Physics 123 – February 25, 2013

- Exam 1 - Thursday 8 am - B+L 106
- Calculator
- 1 side 8.5 x 11 inch sheet w/ formulas/notes
- Q + A session

~~Last Time~~

Fourier Analysis

$$\underbrace{f(x)}_{\text{periodic function}} = \frac{A_0}{2} + \sum_{m=1}^{\infty} A_m \cos(mkx) + \sum_{m=1}^{\infty} B_m \sin(mkx)$$

More detail means Higher frequency components Have more relative importance

as λ becomes large
Function becomes anharmonic (e.g. a wave pulse)

$$F(x) = \frac{1}{\pi}\left[\int_0^{\infty} A(k)\cos(kx)\,dx + \int_0^{\infty} B(k)\sin(kx)\,dx\right]$$

Fourier integrals

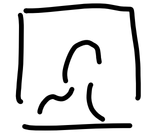
with

$$A(k) = \int_{-\infty}^{\infty} F(x)\cos(kx)\,dx \qquad B(k) = \int_{-\infty}^{\infty} F(x)\sin(kx)\,dx$$

cosine transform   Sine Transform

Describe $F(x)$ in frequency space k

$F(t)$ ω



Narrower functions / sharper details in position or time

require broader functions in k or ω

Very powerful – will see this again + again ... Data compression for Example

Expect you to do a basic Fourier problem

and just be aware of integral + Transforms for now

Essence of the uncertainty principle in Quantum Mechanics

⋮

Electromagnetic Waves

Review ...

Derivation of EM plane waves

Believe!!

Will go thru fast

Want you to see it and

Not an E+M course → you're responsible to understand / use end result

Buckle you seat belts

Note Title 11/18/2008

Integral form of Maxwell's equations

Gauss $\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

No magnetic monopole $\oint_S \vec{B}\cdot d\vec{A} = 0$

Ampère $\oint_c \vec{B}\cdot d\vec{l} = \mu_0 \int_s \vec{J}\cdot d\vec{A} + \boxed{\mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}}$

new term – "Maxwell's Displacement current"

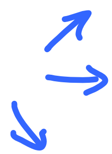
Faraday $\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$

Will now derive "differential" form ...

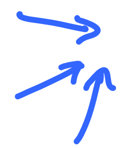
Divergence of vector field $\vec{V}$ (cartesian coordinates)

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

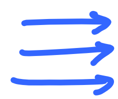
measures divergence or convergence of field



+ Divergence



− Divergence



0 Divergence

Degree of "Divergence" or "convergence"

Can prove to yourself

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right)$$

use $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$

vector calculus

Gauss' Theorem
Green's theorem
divergence theorem

$$\rightarrow \int_{Vol} (\vec{\nabla}\cdot\vec{V})\,dv = \int_{Surf} \vec{V}\cdot d\vec{A}$$

for vector field $\vec{V}$

Gauss

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow \int_S \vec{E}\cdot d\vec{A} = \int_V (\vec{\nabla}\cdot\vec{E})\,dv = \frac{\int_V \rho\,dv}{\epsilon_0}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$$

No magnetic monopole

$$\oint_S \vec{B}\cdot d\vec{A} = 0 \longrightarrow \int_S \vec{B}\cdot d\vec{A} = \int_V (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

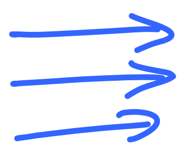
Curl of vector field

$$\text{curl } \vec{V} \equiv \vec{\nabla} \times \vec{V} \longrightarrow \text{Vector}$$

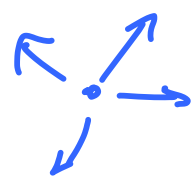
Cartesian

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

large curl



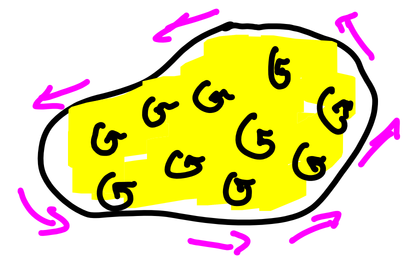
0 curl



degree of "circulation" of vector field about point

Stoke's Theorem

$$\oint_C \vec{v} \cdot d\vec{l} = \oint_S (\vec{\nabla} \times \vec{v}) \cdot d\vec{A}$$



Faraday (induction)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt} = -\frac{d\oint_S \vec{B} \cdot dA}{dt}$$

$$\oint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{A} = -\frac{d\oint_S \vec{B} \cdot d\vec{A}}{dt}$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}}$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \oint_S \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\oint_S \vec{E}\cdot d\vec{A}$$

Ampere's law plus Maxwell's displacement current

$$\oint_S (\vec{\nabla}\times\vec{B})\cdot d\vec{A} = \mu_0 \oint_S \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\oint_S \vec{E}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{j} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\vec{E}}$$

Note Title

Summary

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \longrightarrow \int (\vec{\nabla}\cdot\vec{E})\, d = \int \rho\, dv \longrightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho}$$

$$\oint_S \vec{B}\cdot d\vec{A} = 0 \longrightarrow \int (\vec{\nabla}\cdot\vec{B})\, dv = 0 \longrightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$$\longrightarrow \int_S (\vec{\nabla}\times\vec{B})\cdot dA = \mu_0 \int_S j\cdot dA + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$$\longrightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

$$\longrightarrow \int_S (\vec{\nabla}\times\vec{E})\cdot dA = -\frac{\partial}{\partial t}\int B\cdot dA \longrightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

Differential form of Maxwell's eqns

region w/ NO current

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Take curl of Both sides

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 \epsilon_0 \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t}$$

and it was
Said . . .

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla}(\cancel{\vec{\nabla} \cdot \vec{B}}) - \underbrace{\vec{\nabla} \cdot (\vec{\nabla} \vec{B})}_{\text{Known as the Laplacian } \nabla^2 \vec{B}}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Laplacian of Scalar field T

$$\nabla^2 T = \vec{\nabla}\cdot(\vec{\nabla}T) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

Laplacian of vector field $\vec{v}$

$$\nabla^2 \vec{v} = (\nabla^2 v_x)\hat{i} + (\nabla^2 v_y)\hat{j} + (\nabla^2 v_z)\hat{k}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\epsilon_0 \frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$$

from another Maxwell eqn

$$\vec{\nabla}\cdot(\vec{\nabla}\vec{B}) =$$

$$\vec{\nabla}^2\vec{B} = \mu_0\epsilon_0 \frac{\partial}{\partial t}(\nabla\times E) = \mu_0\epsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\nabla^2\vec{B} = -\mu_0\epsilon_0 \frac{\partial^2\vec{B}}{\partial t^2}$$

$$\nabla^2 B_x = -\mu_0\epsilon_0 \frac{\partial^2 B_x}{\partial t^2}$$

$$\nabla^2 B_y = -\mu_0\epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

$$\nabla^2 B_z = -\mu_0\epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\epsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$$

from another Maxwell eqn

$$\vec{\nabla}\cdot(\vec{\nabla}\vec{B}) =$$

$$\vec{\nabla}^2\vec{B} = \mu_0\epsilon_0\frac{\partial(\nabla\times E)}{\partial t} = \mu_0\epsilon_0\frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\nabla^2\vec{B} = -\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\nabla^2 B_x = -\mu_0\epsilon_0\frac{\partial^2 B_x}{\partial t}$$

$$\nabla^2 B_y = -\mu_0\epsilon_0\frac{\partial^2 B_y}{\partial t^2}$$

$$\nabla^2 B_z = -\mu_0\epsilon_0\frac{\partial^2 B_z}{\partial t^2}$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = -\mu_0 \epsilon_0 \frac{\partial B_x}{\partial t^2}$$

Similarly

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \Big\downarrow \nabla \times$$

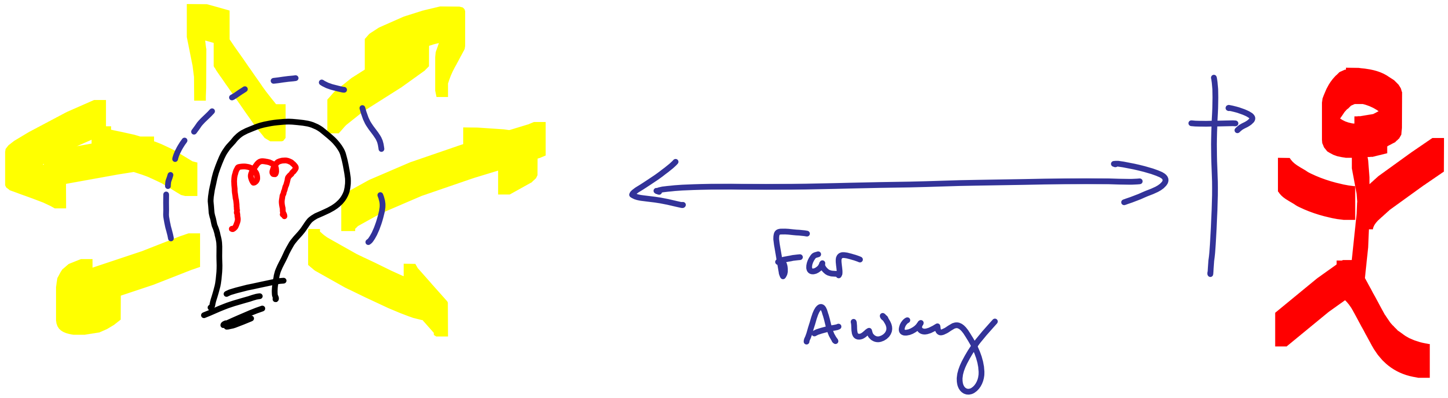
$$\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

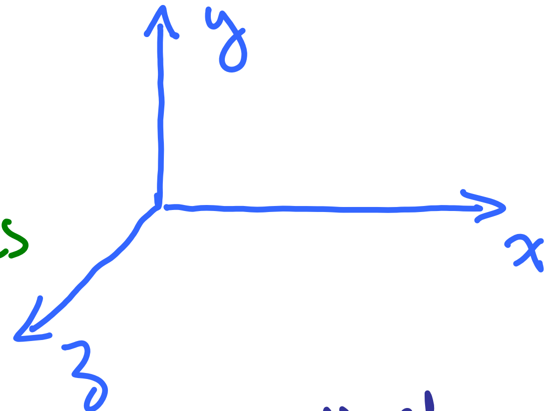
1d wave prop in x

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

wave eqns for $\vec{E}, \vec{B}$
w/ velocity of
Propagation

$$\sqrt{\frac{1}{\mu_0 \epsilon_0}} = c$$





Impose Boundary Conditions

$$\vec{E} = \vec{E}(x,t)$$

No charge $\rightarrow$ $\rho = 0$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \frac{\partial E_x}{\partial x} + \underbrace{\cancel{\frac{\partial E_y}{\partial y}}}_{0} + \underbrace{\cancel{\frac{\partial E_z}{\partial z}}}_{0} = 0$$

So E_x is const for all x $\rightarrow$ $E_x = 0$

E_y or E_z might be nonzero

■ E is Transverse to Direction of Propagation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Equivalent to this

$E = E(x,t)$ only

$$\underbrace{\frac{\partial E_z}{\partial y}}_{0} - \underbrace{\frac{\partial E_y}{\partial z}}_{0} = -\frac{\partial B_x}{\partial t}$$

$$-\left(\underbrace{\cancel{\frac{\partial E_z}{\partial x}}}_{0} - \underbrace{\frac{\partial E_x}{\partial z}}_{0}\right) = -\frac{\partial B_y}{\partial t}$$

$$\left(\frac{\partial E_y}{\partial x} - \underbrace{\frac{\partial E_x}{\partial y}}_{0}\right) = -\frac{\partial B_z}{\partial t}$$

Impose Boundary Condition

$\vec{E} = E(x,t)\,\hat{j}$

"polarization"

B_x is constant in time

B_y is constant in time

Time dependent B field

$\perp$ to $\vec{E}$

$\vec{E}, \vec{B}$ are Transverse and mutually $\perp$

This says:
for P123, know how to do
and use below. above is so
you have seen it before

$$E_y(x,t) = E_{oy} \cos(kx - \omega t + \phi)$$

k: $\frac{2\pi}{\lambda}$

ω: $\frac{2\pi}{T}$

ϕ: Phase (initial condition)

$$B_z = -\int \frac{\partial E_y}{\partial x}\, dt$$

set $\phi = 0$ for now

$$B_z = \int k E_{oy} \sin(kx - \omega t)\, dt$$

$$B_z = \frac{k}{\omega} E_{oy} \cos(kx - \omega t)$$

$$\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{\frac{2\pi}{T}} = \frac{T}{\lambda} = \frac{1}{c}$$

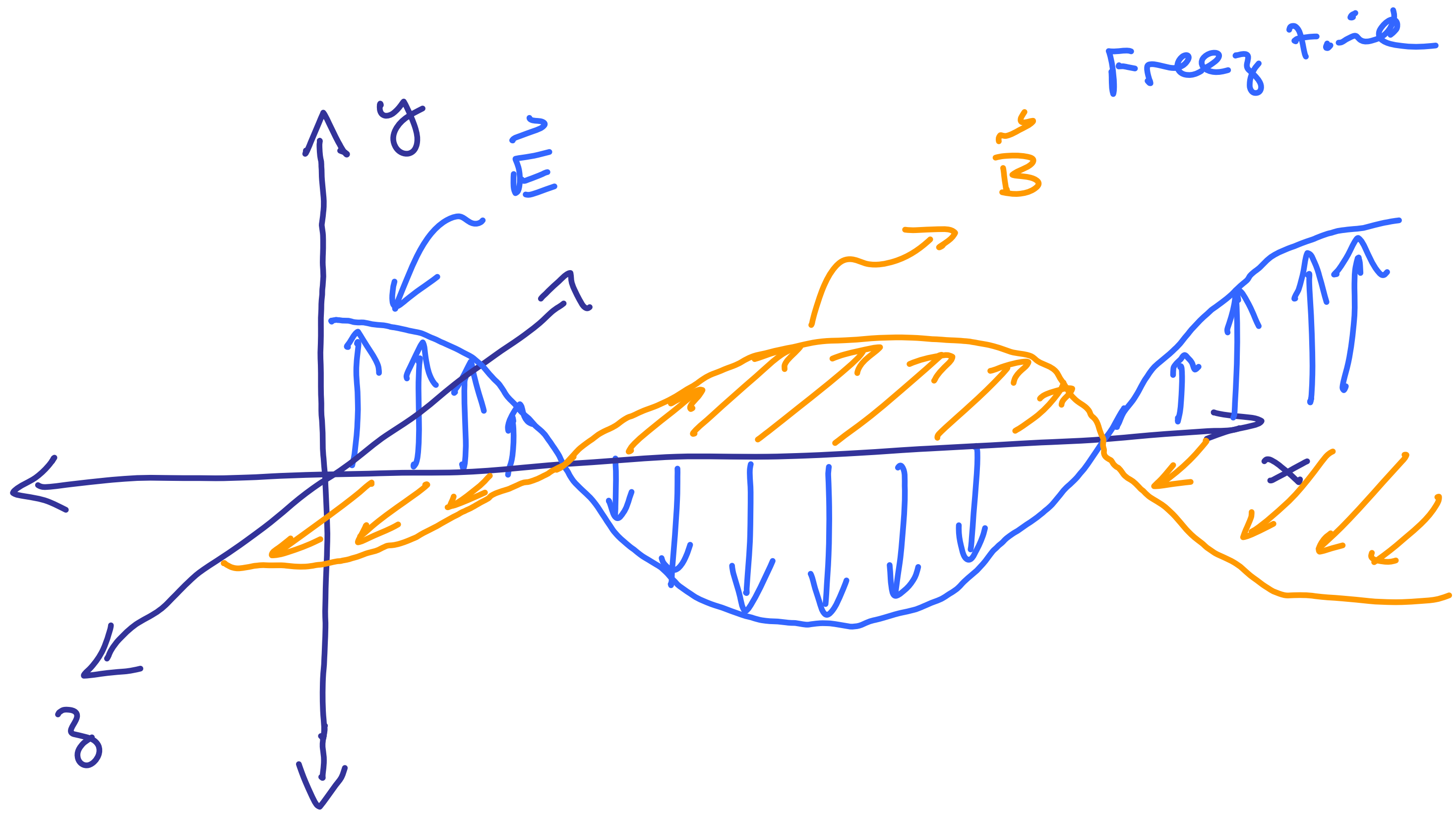
$$B_z = \frac{1}{c} E_y$$

E, B are coupled, time dependent

Mutually $\perp$

in phase in Time + space

$$|\vec{E}| = c\,|\vec{B}|$$



Freeze time
y
$\vec{E}$
$\vec{B}$
x
z