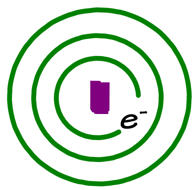
Physics 123 - April 18, 2013

- Final Exam Location → Lower Strong (May 7)
- Exam 2 This Thursday 0800 Dewey 1101
  Full 8.5x11 sheet
  calculator
- Tuesday
  Q+A session 4:30-6:00 pm Meliora 208
  overlap w/ workshops - also review
  NOT ideal - Sorry
- P.S. 9 Solutions
- Wednesday's Lecture

Last Week



Bohr Model

- e^- in circular orbits
- e^- held in atom via Coulomb attraction
- Single e^-
- e^- exists in discrete stable orbits
- photons emitted as e^- jumps between orbits
- Photon energy corresponds to difference in energy between the two Energy levels

quantized circular orbits

$n = 1, 2, 3 \ldots$

$$r_n = \frac{n^2 \hbar^2}{k z e^2 m}$$

with quantized energies

$$E_{n_{TOTAL}} = -\frac{m k^2 z^2 e^4}{2 n^2 h^2}$$

Particles
R
Waves
Wierdness

$$\Delta x \Delta p \geq \hbar$$

$$\Delta E \Delta t \geq \hbar$$

uncertainty Principle

Downfall of the
Deterministic universe!
Here comes Harry Potter!

Quantum
mechanics

1-d
nonrelativistic
time-dependent
Schrödinger
equation

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V\psi(x,t)$$

Time independent
1d nonrelativistic
Schrödinger eqn

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

$$\psi(x,t) = \psi(x)\, e^{-i\frac{E}{\hbar}t}$$

<u>Born's postulate</u> : at time t, $\psi^*_{(x,t)}\,\psi_{(x,t)}\,dx = |\psi^2_{(x,t)}|\,dx$ gives the probability of finding the particle between x and $x+dx$

Normalize $\psi(x,t)$ so that the total probability is 1

$$\int_{-\infty}^{\infty} |\psi^2(x,t)|\,dx = 1$$

The particle must be <u>somewhere</u>

Examples

Free particle

$V(x) = 0$ for all x

Time ind. Schr. eqn

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + \cancel{V(x)}\,\psi(x) = E\,\psi(x)$$

$\cancel{V(x)} \to 0$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

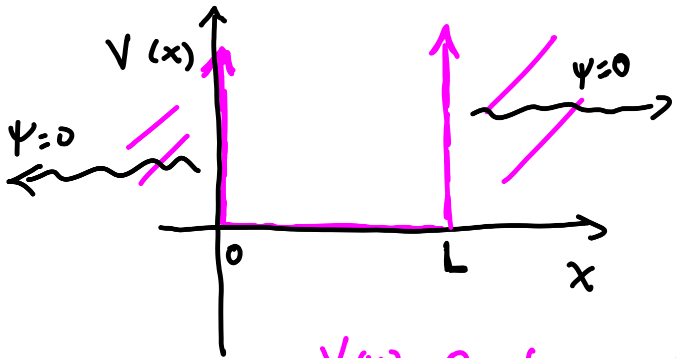
where $k^2 = 2mE/\hbar^2$

-or-

$$\psi(x) = A'\sin(kx) + B'\cos(kx)$$

$$k^2 = \frac{2mE}{\hbar^2}$$

1-d
Particle in box
"square well"



as $V \to \infty$ Schr eqn makes no sense
so $\psi = 0$ there

$V(x) = 0$ for $0 < x < L$
$V(x) = \infty$ for $x < 0,\ L < x$

for $0 < x < L$ $V(x) = 0$
particle is free
inside the Box

$\psi(x) = A \sin kx + B \cos kx$

Boundary conditions

at $x = 0,\ \psi = 0 \rightsquigarrow B = 0$

at $x = L,\ \psi = 0 \rightsquigarrow \sin kL = 0 \quad kL = n\pi \quad n = 1, 2, 3 \ldots$

Energy
quantized

$k = \frac{n\pi}{L}$ So, $E_n = \frac{n^2 \pi^2 \hbar^2}{L^2\, 2m}$ where $n = 1, 2, 3 \cdots$

Normalization

Determine "A"

$$\int \psi^*_{(x)}\,\psi(x) = 1 \quad\longrightarrow\quad \int_0^L A^2 \sin^2(kx)\,dx = 1$$

$$\psi_n(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi}{L}x\right) \quad \text{for } n = 1, 2, 3 \cdots \quad \text{in } 0 < x < L$$

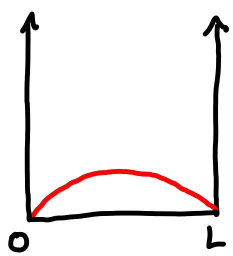
$kL = n\pi$

$\frac{2\pi}{\lambda} L = n\pi$

$\lambda = \frac{2L}{n}$

$n=1$

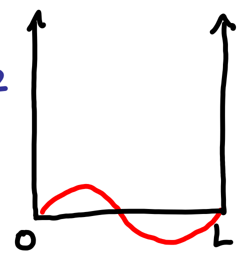
ψ_1



$\lambda = 2L$

$n=2$

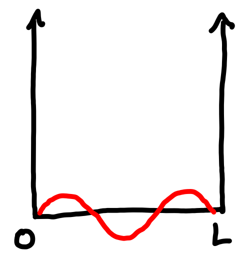
ψ_2



$\lambda = L$

$n=3$

ψ_3



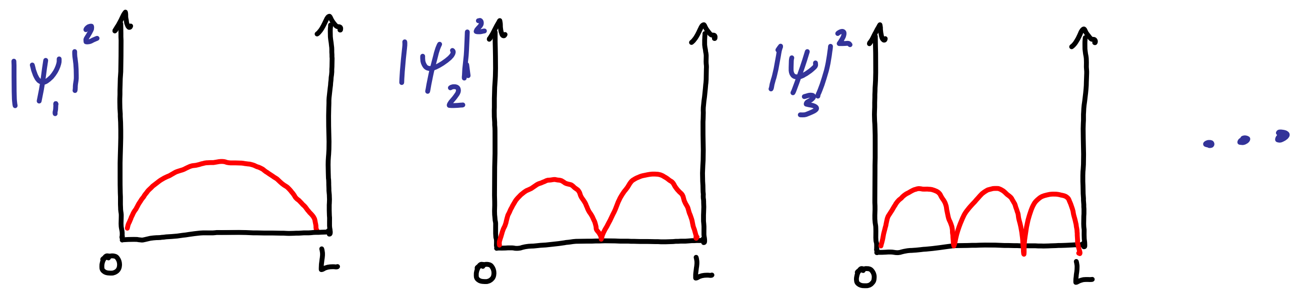
$\lambda = \frac{2L}{3}$

...

$\psi_1 = \sqrt{\frac{2}{L}}\sin\frac{\pi}{L}x \qquad \psi_2 = \sqrt{\frac{2}{L}}\sin\frac{2\pi}{L}x \qquad \psi_3 = \sqrt{\frac{2}{L}}\sin\frac{3\pi}{L}x \qquad \cdots$

What is the probability density for seeing particle at position x

$n=1$ $\quad$ $n=2$ $\quad$ $n=3$



$\lambda = 2L$ $\quad$ $\lambda = L$ $\quad$ $\lambda = \frac{2L}{3}$

$$|\psi_1|^2 = \frac{2}{L}\sin^2\frac{\pi}{L}x \qquad |\psi_2|^2 = \frac{2}{L}\sin^2\frac{2\pi}{L}x \qquad |\psi_3|^2 = \frac{2}{L}\sin^2\frac{3\pi}{L}x \qquad \cdots$$

What is the average position of particle in state n ?

$$\frac{\sum Px}{\sum P}.$$

$$\underbrace{\bar{x} = \langle x \rangle}_{\text{Expectation value of } x} = \frac{\int_{-\infty}^{\infty} \overbrace{\psi^* x \psi}^{\text{prob. weighted position}} \, dx}{\underbrace{\int_{-\infty}^{\infty} \psi^* \psi \, dx}_{=1}}$$

Expectation value of $f(x)$

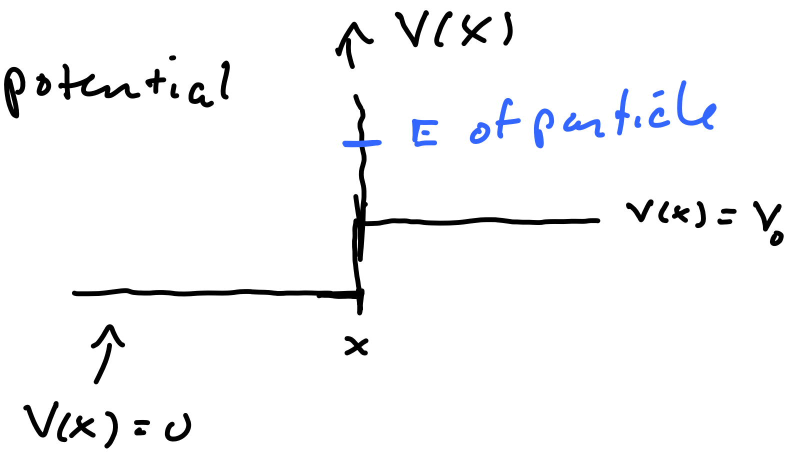
$$\langle f(x) \rangle = \int_{-\infty}^{\infty} \psi^*_{(x)} f(x) \psi(x) \, dx$$

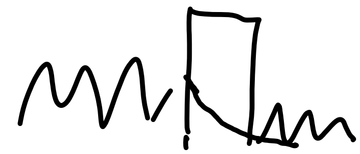
for ∞ square well, $n=2$ $\qquad \psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi \, dx = \frac{2}{L} \int_0^L \sin^2\left(\frac{2\pi x}{L}\right) x \, dx = \frac{L}{2}$$

Step or Barrier Potential problems

Step potential





Classically $E = p^2/2m$ ⊕ x direction →

encounter a potential — repulsive force

$F = -\frac{dV}{dx}$

Slows particle down

$x<0 \quad p = \sqrt{2mE}$ $\qquad x>0 \quad p = \sqrt{2m(E-V_0)}$

in QM

$x<0 \qquad -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\,\psi(x)$ free particle

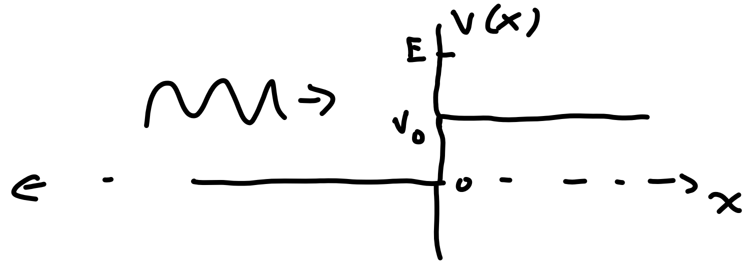
$x>0 \qquad -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = (E-V_0)\psi(x)$ free particle

$E>V_0$

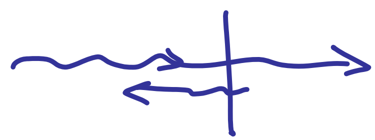
$E-V_0>0$

$x<0 \qquad \psi(x) = Ae^{ik_1x} + Be^{-ik_1x} \qquad k_1 = \frac{\sqrt{2mE}}{\hbar}$

$x>0 \qquad \psi(x) = ce^{ik_2x} + De^{-ik_2x} \qquad k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$



particle coming from left incident on step potential



$x<0$

$$\psi(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

$x>0$

$$\psi(x) = Ce^{ik_2x}$$

$$D=0$$

Boundary conditions $\psi_{x<0}(0) = \psi_{x>0}(0) \Rightarrow A + B = C$

$$\left.\frac{d\psi(0)}{dx}\right|_{x<0} = \left.\frac{d\psi(0)}{dx}\right|_{x>0}$$

$$Ak_1 i e^{ik_1x} - Bik_1 e^{ik_1x} = C i k_2 e^{ik_2x}$$

$$k_1(A - B) = k_2 C$$