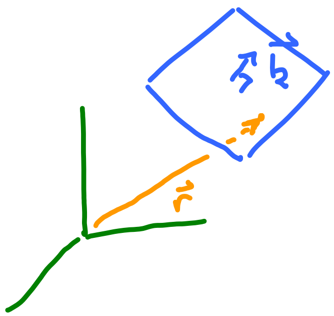
Physics 123 - March 25, 2013

Last Time



Plane wave in 3-D

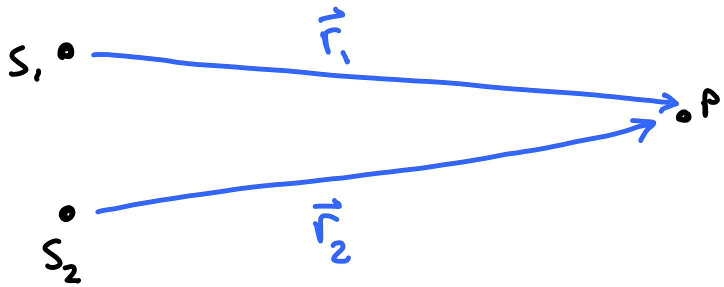
$$\psi(\vec{r},t) = e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

Satisfies 3D Wave equation

$$\nabla^2 \psi(\vec{r},t) = \frac{1}{v^2}\frac{\partial^2 \psi(\vec{r},t)}{\partial t^2}$$

Interference



distance large (plane waves okay)

$$I_p \sim A_1^2 + A_2^2 + 2A_1A_2\cos\delta$$

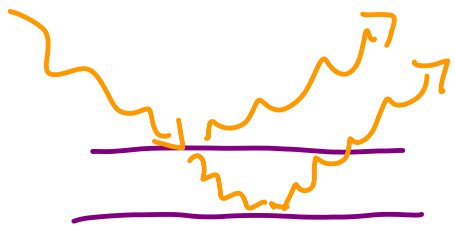
$$\delta = \vec{k}_2\cdot\vec{r} - \vec{k}_1\cdot\vec{r} + \varphi_2 - \varphi_1$$

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\delta$$

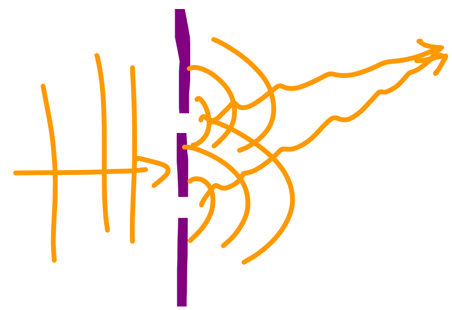
($\cos\delta$) ranges bet. ± 1

Interference Stable when Sources are coherent

→ Same frequency, CONSTANT phase Difference



Amplitude splitting



wavefront Splitting

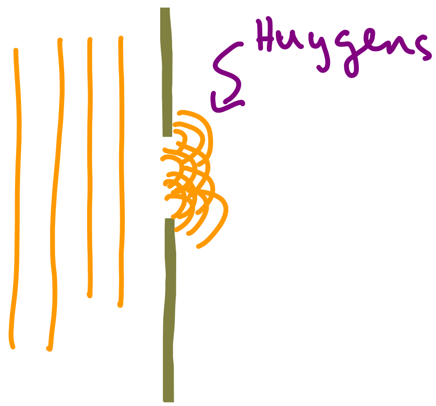
looked at

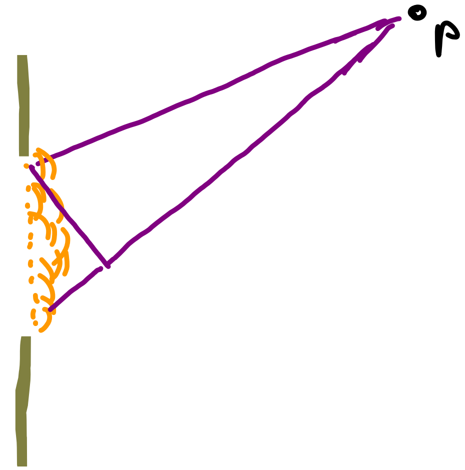
Thin film interference

won't repeat here

Double slit

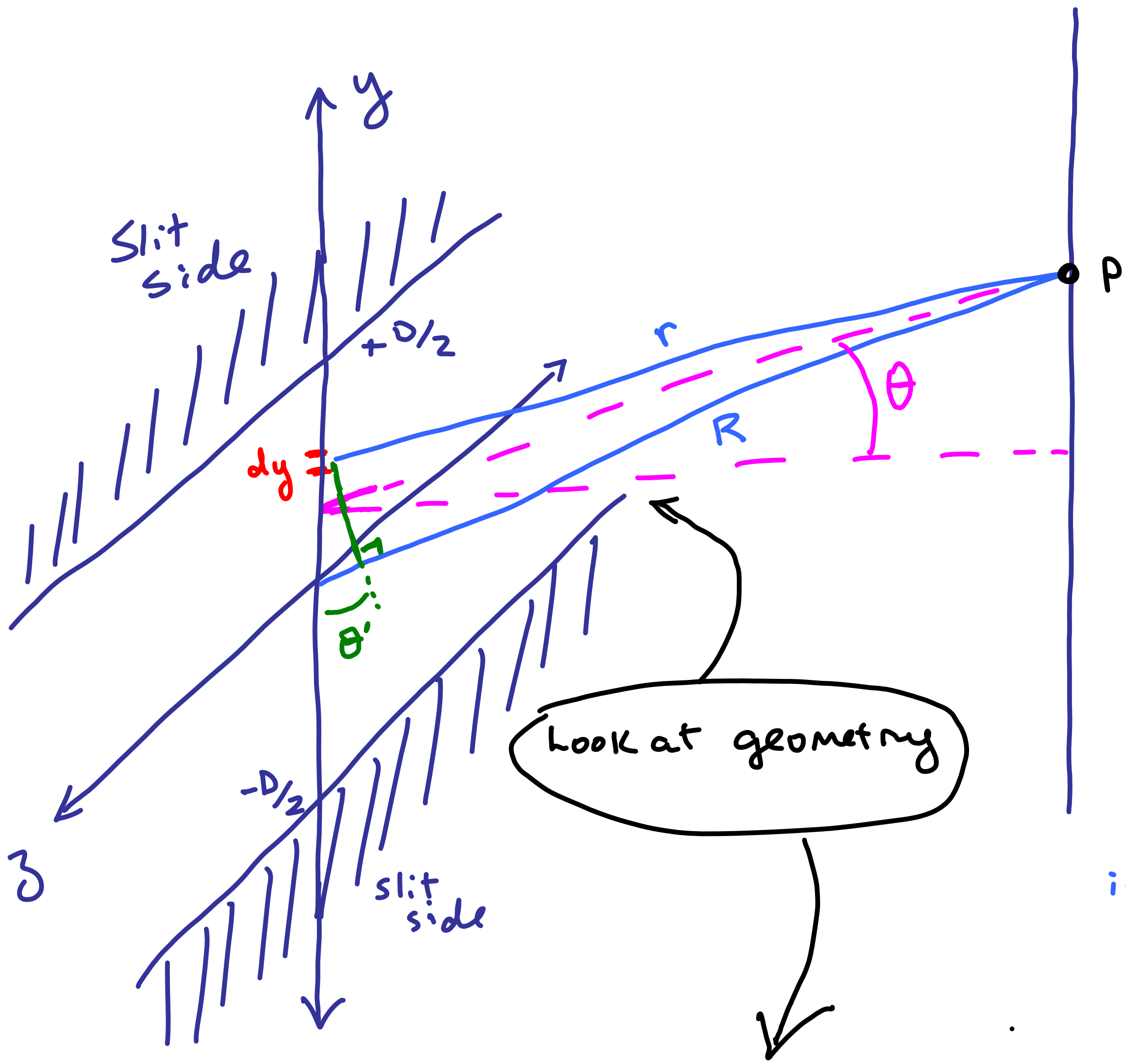
Diffraction



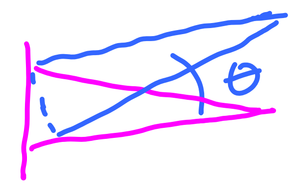


Single Slit Diffraction

Follows Discussion in Hecht

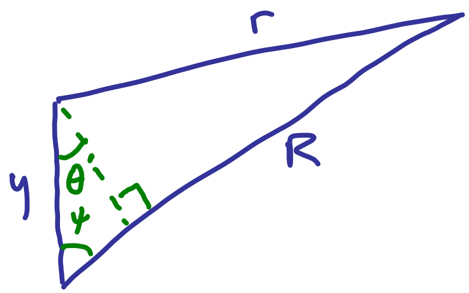


Slit of width D



$\theta \neq \theta'$

if $\theta \sim \theta' \rightsquigarrow r \sim R - y\sin\theta$



$r^2 = R^2 + y^2 + 2Ry\cos\psi$ (Law of Cosines)

$\theta' = 90° - \psi$

$\cos\psi = \sin\theta'$

÷ by R^2

$$\frac{r}{R} = \left[1 + \left(\frac{y}{R}\right)^2 - \frac{2y}{R}\sin\theta'\right]^{1/2}$$

Expand using

$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 \ldots$

Let

$x = \left(\frac{y}{R}\right)^2 - 2\left(\frac{y}{R}\right)\sin\theta$

$\frac{y}{R}$ small, Drop terms $O\left[\left(\frac{y}{R}\right)^3\right]$

$$r = R - y\sin\theta' + \frac{y^2}{2R}\cos^2\theta + \ldots$$

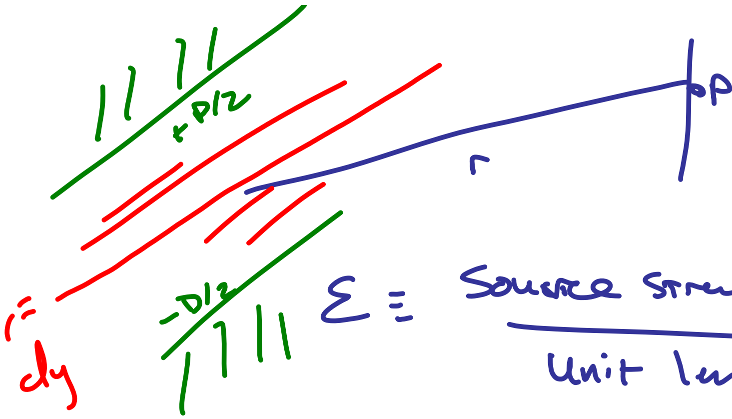
negligible if $R >> D$

Same as $\theta \sim \theta'$

$r = R - y\sin\theta$

Fraunhofer condition

phase of Amplitude contributions linear in Aperture variables



$$\mathcal{E} \equiv \frac{\text{Source strength}}{\text{unit length}}$$

Amplitude contr.

$$dE = \frac{\mathcal{E}}{r} \sin(\omega t - kr)\,dy$$

$r \sim R$ in Amplitude (Not in phase!)

$$dE = \frac{\mathcal{E}}{R} \sin(\omega t - kr)\,dy$$

$$E_p = \int_{-D/2}^{D/2} \frac{\mathcal{E}}{R} \sin\left[\omega t - k(R - y\sin\theta)\right] dy$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

let $A = \omega t - kR$ $\quad B = ky\sin\theta$

$$\sin(\omega t - kR)\cos(ky\sin\theta) + \cos(\omega t - kR)\sin(ky\sin\theta)$$

get cos in $\int$ even in y

Term $\to 0$ as integrate bet $-D/2 \to D/2$

$$E_p = \int_{-D/2}^{D/2} \frac{\varepsilon}{R}\sin(\omega t - kR)\cos(ky\sin\theta)\,dy$$

$$E_p = \frac{\varepsilon}{R}\sin(\omega t - kR)\left.\frac{\sin(ky\sin\theta)}{k\sin\theta}\right|_{-D/2}^{D/2}$$

$$E_p = \frac{\varepsilon}{R} D \frac{\sin\left(\frac{kD}{2}\sin\theta\right)}{\frac{kD}{2}\sin\theta}\sin(\omega t - kR)$$

$$E_p = \frac{\varepsilon D}{R} \frac{\sin \beta}{\beta} \sin(\omega t - kR)$$

$$\beta \equiv \frac{kD}{2} \sin \theta$$

$$I(\theta) = \langle E^2 \rangle = \underbrace{\frac{1}{2}\left(\frac{\varepsilon D}{R}\right)^2}_{I(0)} \left(\frac{\sin \beta}{\beta}\right)^2$$

Single-slit Diffraction intensity pattern

$$I(\theta) = I(0)\left(\frac{\sin \beta}{\beta}\right)^2$$

Look at Extrema

$$\frac{dI}{d\beta} = I(0)\left\{ \frac{2\sin\beta}{\beta^3}\left(\beta\cos\beta - \sin\beta\right)\right\} = 0$$

$\frac{dI}{d\beta} = 0$ if $\sin\beta = 0$ or $\beta\cos\beta - \sin\beta = 0$

intensity minima: True $\beta = m\pi$, $m = \pm 1, \pm 2 \ldots$

$\beta = \tan\beta$ — intensity maxima

at $m = 0$

$$\frac{\lim_{\beta\to 0} \sin\beta}{\lim_{\beta\to 0} \beta} \sim \frac{\cos\beta}{1} = 1$$

intensity max

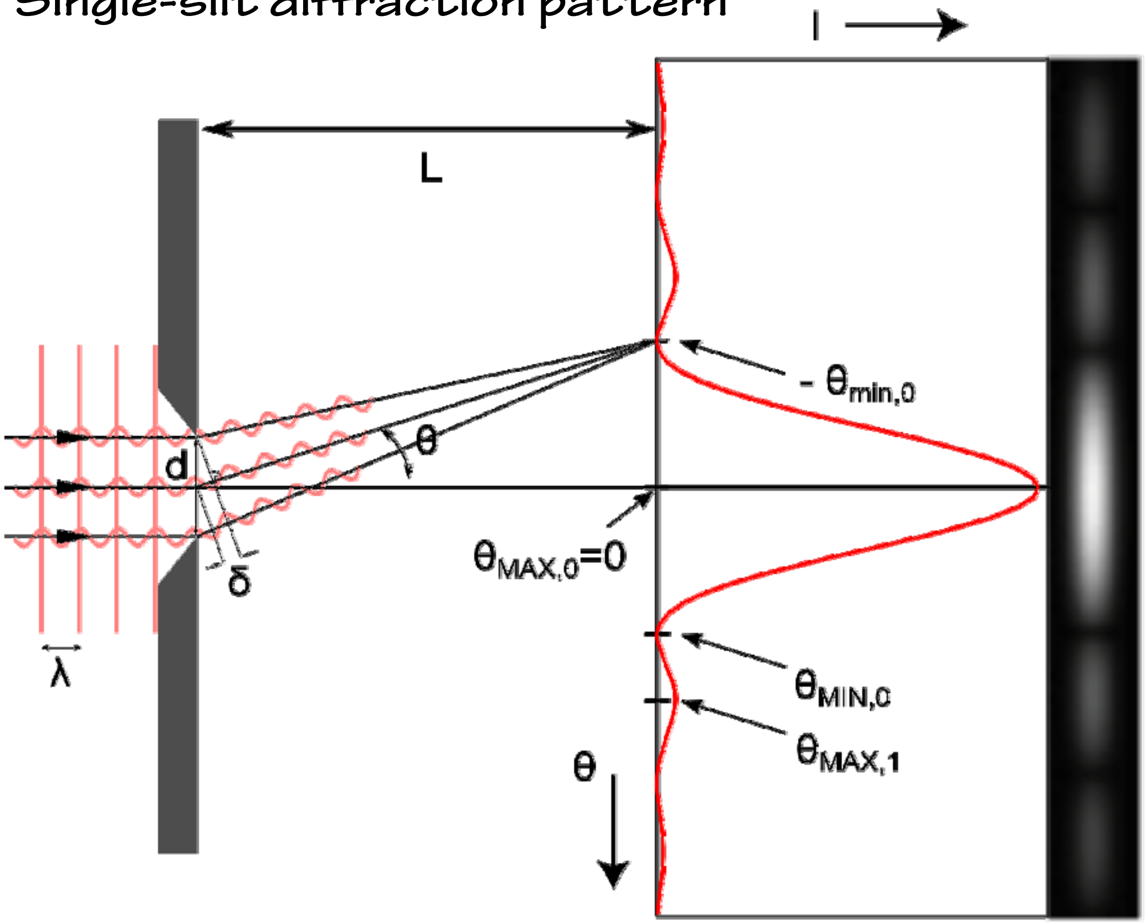
minima

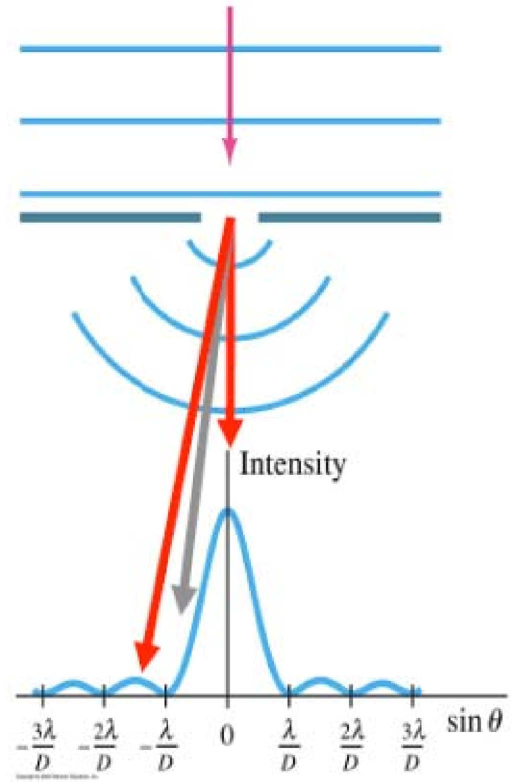
$$\beta = m\pi$$

$$\frac{k}{2} D \sin\theta = m\pi$$

$$\frac{2\pi}{\lambda} \frac{D}{2} \sin\theta = m\pi \rightarrow D\sin\theta = m\lambda$$

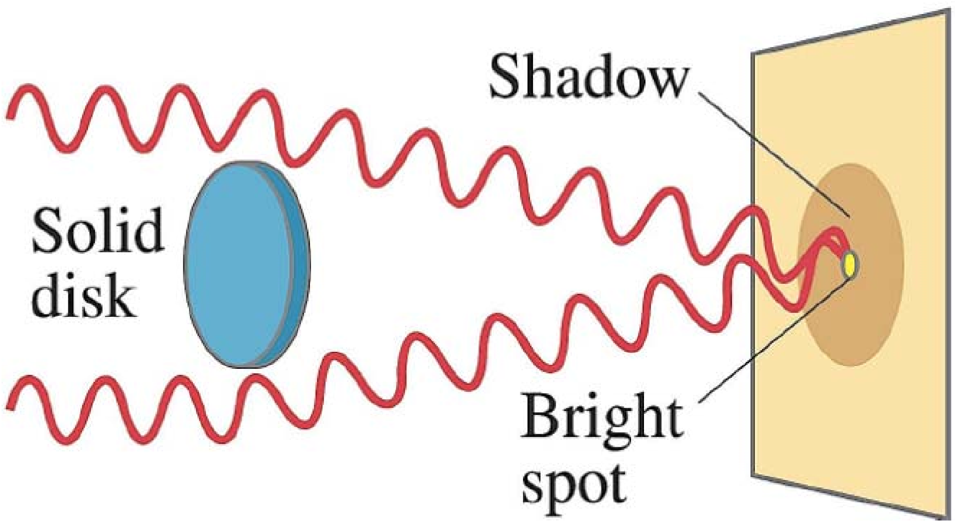
Single-slit diffraction pattern

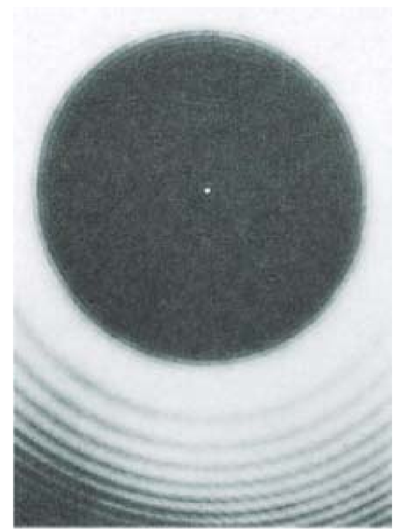




Minima

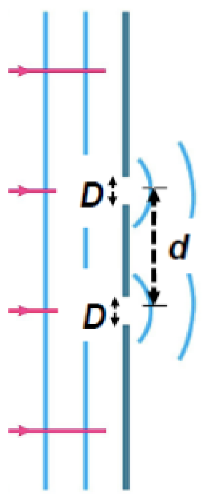
$$D \sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \ldots$$


Shadow
Solid
disk
Bright
spot



Diffraction and two slits ...

In Young's double-slit experiment we assumed *infinitesimally narrow* slits. This can never be the case for real slits and diffraction must be included.

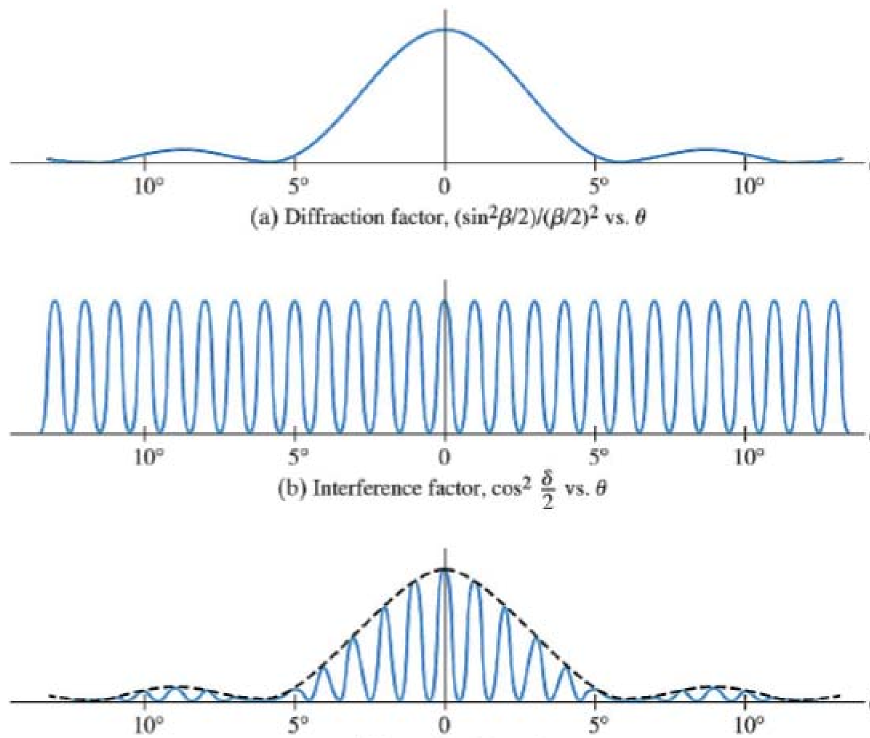


$$\begin{cases} E_\theta = E_0 \dfrac{\sin\beta/2}{\beta/2} \\ \beta = \dfrac{2\pi}{\lambda} D \sin\theta \end{cases} \quad ; \quad \begin{cases} E_{\theta,0} = 2E_0 \cos\dfrac{\delta}{2} \\ \delta = \dfrac{2\pi}{\lambda} d \sin\theta \end{cases}$$

Diffraction factor **Interference factor**

$$\rightarrow \quad E_{\theta,0} = 2E_0\left(\frac{\sin\beta/2}{\beta/2}\right)\cos\frac{\delta}{2}$$

$$\rightarrow \quad I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2\left(\cos\frac{\delta}{2}\right)^2$$

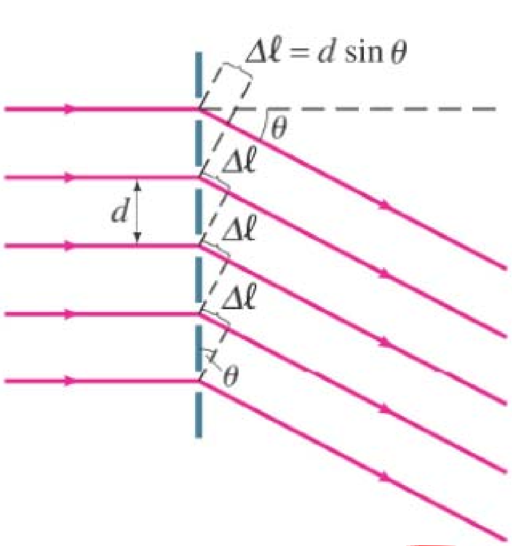


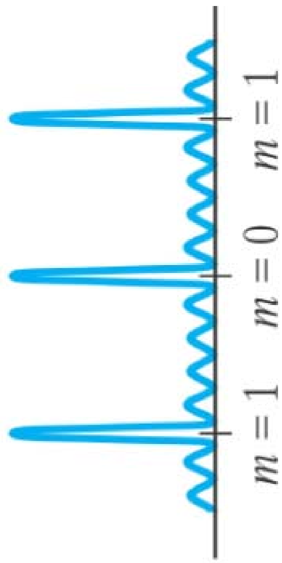
(a) Diffraction factor, $(\sin^2\beta/2)/(\beta/2)^2$ vs. θ

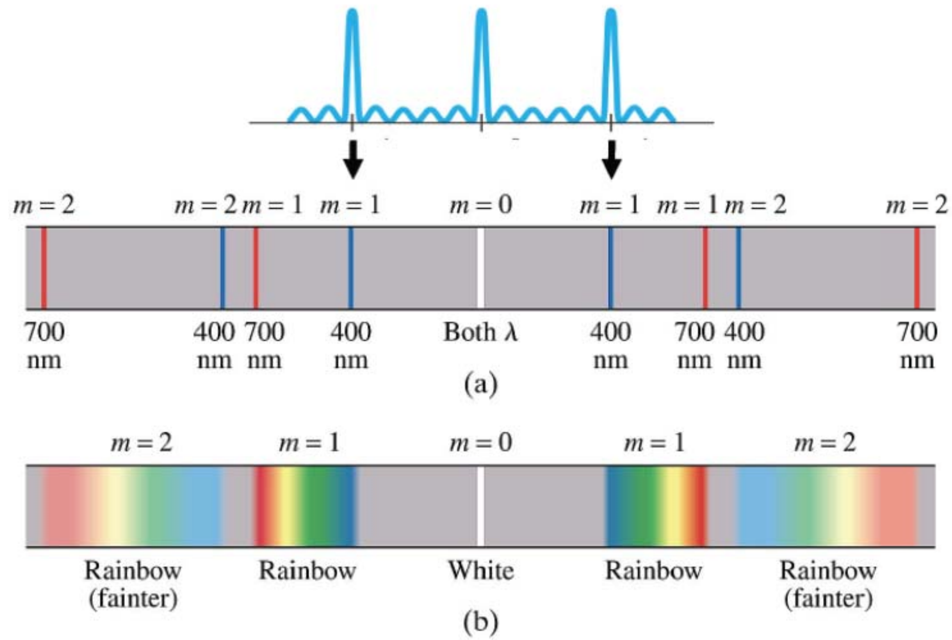
(b) Interference factor, $\cos^2\frac{\delta}{2}$ vs. θ

(c) Intensity, I_θ vs. θ

Many slits ... Diffraction gratings



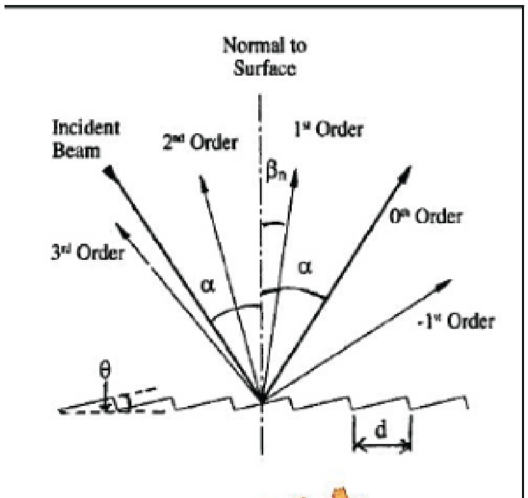


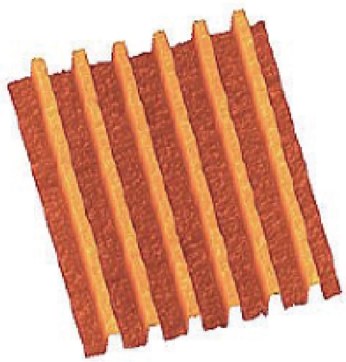


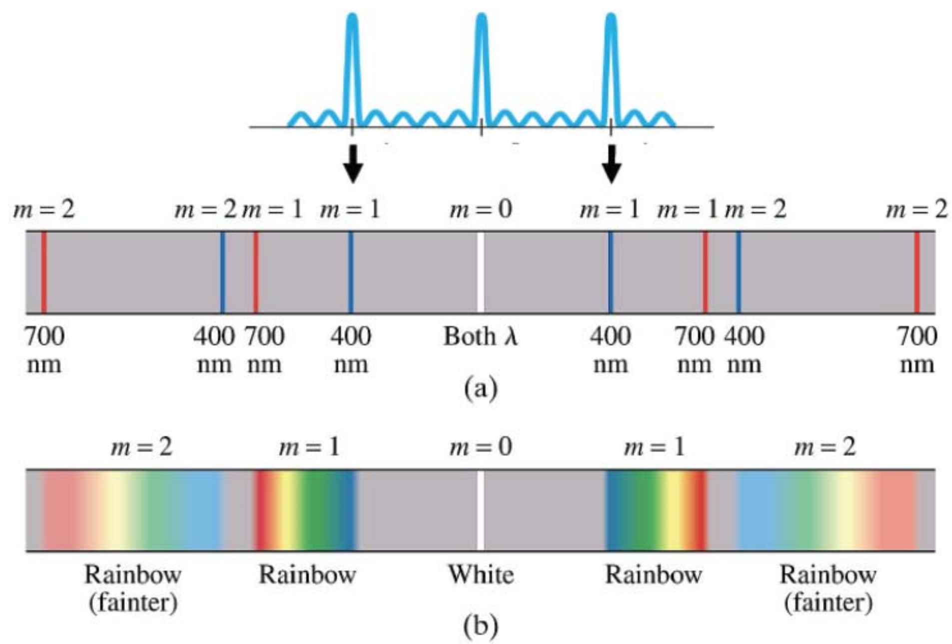
$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots$$

m is called the **order of diffraction**

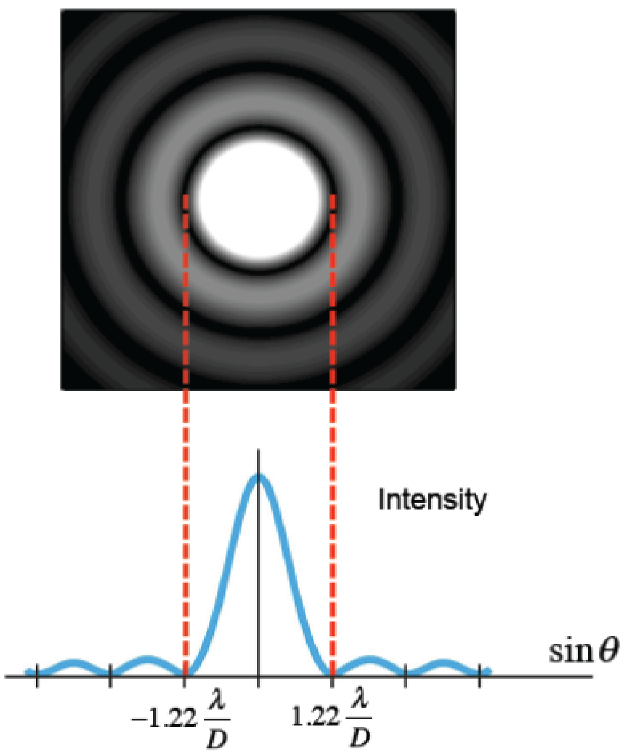
Many slits ... Diffraction gratings



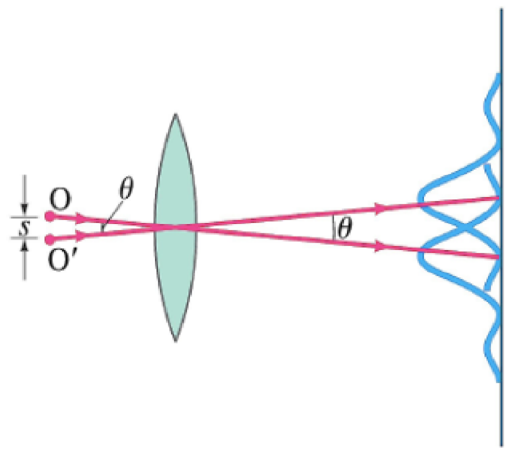




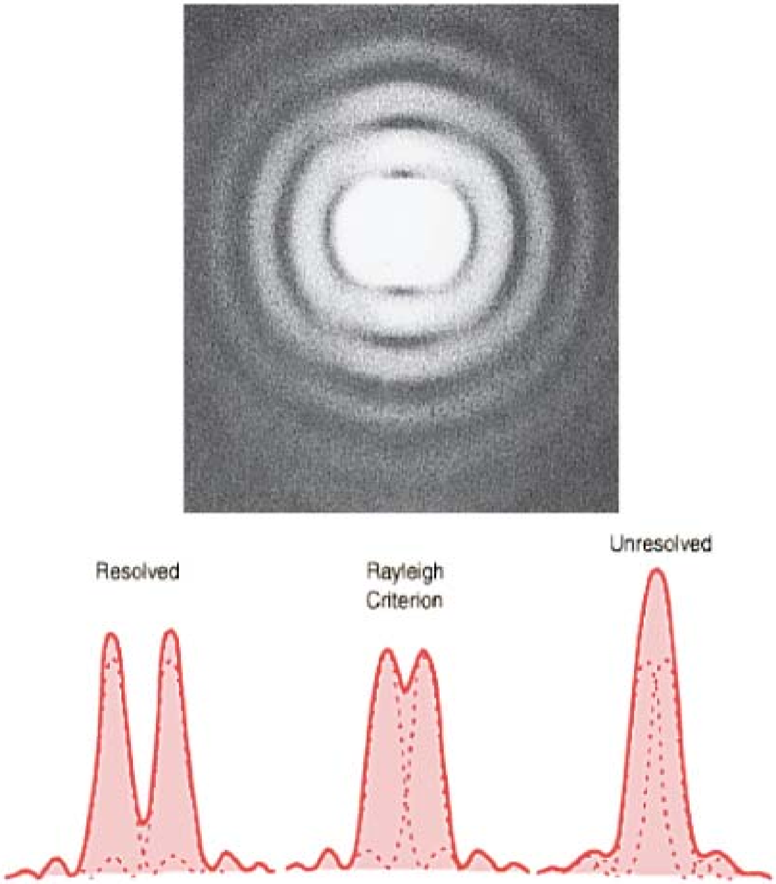
Resolution limits



The **Rayleigh criterion** states that two images are just resolvable when the center of one peak is over the first minimum of the other.



$$\theta = 1.22\frac{\lambda}{D}$$



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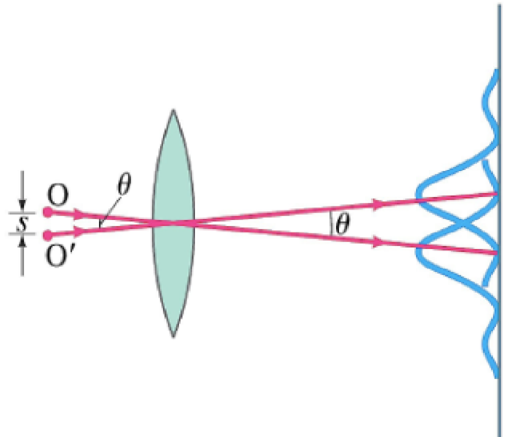
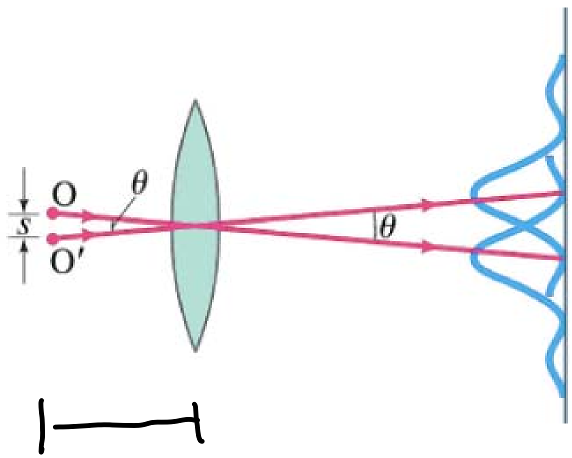


$$\theta = 1.22\frac{\lambda}{D}$$



$\sim f$

for microscope

$s = f\theta$

$\theta = 1.22 \frac{\lambda}{D}$

$\frac{s}{f} = 1.22 \frac{\lambda}{D}$

$s = 1.22 \frac{\lambda}{D} f$

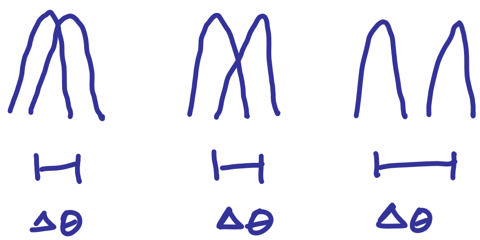
$f \sim \frac{D}{2}$ at best

$s \sim \frac{1.22 \lambda}{2}$

Cannot image things smaller than the wavelength used

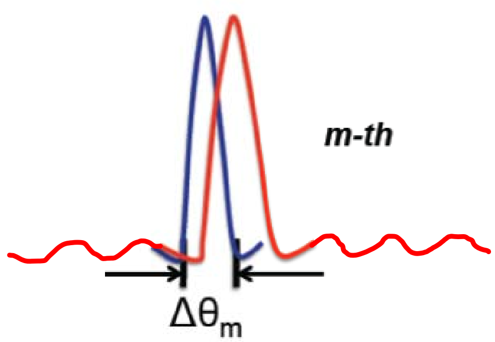
Resolving Power for grating

Gratings used for spectral analysis – important to clearly separate two spectral lines



at what Δθ are lines clearly resolved?

principle max. of one line falls ≥ distance to first minimum for other (Rayleigh again)



$$R \equiv \frac{\lambda}{\Delta\lambda} = N m$$

Resolving Power for grating

N: # of "Rulings"


Let there be Light...
And light is A wave!!
Do you Believe?!