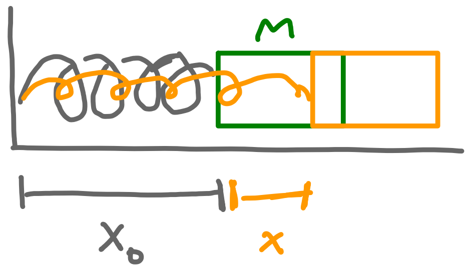
Physics 123 - February 6, 2013

Last Time



$\vec{F} = -k(\vec{x} - \vec{x}_0)$

define $\vec{x}_0 = 0$

1d motion, sign gives direction

restoring force

$F = -kx$

$$\frac{dx^2}{dt^2} + \frac{k}{m}x = 0$$

SHM w/ $\omega^2 = \frac{k}{m}$

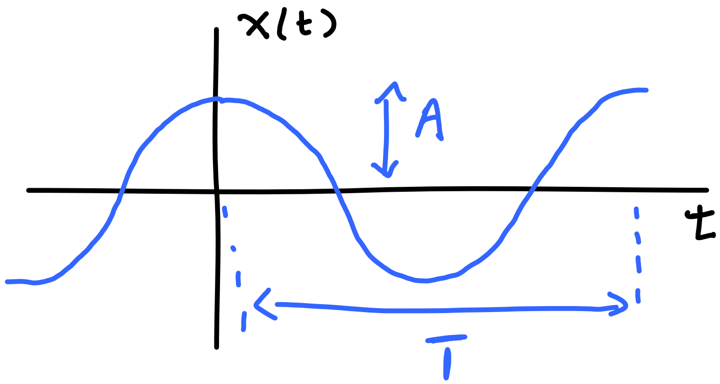
Amplitude

$$x(t) = A\cos(\omega t + \phi)$$

initial phase

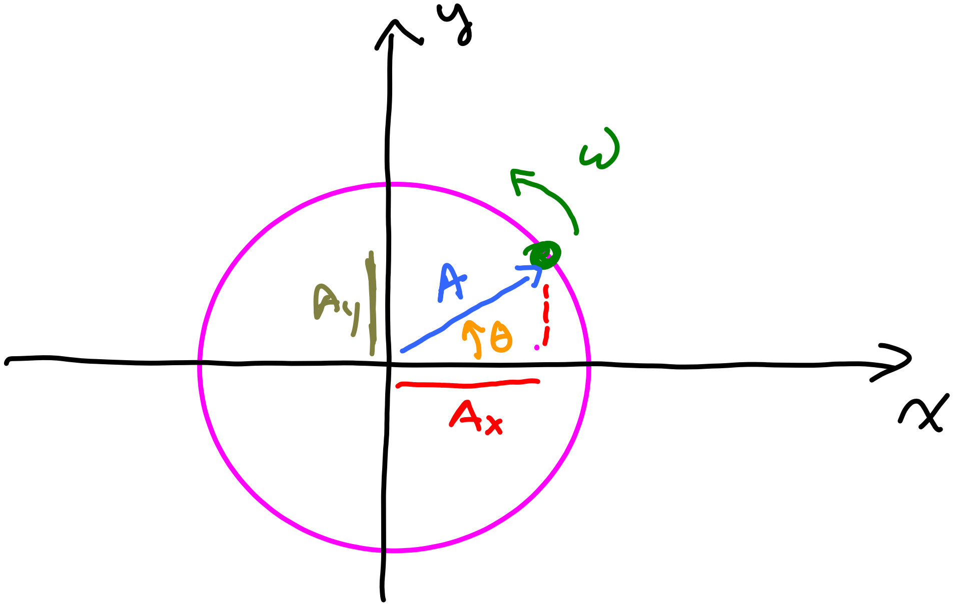
$\omega = \frac{2\pi}{T}$

Period of oscillation



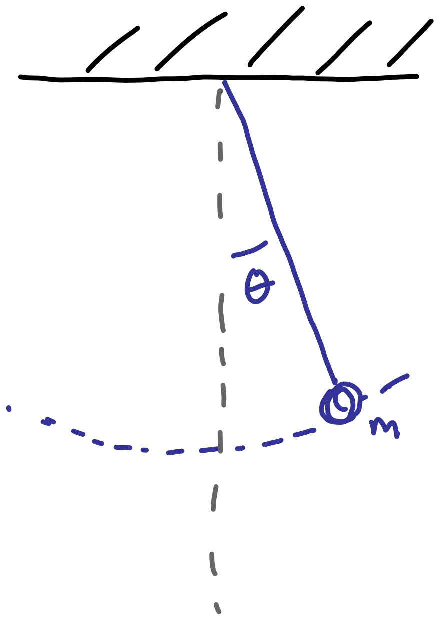
second deriv. + const × Linear = 0

SHM w/ ω^2 = const



$$A_x = A\cos\theta = A\cos\omega t$$

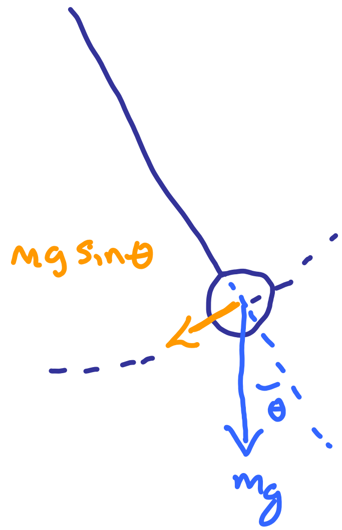
$$A_y = A\sin\theta = A\sin\omega t$$



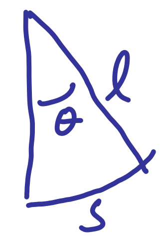
Simple pendulum

$$F = -mg\sin\theta$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta$$



$$\frac{d^2s}{dt^2} = -g\sin\theta$$



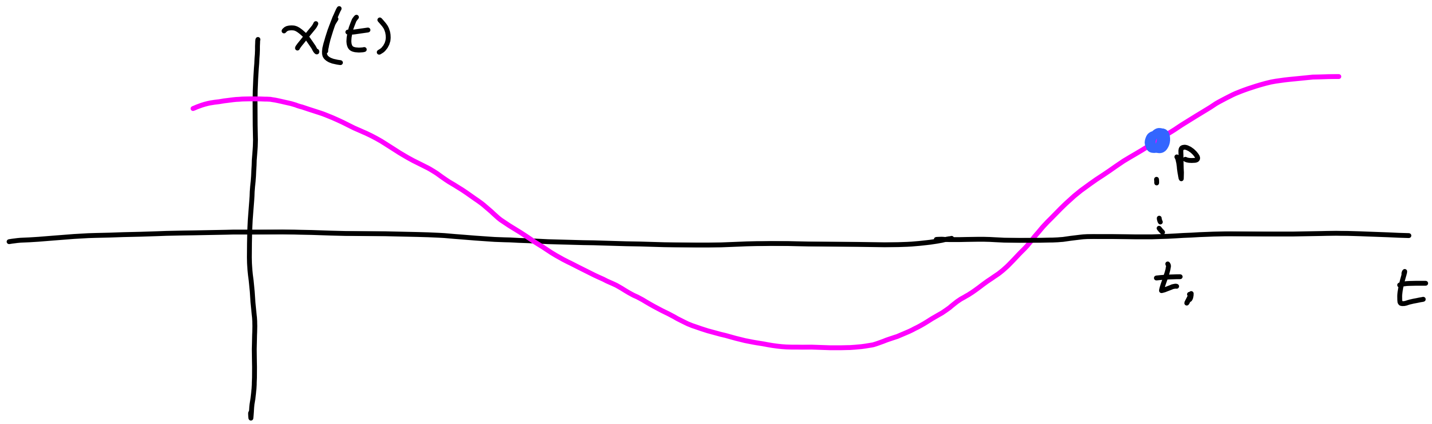
$$s = \ell\theta$$

$$\Big\downarrow \text{ small } \theta$$

$$\frac{d^2s}{dt^2} = -g\theta$$

$$\frac{d^2s}{dt^2} + \frac{g}{\ell}s = 0$$

SHM w/ $\omega^2 = \frac{g}{\ell}$



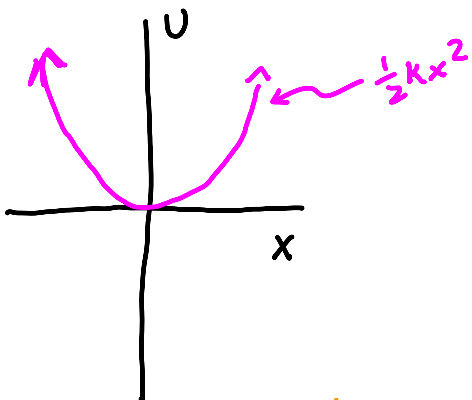
Particle P executes SHM

at time t, P has (A) + velocity + accel
(B) − vel − accel
(C) + vel − accel
(D) − vel + accel

?

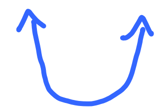
Why is SHM <u>so</u> important?

recall potential energy $U_{spring} = \frac{1}{2} k x^2$



$F_x = -\frac{dU}{dx}$

see how potential like



gives restoring force about equilibrium point

$F_s = -\frac{dF}{ds}$ <u>or</u>

gradient

$-\overrightarrow{\text{grad}}\, U$

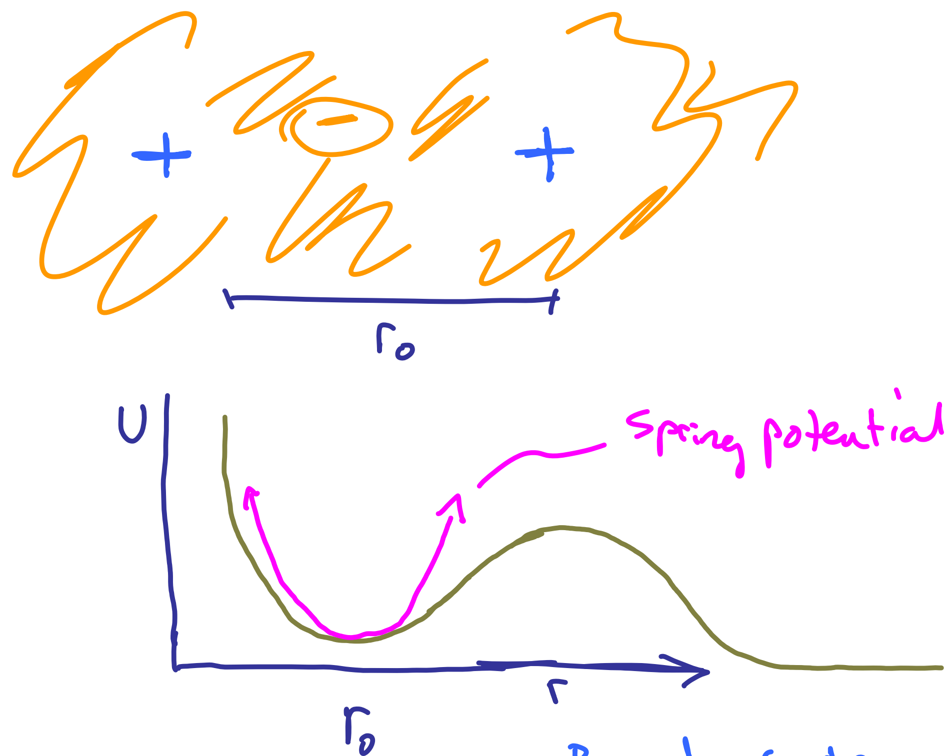
$\vec{F} = -\vec{\nabla} U$ <u>or</u> $\vec{F} = -\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$

vector operator

$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

"Del"

molecule



Bound state and

Expect SHM for small displacements about equilibrium position

Damped harmonic oscillations

$$F = m\frac{d^2x}{dt^2} = -kx - \underbrace{b\frac{dx}{dt}}_{\text{Damping term}}$$

b: Damping constant

Damping term
why this form?

$$x(t) = A\, e^{-bt/2m} \cos(\omega' t + \varphi)$$

Think of this as time Dependent Amplitude ($A\,e^{-bt/2m}$)

Exponential Damping depends on b ($e^{-bt/2m}$)

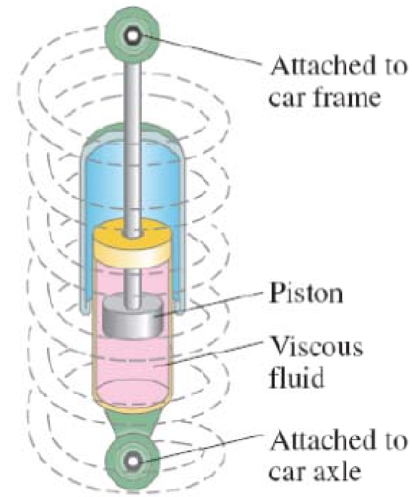
regular SHM ($\cos(\omega' t + \varphi)$)

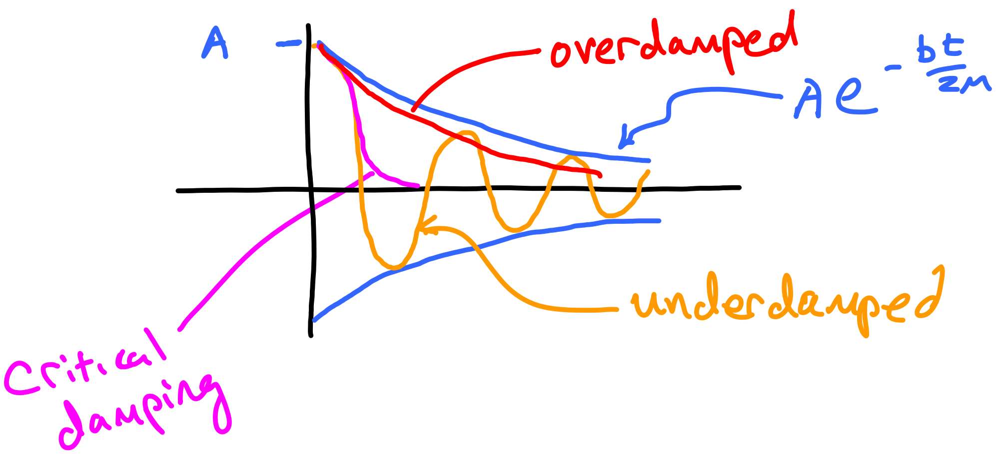
$$\omega' = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \omega_0 \sqrt{1 - \left(\frac{b}{2m\omega_0}\right)^2}$$

$\omega_0 \equiv$ frequency in no Damping limit, $\omega_0^2 = \frac{k}{m}$

For small damping $\omega' \approx \omega_0$

do demo





Energy in system

$$E(t) = \frac{1}{2}k\underbrace{A^2 e^{-bt/m}}$$

A^2 w/ time degradation of A tossed in

E lost to friction in damping

Forced oscillations

Think: child on swing
- Earthquake

$$F = m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \cos \omega t$$

driving force ... frequency ω

if no damping
Natural frequency is ω_0

$$x = A\cos(\omega t + \delta)$$

ω of driving force

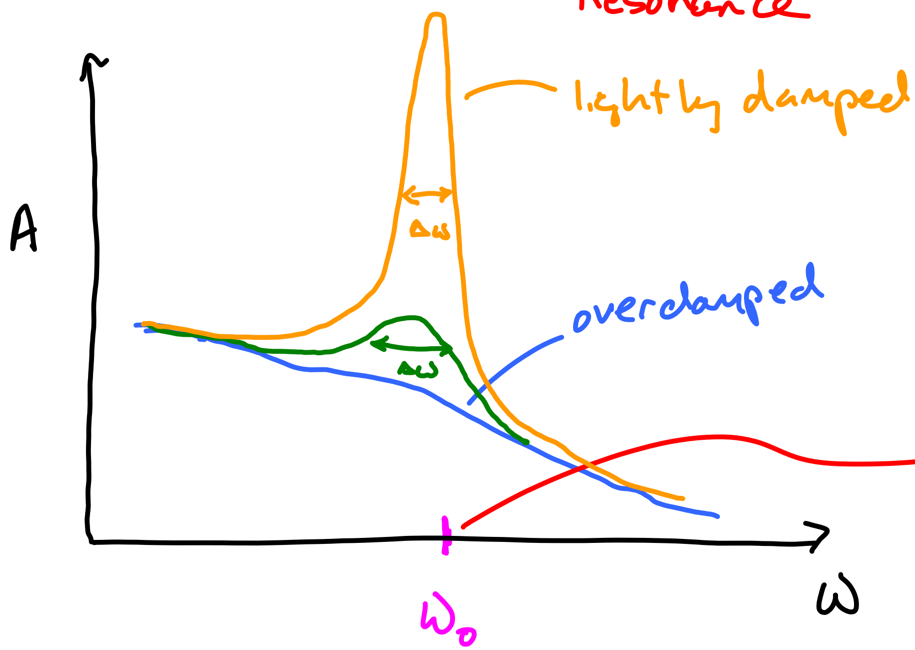
I used soln form from Tipler
Giancoli uses a slightly diff. form

$$A = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + b^2\omega^2/m^2}}$$

$$\tan\delta = \frac{b\omega}{m(\omega_0^2 - \omega^2)}$$

natural frequency
e.g., $\sqrt{\frac{k}{m}}$ for mass on spring

Resonance



$$A = \frac{F_0}{m\sqrt{(\omega^2-\omega_0^2)^2 + b^2\omega^2/m^2}}$$

Term $\rightarrow 0$ when $\omega \rightarrow \omega_0$
Amplitude peaks

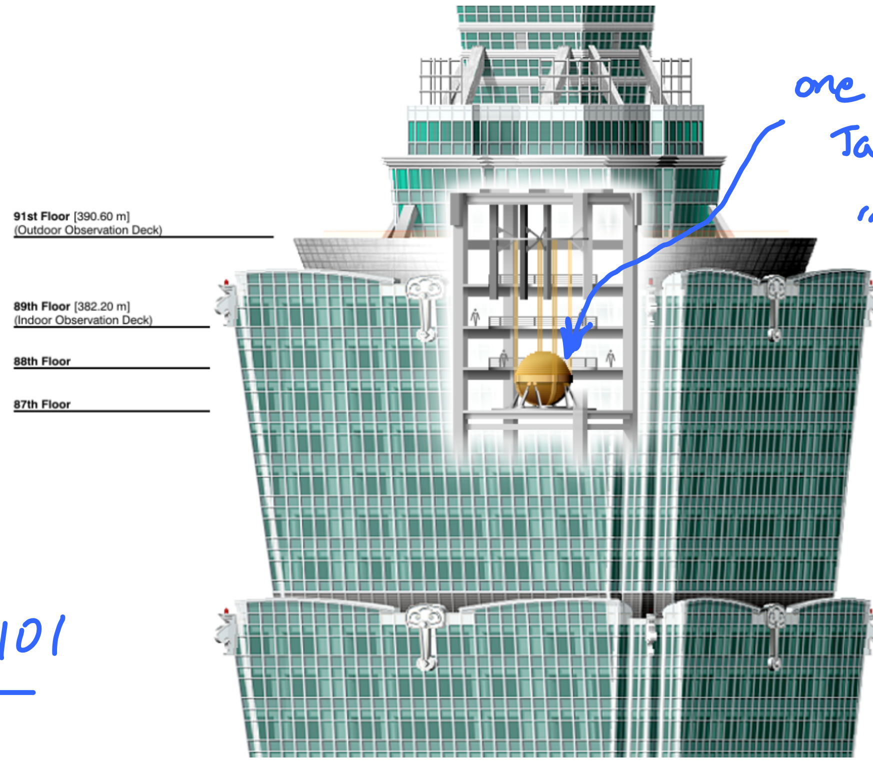
$\frac{\omega_0}{\Delta\omega} \equiv Q$ Quality factor

Measure of the width
or "peakiness"
of resonance

Forced oscillations Make us Happy!

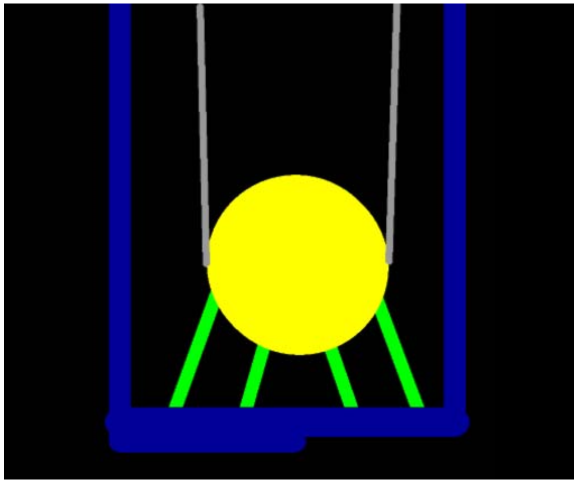
If they don't get too out of hand

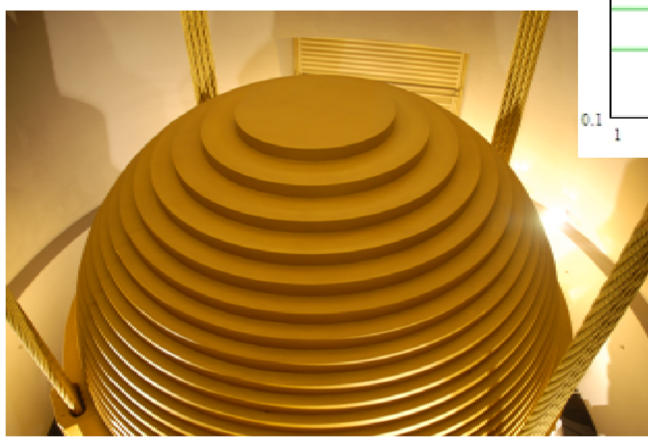
Taipei 101

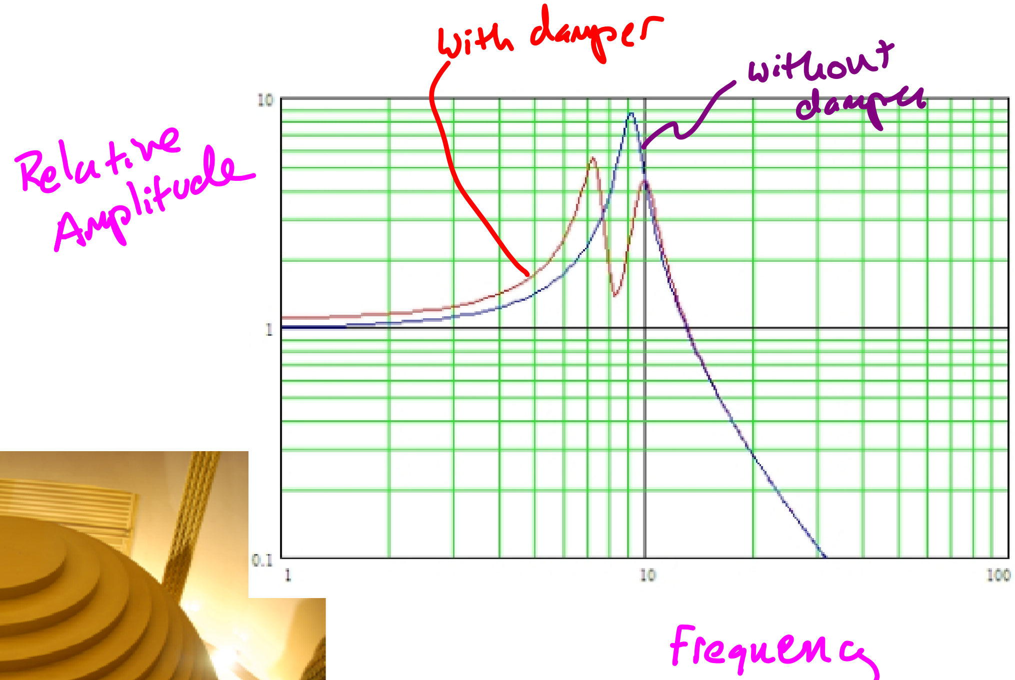


one of Taipei 101's "Tuned mass dampers"

730 tons



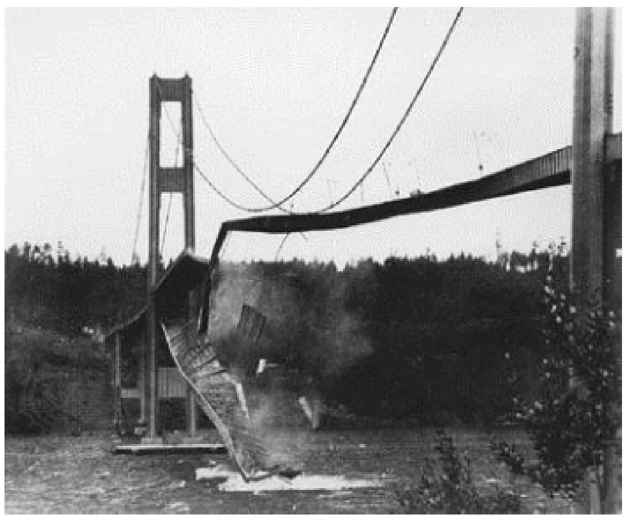




Illustrative plot
(not nec. Taipei 101)

Because when forced oscillations get out of hand

we get unhappy



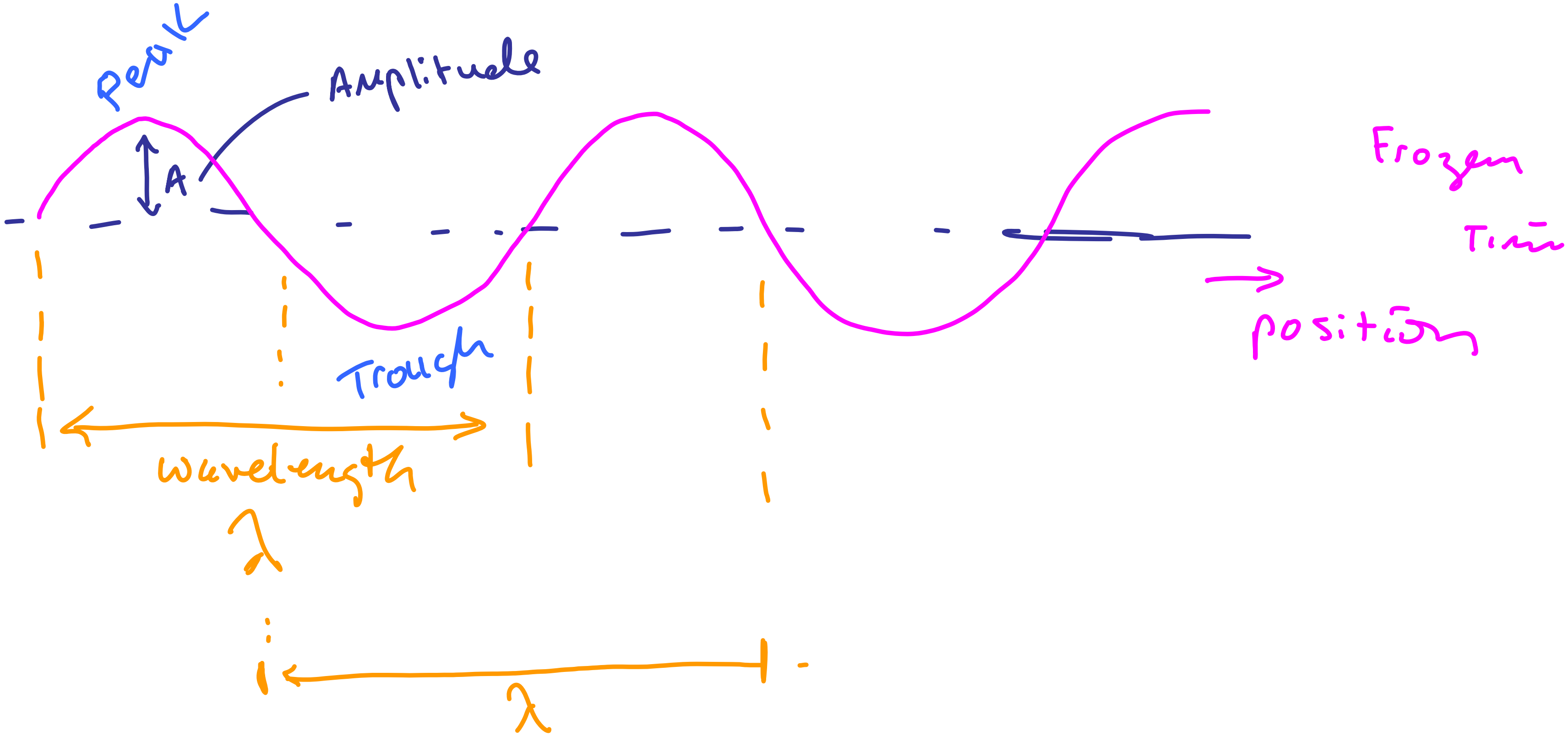
Tacoma Narrows

Oakland 1989
Loma Prieta Earthquake
Interstate 880

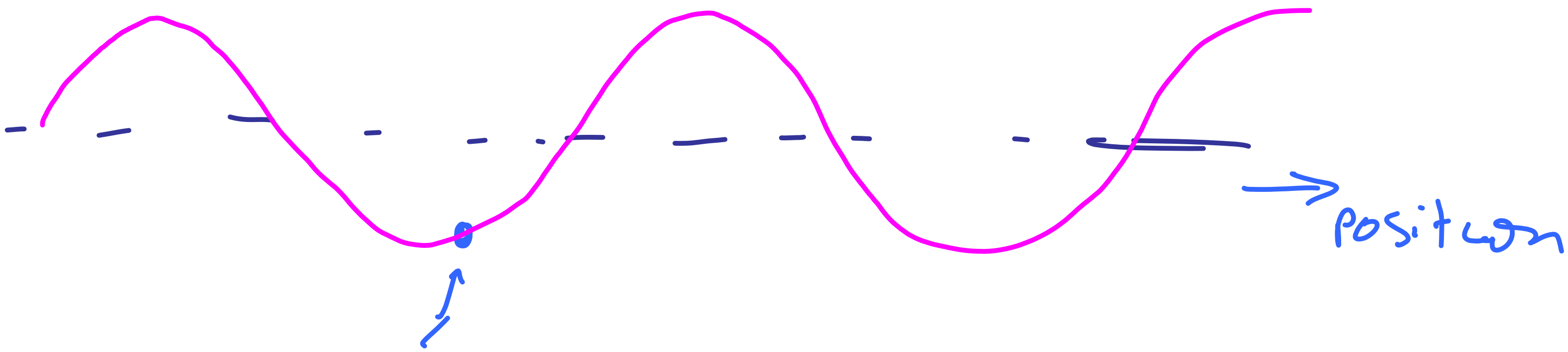
7.1

Be nice to your mechanical engineering friends

Waves



Peak
Amplitude
A
Trough
wavelength
λ
λ
Frozen
Time
position



T = time for element to execute one full cycle

frequency $= \frac{1}{T} = f$

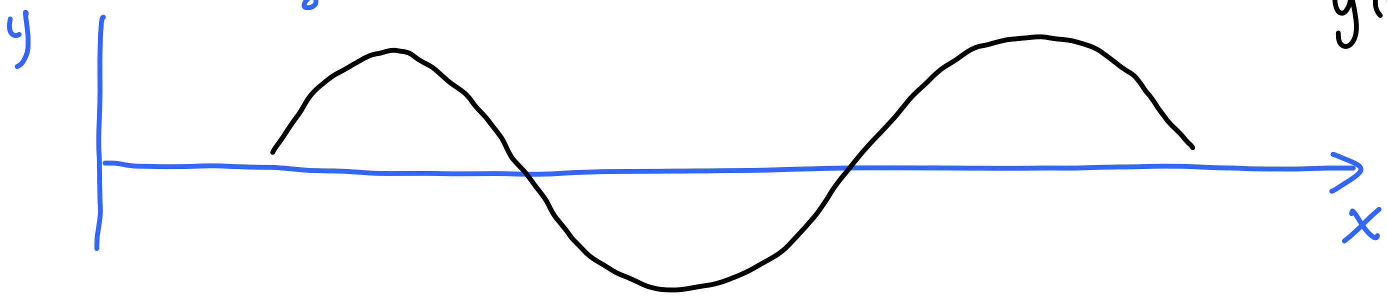
ν

$$v = \frac{\lambda}{T} = \lambda f = \lambda \nu$$

velocity $= \frac{\lambda}{T}$

v

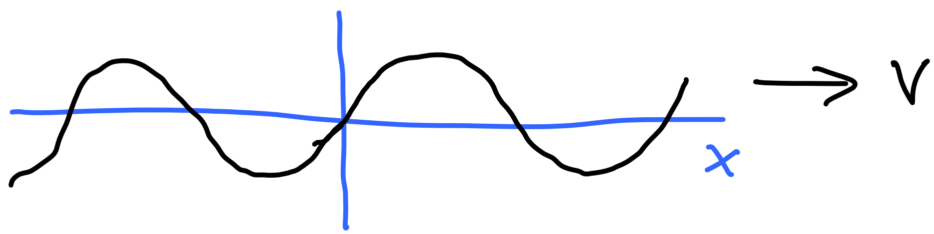
Freeze time



$$y(x) = A\,\mathrm{Sin}(kx)$$

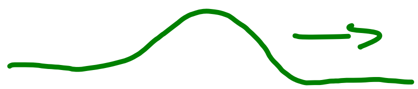
wave # $\equiv \frac{2\pi}{\lambda}$

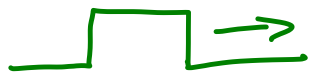
allow time to move, what is y at $x = 0$



$2\pi / T$

$$y \sim A\,\mathrm{Sin}(\omega t)$$





Can Have different Shapes

Fourier Analysis

const phase

$kx - \omega t = 0$

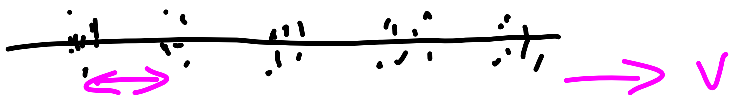
$x = \frac{\omega}{k} t$

$\frac{dx}{dt} = \frac{\omega}{k}$ $+$

Try $y(x,t) = A \sin(\underbrace{kx - \omega t}_{\text{fix this}})$ wave to right

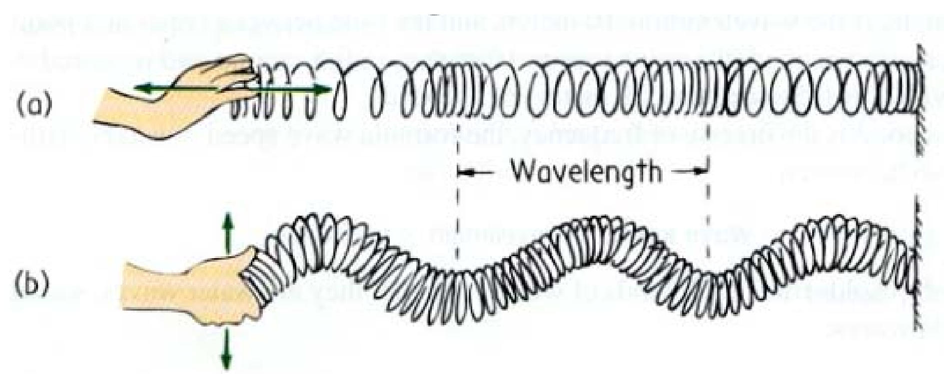
$y(x,t) = A \sin(kx + \omega t)$ wave to left

Transverse Wave

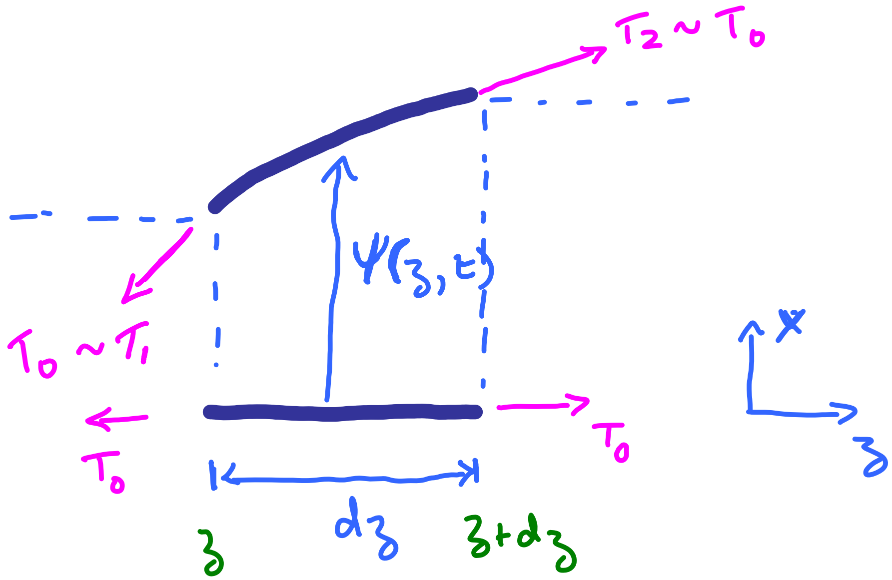


Longitudinal wave

Think of a wave on a "slinky"



1d wave equation



Wave on string

Small displacement

Continuous string

Transverse wave

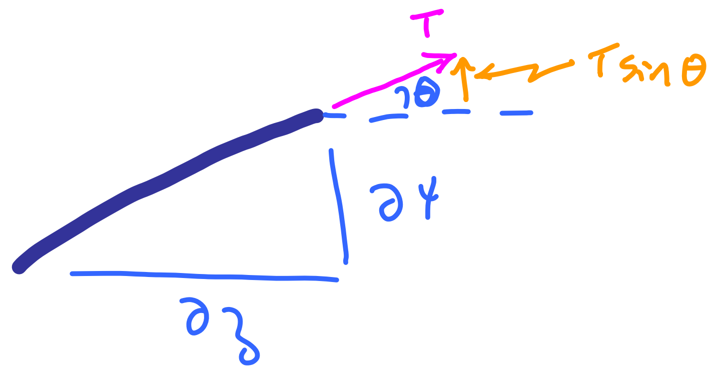
let $dM/dz = \mu$, linear mass density

μ uniform

Velocity at any point $= \dfrac{\partial \psi}{\partial t}$

Slope at any point $= \dfrac{\partial \psi}{\partial z}$

Do you guys Dig partial derivatives?



$$T\sin\theta \approx T\tan\theta \approx T\theta$$

$$T\frac{\partial\psi}{\partial z}$$

$$dF_x = T\frac{\partial\psi}{\partial z}\bigg|_{z+dz} - T\frac{\partial\psi}{\partial z}\bigg|_{z} = \frac{\partial}{\partial z}\left(T\frac{\partial\psi}{\partial z}\right)dz$$

$$dF = d(ma) = \mu\, dz\, \frac{\partial^2\psi}{\partial t^2}$$

$$\mu\, dz \frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial z}\left(T \frac{\partial \psi}{\partial z}\right) dz$$

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial z^2}} \quad \text{where } c = \sqrt{\frac{T}{\mu}}$$

1d wave equation

This is velocity of a wave on a string (see sect 15.2 in Giancoli)