelion advance of Mercury

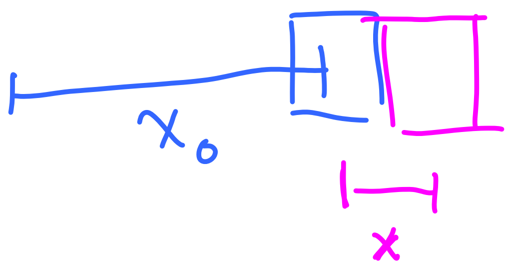
- Gravitational Waves ?

Amplitude ~ 10^{-16} m

LIGO

Simple Harmonic Motion





$F \propto x$

$F = kx$

Spring constant

$\vec{F} = -k\vec{x}$

$F = kx$

$$F = -kx$$

$$ma = m\frac{d^2x}{dt^2} = -kx$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$$

Simple Harmonic Motion

$$\omega^2 = \frac{k}{m}$$

$$x = A\cos(\omega t + \phi)$$

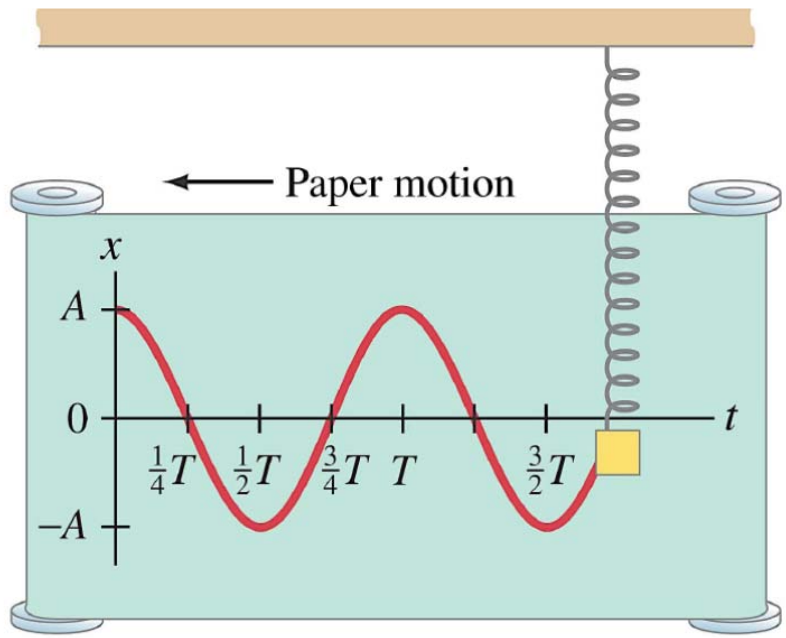
Amplitude

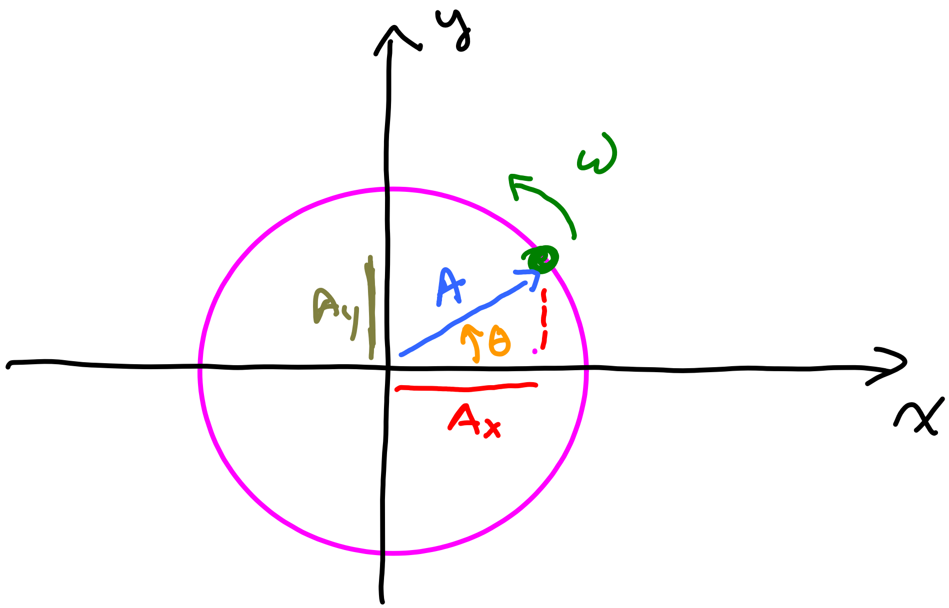
frequency

initial phase Angle

$\omega = \frac{2\pi}{T}$

cos is periodic in $\underline{\underline{2\pi}}$

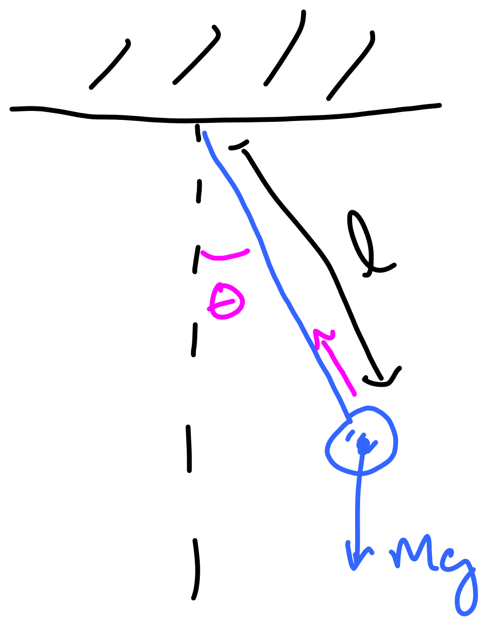


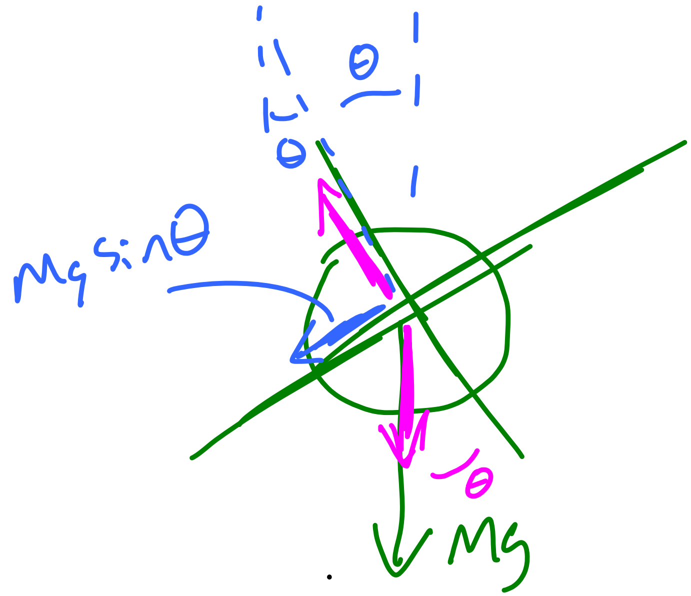


$$A_x = A\cos\theta = A\cos\omega t$$

$$A_y = A\sin\theta = A\sin\omega t$$

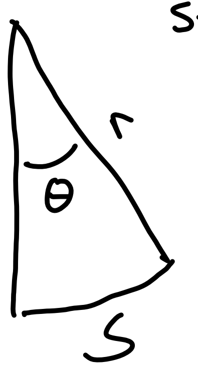
simple pendulum





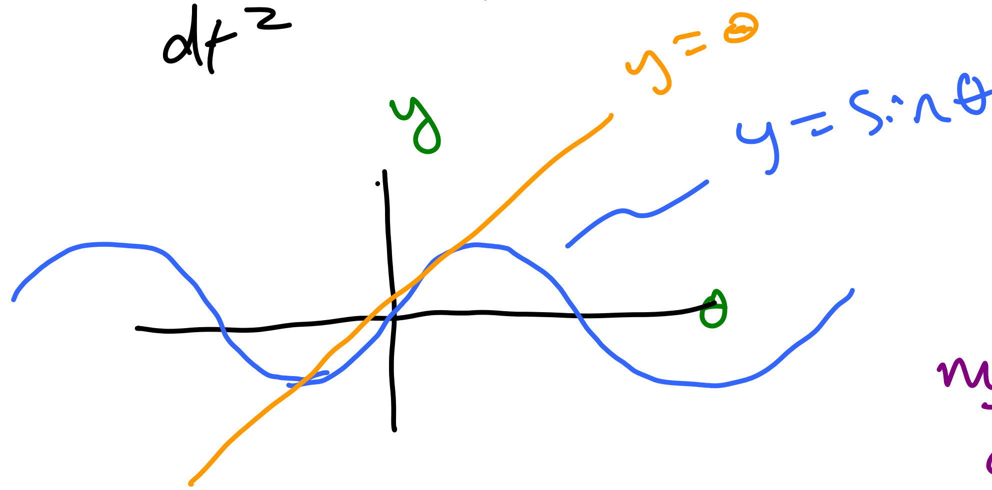
$$ma_{\perp} = -mg\sin\theta$$

$s = r\theta$



$s = \ell\theta$

$$m\frac{d^2 s}{dt^2} = -mg\sin\theta$$



For θ small

$-mg\frac{s}{\ell}$

$$m\frac{d^2 s}{dt^2} = -mg\theta$$

$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots \qquad -\infty < x < \infty; \quad n \in \mathbb{N}$$

$$\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots \qquad -\infty < x < \infty; \quad n \in \mathbb{N}$$

$$m\frac{d^2s}{dt^2} = -\frac{mgs}{\ell}$$

$$\frac{d^2s}{dt^2} + \frac{g}{\ell}s = 0$$

SHO w/ $\omega^2 = \frac{g}{\ell}$