Physics 123 - February 11, 2013

Workshop 2 Problems

In beta decay, a neutron decays into a proton and an electron. What kinetic energy would you expect the electron to have in such a decay (ignoring effects due to the surrounding nucleus)? In reality, the kinetic energy distribution is smeared out and has a maximal value which is what you calculate in the first part of this problem. Can you imagine a reason for this to be true?

initial STATE: A (nucleus A at rest) → Final STATE: ← B, e →

Relativistic Energy + Momentum cons.

$$P_A = P_B + P_e$$

$$-P_B = P_e - P_A$$

$$P_B^2 = (P_e - P_A)^2$$

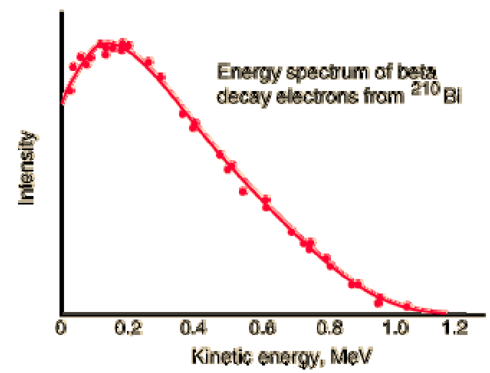
$$-m_B^2c^2 = P_e^2 + P_A^2 - 2P_eP_A$$

$$-m_B^2c^2 = -m_e^2c^2 - m_A^2c^2 - 2\frac{E_e}{c}\frac{E_A}{c} - 2\vec{P}_e\cdot\vec{P}_A$$

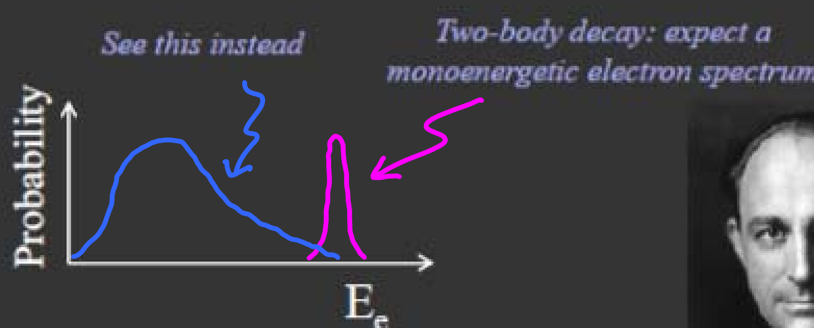
A at rest: m_Ac^2; 0, since $\vec{P}_A = 0$

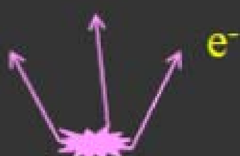
$$E_e = \frac{[m_e^2 + m_A^2 - m_B^2]c^2}{2m_A}$$

↑ Fixed # for nuclei A and B



"Neutron" proposed by Wolfgang Pauli in 1930 to explain electron spectrum in β-decay.





Renamed the "neutrino" by Enrico Fermi in 1933

$n \rightarrow p + e^- + ?$

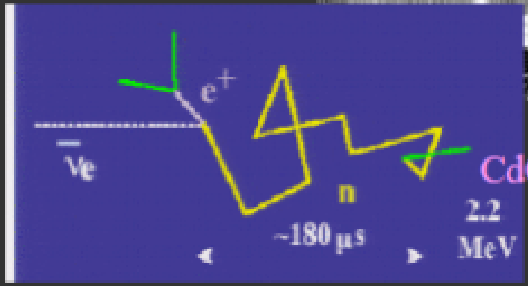
The *Elusive* Neutrino

Go big or go home!

- Reines and Cowan (1955)
  - Nobel Prize 1995
  - 1 ton detector
  - Neutrinos from a nuclear reactor $\bar{\nu} p \rightarrow e^+ n$

~10^{13} neutrinos $s^{-1}cm^{-1}$

Reines and Cowan at Savannah River

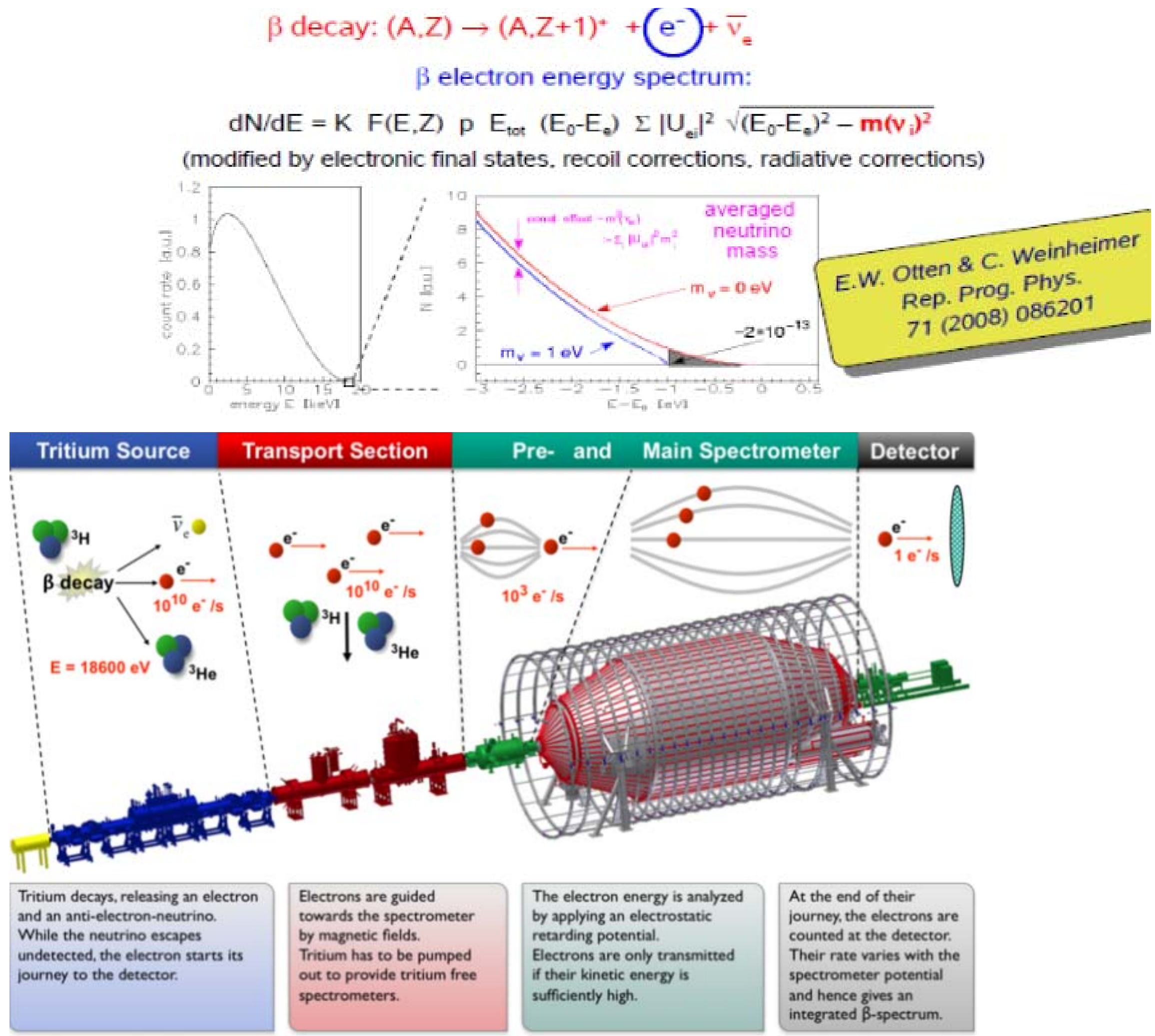


$CdCl_2$ in 200 liters water
+
scintillators

KATRIN Experiment

http://www.katrin.kit.edu/

Attempt to determine the neutrino mass by looking at modification of β-decay spectrum end point



But - if that's not exciting enough for you … check this out

Last Time

$$F = m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

b → Damping constant

$b\frac{dx}{dt}$: Damping term — why this form?

$$x(t) = A\, e^{-bt/2m} \cos(\omega' t + \varphi)$$

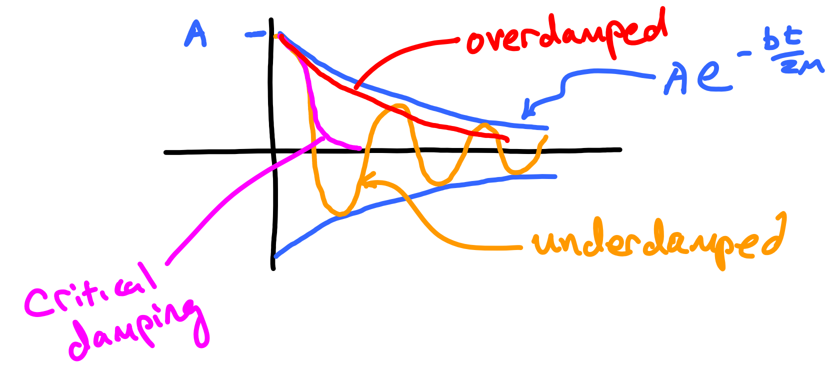
A: Think of this as time dependent Amplitude

$e^{-bt/2m}$: Exponential Damping depends on b

$\cos(\omega' t + \varphi)$: ↳ regular SHM

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

ω_0 = frequency in no Damping limit, $\omega_0^2 = \frac{k}{m}$



Forced oscillations

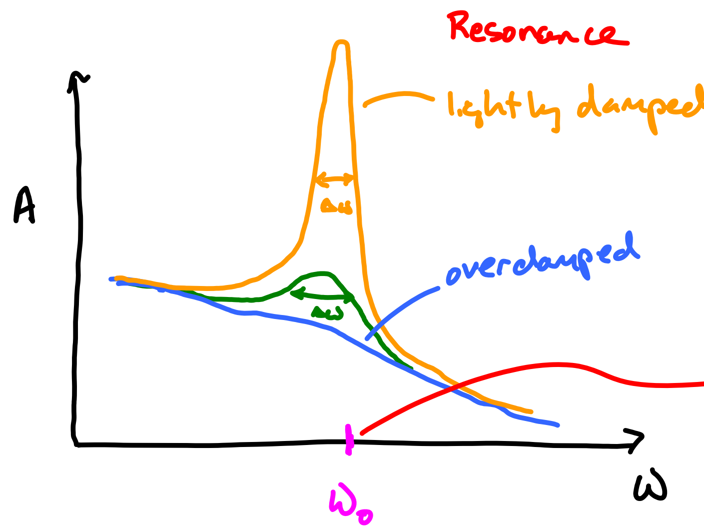
$$F = m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \cos \omega t$$

$+ F_0 \cos \omega t$ → driving force ... frequency ω

$-kx$ → if no damping Natural frequency is ω_0

$$x = A\cos(\omega t + \delta)$$

ω → ω of driving force



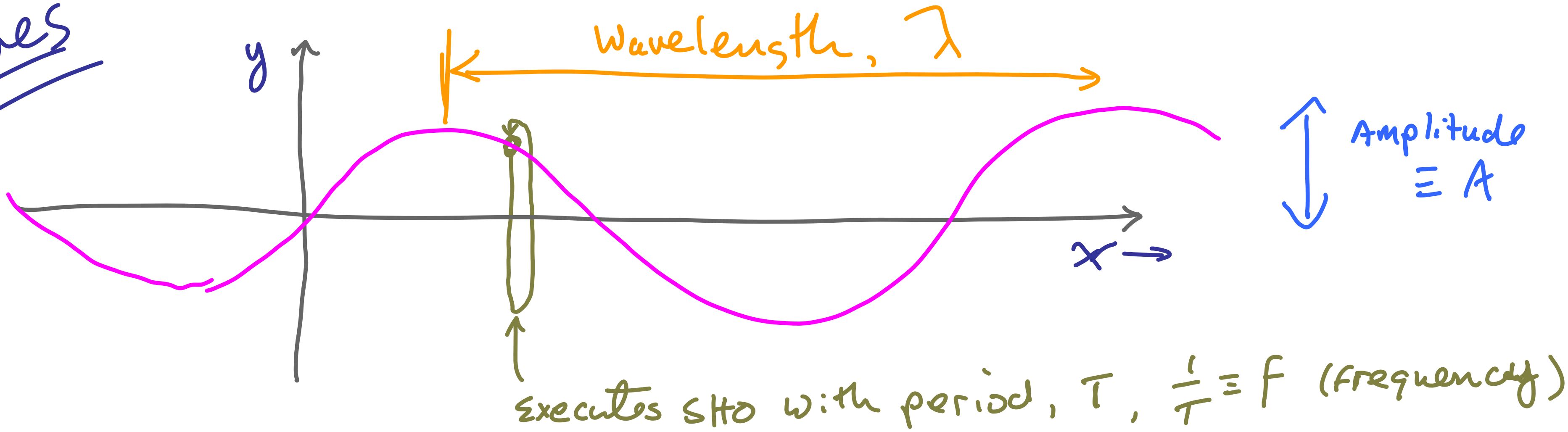
$$A = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + b^2\omega^2/m^2}}$$

Term → 0 when $\omega \to \omega_0$

Amplitude peaks

ω_0: natural frequency e.g., $\sqrt{\frac{k}{M}}$ for mass on spring

Waves



$$y(x,t) = A\sin(kx - \omega t + \varphi)$$ Wave traveling to +x right

(φ ← initial phase)

$$y(x,t) = A\sin(kx + \omega t + \varphi)$$ wave traveling to left toward $-x$

$k = 2\pi/\lambda$, $\quad \omega = \dfrac{2\pi}{T} = 2\pi f$

Look at constant phase $\quad kx - \omega t + \varphi = \text{const}$

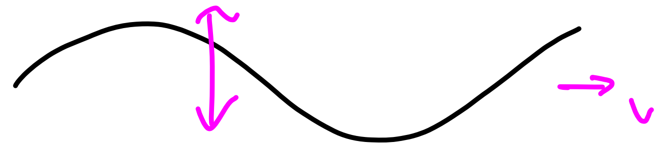
$$\frac{d}{dt}(kx - \omega t + \varphi) = 0$$

$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T} = v$$

$$v = \lambda f \quad \text{-or-} \quad v = \lambda \nu$$

Transverse waves



string

Longitudinal waves

sound in air or slinky

v depends on what is vibrating

e.g. For Transverse wave on String

$$v = \sqrt{\frac{T}{\mu}}$$

T: Tension ... just looks like Period ☺

μ: Mass/length

See Giancoli Sect 15.2 if interested

Won't derive ... but these are useful

longitudinal vibrations in Material (sound)

$$v = \sqrt{B/\rho}$$

B: Bulk Modulus

ρ: mass/vol

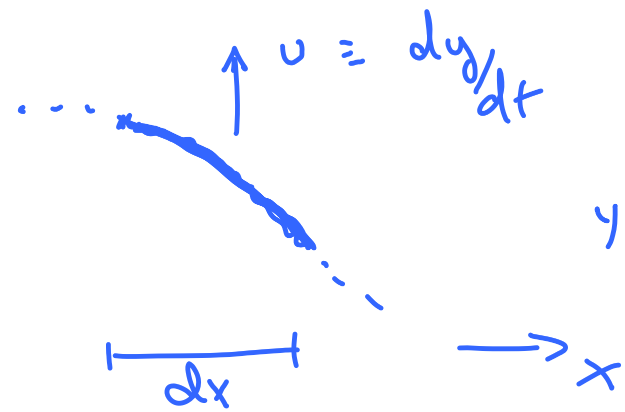
$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial z^2}$$

1d wave equation

Energy + Power in waves

$dm \to \mu\, dx$

$\mu = \frac{dm}{dx}$



$y = A\cos(kx - \omega t)$

$$dK = d(\text{kinetic energy})$$

$$dK = \frac{1}{2}\, dm\, u^2 \qquad u = \frac{dy}{dt} = -A\omega \sin(kx - \omega t)$$

$$dK = \frac{1}{2}\mu\, dx\, (-A\omega)^2 \sin^2(kx - \omega t)$$

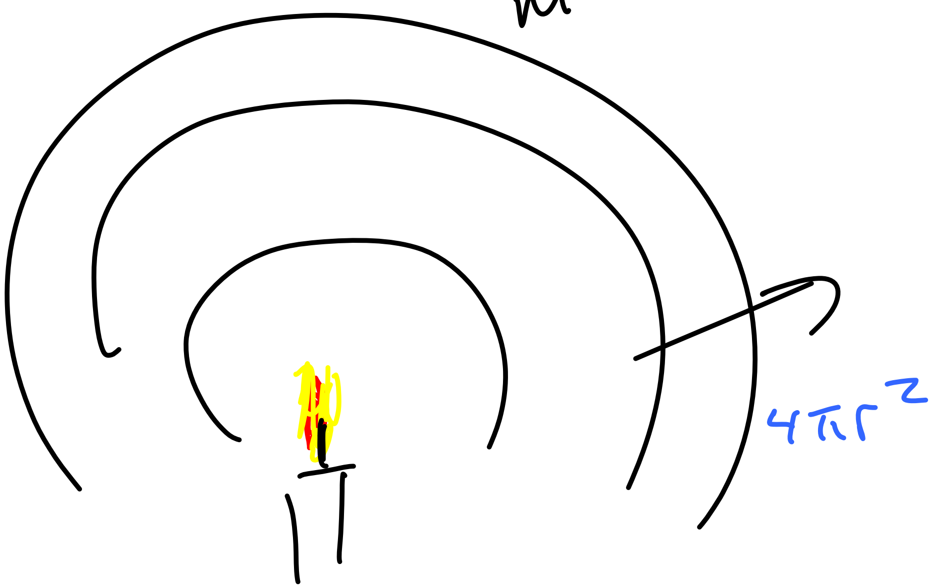
$$\frac{dK}{dt} = \frac{1}{2}\mu \underbrace{\frac{dx}{dt}}_{v} A^2\omega^2 \sin^2(kx - \omega t)$$

$$\overline{\sin^2(\ \ )} = \frac{1}{2}$$

$$\overline{\frac{dK}{dt}} = \frac{1}{4}\mu v A^2 \omega^2$$

$$\overline{\frac{dE}{dt}} = \frac{1}{2} \mu v A^2 \omega^2 \equiv \text{Average Power in Wave}$$

$$\overline{\text{Power}}/m^2 \equiv \text{Intensity of a Wave}$$



$$\text{Intensity} \sim \frac{\text{Power}}{m^2} \sim \frac{1}{r^2}$$

Principle of Superposition

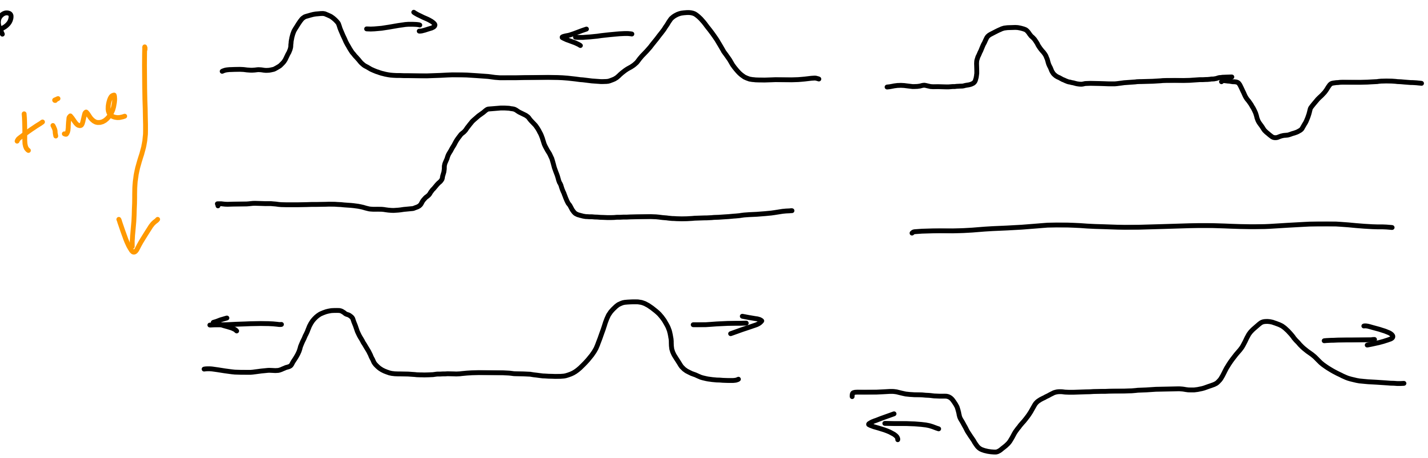
If ψ_1 and ψ_2 are solns of the wave equation, then $\psi_1 + \psi_2$ is a solution of the wave equation.

$$\frac{\partial^2 \psi_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2} \qquad \frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\left(\frac{\partial^2 \psi_1}{\partial t^2} + \frac{\partial^2 \psi_2}{\partial t^2}\right)$$

$$\frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 (\psi_1 + \psi_2)}{\partial t^2}$$

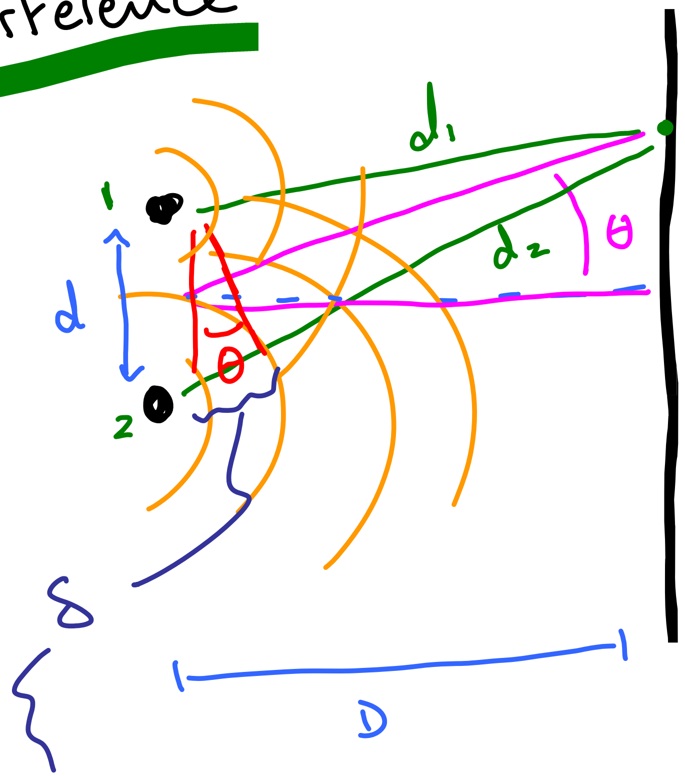
Interference



Constructive Interference

Destructive Interference

Interference



$\delta = d \sin\theta$

if $d_2 - d_1 = m\lambda$

↗ $0, 1, 2 \ldots$

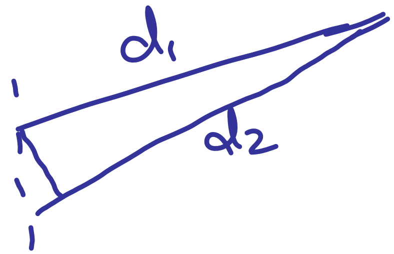
constructive interference

condition for waves adding together maximally

$d_2 = d_1 + \delta$

$$d \sin\theta = m\lambda$$

for destructive interference



$$d_2 - d_1 = \left(m + \tfrac{1}{2}\right)\lambda$$

$m = 0, 1, 2 \ldots$

For light this is usually called "Double-Slit interference"

Beats

Two waves
pass a fixed point
Differ Slightly in frequency

$$\psi_1(x,t) = A\sin(k_1x + \omega_1 t) = A\sin\omega_1 t$$

$$\psi_2(x,t) = A\sin(k_2x + \omega_2 t) = A\sin\omega_2 t$$

use superposition

$$\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$$

$$\psi(x,t) = A\sin(\omega_1 t) + A\sin(\omega_2 t)$$

use Trig identity

$$\sin A + \sin B = 2\sin\left[\tfrac{1}{2}(A+B)\right]\cos\left[\tfrac{1}{2}(A-B)\right]$$

$$\psi(x,t) = 2A\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t\right]\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$

Ave frequency

Difference in frequency

Amplitude

what you hear