Physics 123 - March 20, 2013

last time

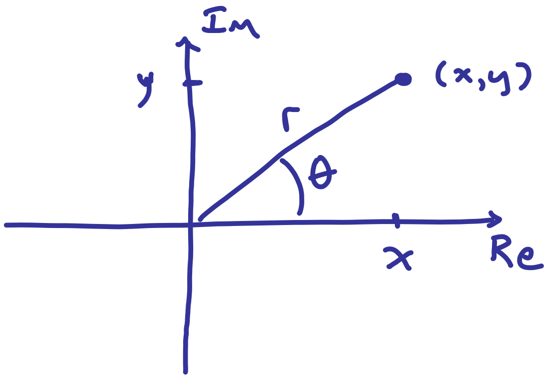
Physical Optics

Huygens-Fresnel principle

we'll use this as we go . . .

Every unobstructed point in a wavefront, at a given instant in time, acts as a source of secondary spherical wavelets

Complex representation of waves



$$z = x + iy$$

$$z^* = x - iy$$

$$z = re^{i\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\mathrm{Re}(z) = r\cos\theta$$

$$\mathrm{Im}(z) = r\sin\theta$$

$$\psi(x,t) = A\cos(kx - \omega t) = \mathrm{Re}\left[Ae^{i(kx-\omega t)}\right]$$

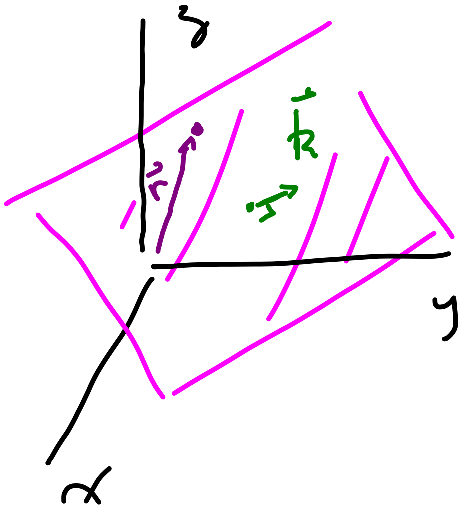
understood . . . ☺

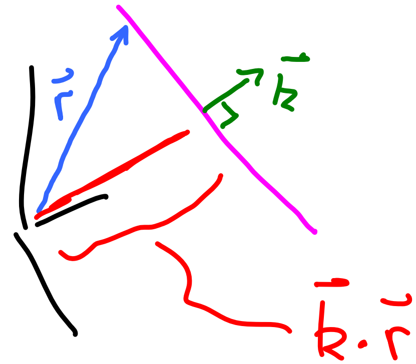
$e^{i(kx-\omega t)}$ is a plane wave moving in +x direction

$e^{i(kx+\omega t)}$ " " " " -x "

$e^{i(ky-\omega t)}$ " " " " +y "

$\vdots$





ψ moving in $\vec{k}$ direction

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

$$\psi(\vec{r}) = A\cos(\vec{k}\cdot\vec{r} - \omega t)$$

general plane wave $\quad \psi(\vec{r}) = A\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

$$\psi = e^{i(\vec{k}\cdot\vec{r} - \omega t)} = e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = -k_x^2 \psi$$

$$\frac{\partial^2 \psi}{\partial y^2} = -k_y^2 \psi$$

$$\frac{\partial^2 \psi}{\partial z^2} = -k_z^2 \psi$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$$

$$\underbrace{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}}_{} = -k^2 \psi$$

Laplacian

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 \psi = -k^2 \psi = \frac{-k^2}{-\omega^2} \frac{\partial^2 \psi}{\partial t^2}$$

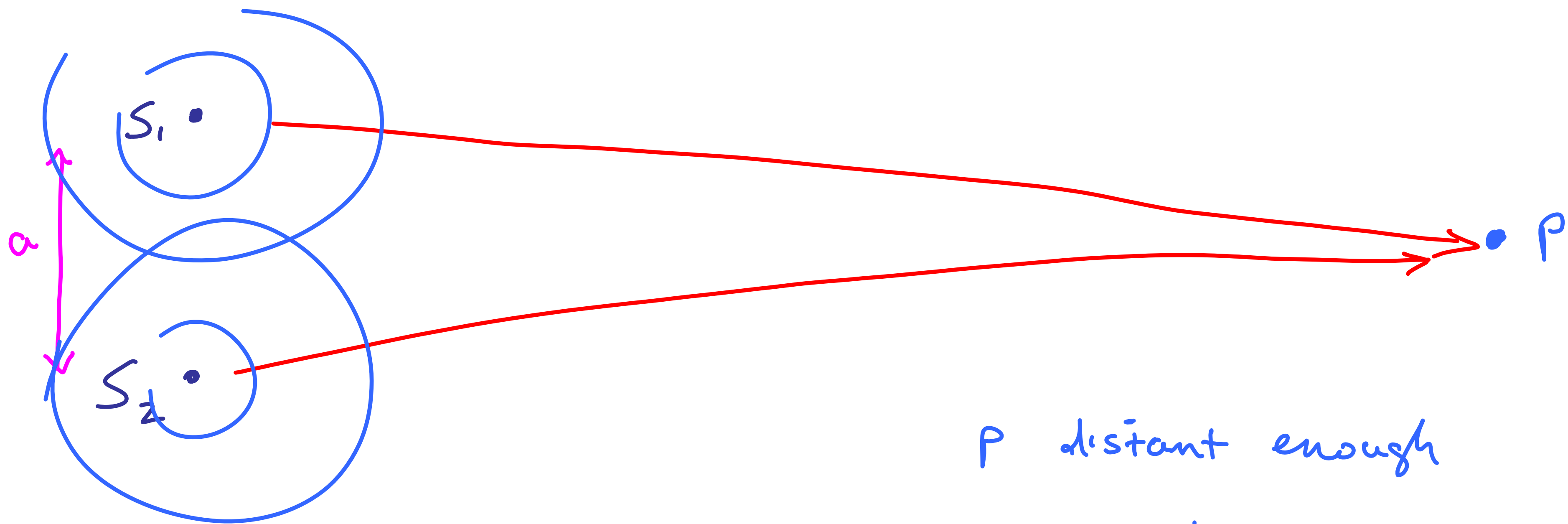
$$\nabla^2 \psi = \frac{k^2}{\omega^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\frac{\frac{2\pi}{\lambda}}{\frac{2\pi}{T}}$$

$$\frac{1}{v^2}$$

$$\boxed{\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}}$$

3d wave equation



P distant enough

use plane wave approx.

2 pt sources

monochromatic

$$\vec{E}_1(\vec{r},t) = \vec{A}_1 e^{i(\vec{k}_1\cdot\vec{r} - \omega t + \varphi_1)}$$

$$\vec{E}_2(\vec{r},t) = \vec{A}_2 e^{i(\vec{k}_2\cdot\vec{r} - \omega t + \varphi_2)}$$

$$I_p \sim \langle E^2 \rangle$$

$$= \langle (\vec{E}_1 + \vec{E}_2)^* \cdot (\vec{E}_1 + \vec{E}_2) \rangle$$

$$= \langle \vec{E}_1^* \cdot \vec{E}_1 + \vec{E}_2^* \cdot \vec{E}_2 + \underbrace{\vec{E}_1^* \cdot \vec{E}_2 + \vec{E}_1 \cdot \vec{E}_2^*} \rangle$$

$$= \langle A_1^2 + A_2^2 + \ldots \rangle$$

$$\vec{E}_1^* \cdot \vec{E}_2$$

$$\underbrace{\vec{A}_1 \cdot \vec{A}_2}_{A_1 A_2} \, e^{-i(\vec{k}_1 \cdot \vec{r} - \omega t + \phi_1)} \, e^{i(\vec{k}_2 \cdot r - \omega t + \phi_2)}$$

$A_1 A_2$ because P distance >> a

$$\vec{E}_1^* \cdot \vec{E}_2 = A_1 A_2 \, e^{i(\vec{k}_2 \cdot \vec{r} - \vec{k}_1 \cdot \vec{r} + \phi_2 - \phi_1)}$$

recall $\cos\phi = \dfrac{e^{i\phi} - e^{-i\phi}}{2}$

$$I_P \sim A_1^2 + A_2^2 + 2 A_1 A_2 \cos\delta$$

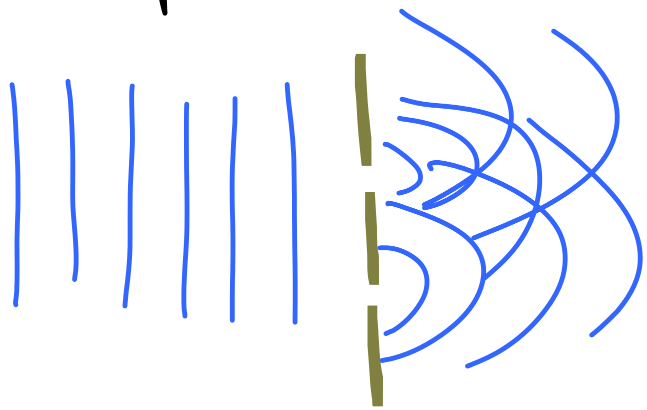
$$\delta = \vec{k}_2 \cdot \vec{r} - \vec{k}_1 \cdot \vec{r} + \phi_2 - \phi_1$$

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

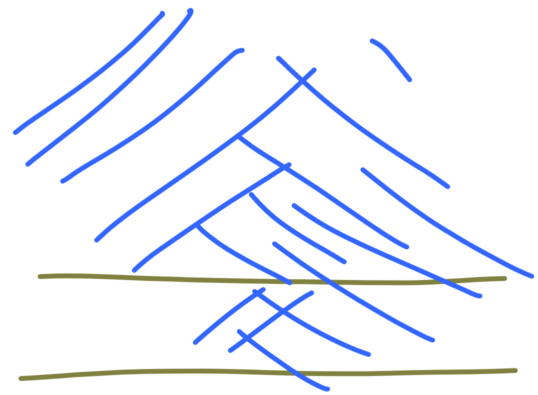
I_p Max if $\cos\delta = +1$ $\qquad \delta = 0, \pm 2\pi, \pm 4\pi \cdots$

I_p Min if $\cos\delta = -1$ $\qquad \delta = \pm\pi, \pm 3\pi, \cdots$

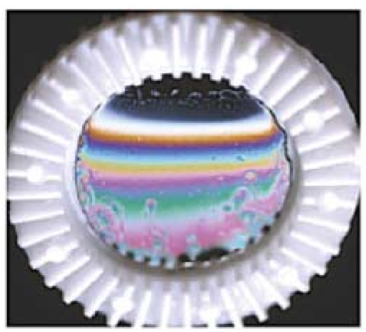
Wavefront splitting

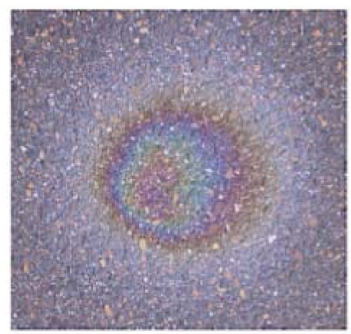


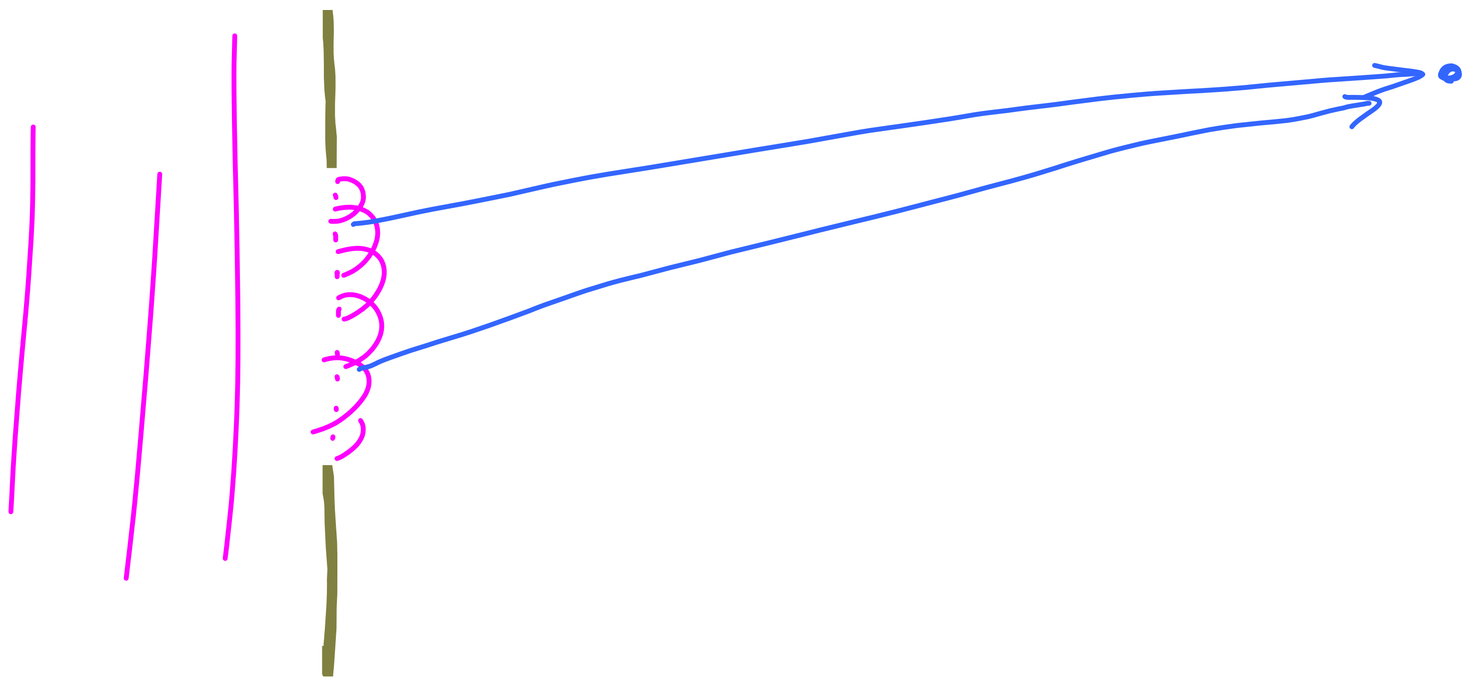
Amplitude splitting

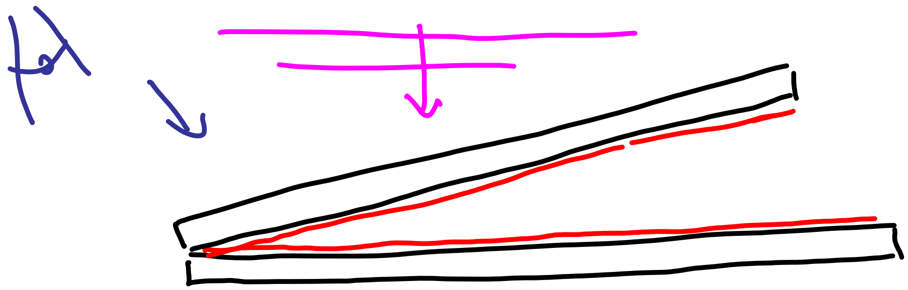


Thin film



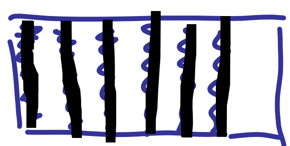


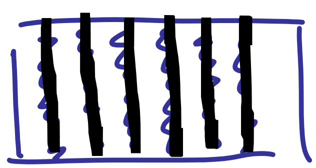


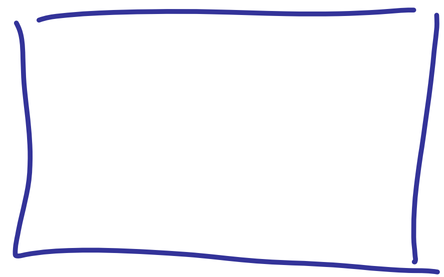


1) All bright

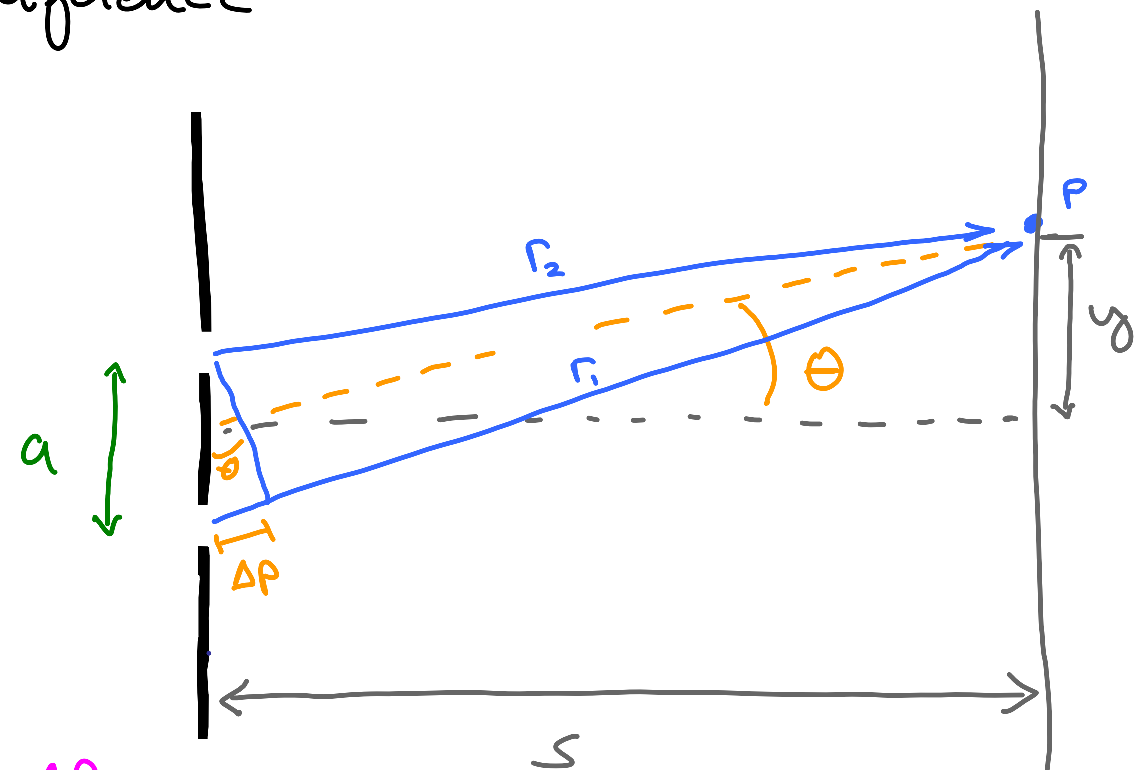
2) All dark

3) 

4) 



Double Slit interference



$y = s\theta$

$\theta = \frac{y}{s}$

$\theta \sim \sin\theta = \frac{\Delta p}{a}$

$\frac{y}{s} = \frac{\Delta p}{a}$ $\qquad$ $\Delta p = \frac{ay}{s}$

$\Delta p = m\lambda \quad m = 0, 1, 2 \ldots$ constr. int

$\frac{ay}{s} = m\lambda \quad \boxed{a \sin\theta = m\lambda}$ Maxima

$\Delta p = (m + \frac{1}{2})\lambda \quad m = 0, 1, 2 \ldots$ destr. int

$$\boxed{a \sin\theta = (m + \tfrac{1}{2})\lambda}$$

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\delta$$

$$I_1 = I_2 = I_0 \qquad \delta = k(r_1 - r_2)$$

$$I_p = 2I_0 + 2I_0 \cos k(r_1 - r_2)$$

$$k = \frac{2\pi}{\lambda}$$

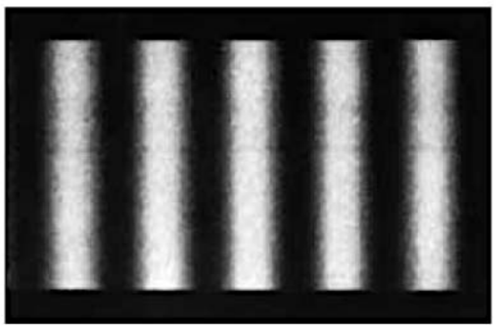
$$r_1 - r_2 = \Delta p = a\frac{y}{s}$$

$$I_p = 2I_0\left(1 + \cos k(r_1 - r_2)\right)$$

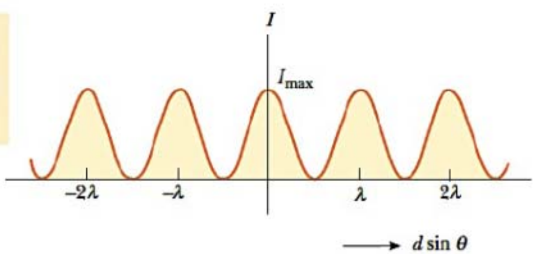
$$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$$

$$I_p = 4 I_0 \cos^2 \frac{k(r_1 - r_2)}{2}$$

$$I_p = 4 I \cos^2\left(\frac{\pi}{\lambda}\frac{ay}{s}\right)$$



$$I = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

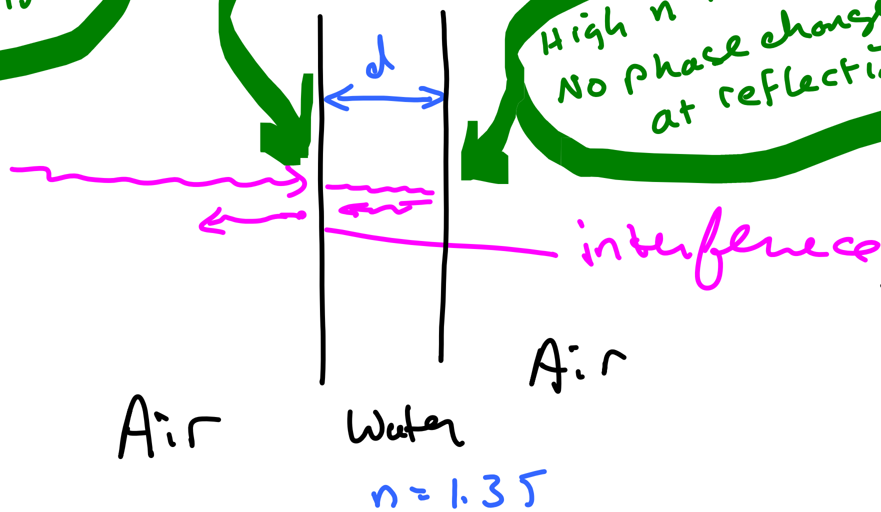


example

what is the smallest thickness of a soap bubble that does not reflect blue light?

Low n to high n
180° phase change
at reflection

High n to low n
No phase change
at reflection



$\lambda_{blue\,vac} = 450\text{ nm}$

$$\lambda_n = \frac{\lambda}{n} = \frac{450\text{nm}}{1.35} = 333\text{nm}$$

$$2d = \lambda_n$$

$$d = \frac{\lambda_n}{2} = 167\text{ nm}$$