Physics 123 - April 8, 2013

- Final Exam Location -> Lower Strong (May 7)
- Exam 2 This Thursday 0800 Dewey 1101
  Full 8.5x11 Sheet Calculator
- Q+A session 4:30-6:00 pm Meliora 208
  Overlap w/ workshops - also review not ideal - sorry
- P.S. 9 Solutions
- Wednesday's Lecture
Last Week

Bohr model

- e⁻ in circular orbits
- e⁻ held in atom via Coulomb attraction
- Single e⁻
- e⁻ exists in discrete stable orbits
- Photons emitted as e⁻ jumps between orbits
- Photon energy corresponds to difference in energy between the two energy levels

- quantized circular orbits
- \( r_n = \frac{n^2 \hbar^2}{k^2 e^2 m} \)
- \( n = 1, 2, 3, \ldots \)

- with quantized energies
- \( E_n = -\frac{m k^2 e^2}{2 n^2 \hbar^2} \)
Uncertainty Principle

\[ \Delta x \Delta p \geq \hbar \]

\[ \Delta E \Delta t \geq \hbar \]

Downfall of the Deterministic Universe!

Here comes Harry Potter!

Quantum Mechanics

1st nonrelativistic time-dependent Schrödinger equation

\[ i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x) \psi(x,t) \]

Time independent 1st nonrelativistic Schrödinger eqn

\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x) \psi(x) = E \psi(x) \]

\[ \psi(x,t) = \psi(x) e^{-i\frac{Et}{\hbar}} \]
Born's postulate: at time $t$, $\Psi^*(x,t) \Psi(x,t) \, dx = |\Psi(x,t)|^2 / dx$
gives the probability of finding the particle between $x$ and $x+dx$

Normalize $\Psi(x,t)$ so that the total probability is 1

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 / dx = 1$$

The particle must be somewhere
Examples

Free particle

Schr. eqn

Time ind.

\[ V(x) = 0 \text{ for all } x \]

\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x) \psi(x) = E \psi(x) \]

\[ \psi(x) = A e^{i k x} + B e^{-i k x} \]

where \( k^2 = \frac{2mE}{\hbar^2} \)

- or -

\[ \psi(x) = A' \sin(kx) + B' \cos(kx) \]

\[ k^2 = \frac{2mE}{\hbar^2} \]
1-D Particle in box

"Square well"

Boundary conditions

\[ V(x) = 0 \text{ for } 0 < x < L \]
\[ V(x) = \infty \text{ for } x < 0, x > L \]

as \( V \to \infty \) Schröd. eqn makes no sense so \( \psi = 0 \) there

for \( 0 < x < L \) \( V(x) = 0 \) particle is free inside the box

\[ \psi(x) = A \sin kx + B \cos kx \]

at \( x = 0, \psi = 0 \) \( \Rightarrow B = 0 \)

at \( x = L, \psi = 0 \) \( \Rightarrow \sin kl = 0 \) \( kl = n \pi \) \( n = 1, 2, 3 \cdots \)

Energy quantized

\[ E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad \text{where} \quad n = 1, 2, 3 \cdots \]
Normalization: Determine $A$.

\[ \int \psi_n^* \psi_k \, dx = 1 \quad \Rightarrow \quad \int_0^L A^2 \sin^2(kx) \, dx = 1 \]

\[ \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n \pi x}{L}\right) \quad \text{for } n = 1, 2, 3, \ldots \quad \text{in } 0 < x < L \]

\begin{align*}
\lambda &= 2L, & n &= 1 \\
\lambda &= L, & n &= 2 \\
\lambda &= \frac{2L}{3}, & n &= 3 \\
\lambda &= \frac{2L}{3}, & n &= \ldots
\end{align*}

\begin{align*}
\psi_1 &= \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \\
\psi_2 &= \sqrt{\frac{2}{L}} \sin\left(\frac{2 \pi x}{L}\right) \\
\psi_3 &= \sqrt{\frac{2}{L}} \sin\left(\frac{3 \pi x}{L}\right) \\
\psi_4 &= \sqrt{\frac{2}{L}} \sin\left(\frac{4 \pi x}{L}\right) \\
&\quad \ldots
\end{align*}
What is the probability density for seeing particle at position $x$

$n=1$

$|\psi_1|^2 = \frac{2}{L} \sin \frac{2\pi x}{L}$

$n=2$

$|\psi_2|^2 = \frac{2}{L} \sin \frac{4\pi x}{L}$

$n=3$

$|\psi_3|^2 = \frac{2}{L} \sin \frac{6\pi x}{L}$

$\lambda = \frac{2L}{3}$

$\lambda = L$

$\lambda = \frac{2L}{3}$

$\ldots$
What is the average position of particle in state \( n \)?

\[
\overline{x} = \langle x \rangle = \frac{\sum_{x} \frac{\Gamma_{n,n}^{*}}{\sum_{i} \Gamma_{n,i}^{*}}} \int_{-\infty}^{\infty} \psi^{*}_{n} \psi \, dx
\]

Expectation value of \( x \)

Expectation value of \( f(x) \)

\[
\langle f(x) \rangle = \int_{-\infty}^{\infty} \psi^{*}_{n} f(x) \psi_{n} \, dx
\]
for a square well, \( n = 2 \)

\[
\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}
\]

\[
\langle x \rangle = \int_{-a}^{a} \psi^* x \psi \, dx = \frac{2}{L} \int_{0}^{L} \sin^2 \left( \frac{2\pi x}{L} \right) x \, dx = \frac{L}{2}
\]
Step or Barrier Potential problems

Step potential

\[ \begin{align*}
V(x) &= 0 \\
E_{\text{of particle}} \\
V(x) &= V_0
\end{align*} \]

Classically, \( E = \frac{p^2}{2m} \), \( \Theta \) in \( x \) direction \( \Rightarrow \)
encounter a potential — repulsive force
\[ p = -\frac{dv}{dx} \]
Slows particle down
\[ x < 0 \quad p = \sqrt{2mE} \]
\[ x > 0 \quad p = \sqrt{2m(E - V_0)} \]
\( \text{in QM} \quad x < 0 \quad -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} = E \psi(x) \)

\( x > 0 \quad -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} = (E - V_0) \psi(x) \)

\( x < 0 \quad \psi(x) = A e^{ik_1 x} + Be^{-ik_1 x} \quad \color{green}{k_1 = \frac{\sqrt{2mE}}{\hbar}} \)

\( x > 0 \quad \psi(x) = Ce^{ik_2 x} + De^{-ik_2 x} \quad \color{green}{k_2 = \frac{\sqrt{2m(E - V_0)}}{\hbar}} \)
Particle coming from left incident on step potential

\[ \psi(x) = A e^{ik_1 x} + B e^{-ik_1 x} \quad x < 0 \]

\[ \psi(x) = C e^{ik_2 x} \quad x > 0 \]
Boundary conditions

\[
\psi(0) = \psi(1) \\
\frac{d\psi(0)}{dx} <0 = \frac{d\psi(1)}{dx} >0
\]

\[\Rightarrow A + B = C\]

\[Ak_i e^{-ikx} - Bk_i e^{ikx} = c_i k_x e^{ikx}\]

\[k_x(A-B) = k_x c\]