Physics 123 - Spring 2013 - Solutions to PS 10

1.
37-55

To ionize the atom means removing the electron, or raising it to zero energy.

$$E_{\text{ionization}} = 0 - E_n = 0 - \frac{(-13.6\text{eV})}{n^2} = \frac{(13.6\text{eV})}{3^2} = \boxed{1.51\text{eV}}$$

2.
37-56

We use the equation that appears above Eq. 37-15 in the text.

(*a*) The second Balmer line is the transition from $n = 4$ to $n = 2$.

$$\lambda = \frac{hc}{(E_4 - E_2)} = \frac{1240\text{eV}\cdot\text{nm}}{[-0.85\text{eV} - (-3.4\text{eV})]} = \boxed{490\,\text{nm}}$$

(*b*) The third Lyman line is the transition from $n = 4$ to $n = 1$.

$$\lambda = \frac{hc}{(E_4 - E_1)} = \frac{1240\text{eV}\cdot\text{nm}}{[-0.85\text{ eV} - (-13.6\text{ eV})]} = \boxed{97.3\,\text{nm}}$$

(*c*) The first Balmer line is the transition from $n = 3$ to $n = 2$.
For the jump from $n = 5$ to $n = 2$, we have

$$\lambda = \frac{hc}{(E_3 - E_2)} = \frac{1240\text{eV}\cdot\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\,\text{nm}}$$

3.
37-57

Doubly ionized lithium is similar to hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\,\text{eV})}{n^2} = -\frac{3^2(13.6\,\text{eV})}{n^2} = -\frac{(122\,\text{eV})}{n^2}$$

$$E_{\text{ionization}} = 0 - E_1 = 0 - \left[-\frac{(122\,\text{eV})}{(1)^2}\right] = \boxed{122\,\text{eV}}$$

4.
37-61

The energy of the photon is the sum of the ionization energy of 13.6 eV and the kinetic energy of 20.0eV. The wavelength is found from Eq. 37-3.

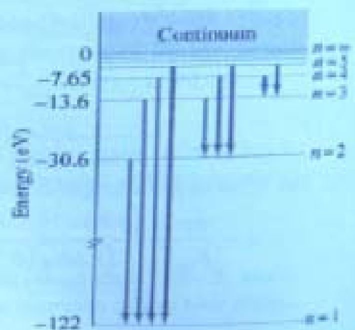
$$hf = \frac{hc}{\lambda} = E_{\text{total}} \rightarrow \lambda = \frac{hc}{E_{\text{total}}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(33.6\,\text{eV})} = 3.70\times10^{-8}\,\text{m} = \boxed{37.0\,\text{nm}}$$

5.
37-64

Doubly ionized lithium is like hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\,\text{eV})}{n^2} = -\frac{3^2(13.6\,\text{eV})}{n^2} = -\frac{(122.4\,\text{eV})}{n^2}$$

$E_1 = -122\,\text{eV}$, $E_2 = -30.6\,\text{eV}$, $E_3 = -13.6\,\text{eV}$,
$E_4 = -7.65\,\text{eV}$



6.
37-70

When we compare the gravitational and electric forces we see that we can use the same expression for the Bohr orbits, Eq. 37-11 and 37-14a, if we replace $Ze^2/4\pi\varepsilon_0$ with $Gm_e m_p$.

$$r_1 = \frac{h^2\varepsilon_0}{\pi m_e Z e^2} = \frac{h^2}{4\pi^2 m_e}\frac{4\pi\varepsilon_0}{Ze^2} \rightarrow$$

$$r_1 = \frac{h^2}{4\pi^2 G m_e^2 m_p} = \frac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^2}{4\pi^2(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\,\text{kg})^2(1.67\times10^{-27}\,\text{kg})}$$

$$= \boxed{1.20\times10^{29}\,\text{m}}$$

$$E_1 = -\frac{Z^2e^4m_e}{8\varepsilon_0^2h^2} = -\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{2\pi^2 m_e}{h^2} \rightarrow E_1 = -\frac{2\pi^2G^2m_e^3m_p^2}{h^2}$$

$$= -\frac{2\pi^2(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)^2(9.11\times10^{-31}\,\text{kg})^3(1.67\times10^{-27}\,\text{kg})^2}{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^2} = \boxed{-4.22\times10^{-97}\,\text{J}}$$

7.
37-71

We know that the radii of the orbits are given by $r_n = n^2 r_1$. Find the difference in radius for adjacent orbits.

$$\Delta r = r_n - r_{n-1} = n^2 r_1 - (n-1)^2 r_1 = n^2 r_1 - (n^2 - 2n + 1) r_1 = (2n-1) r_1$$

If $n \gg 1$, we have $\Delta r \approx 2nr_1 = 2n\frac{r_n}{n^2} = \frac{2r_n}{n}$.

In the classical limit, the separation of radii (and energies) should be very small. We see that letting $n \rightarrow \infty$ accomplishes this. If we substitute the expression for r_1 from Eq. 37-11, we have this.

$$\Delta r \approx 2nr_1 = \frac{2nh^2\varepsilon_0}{\pi m e^2}$$

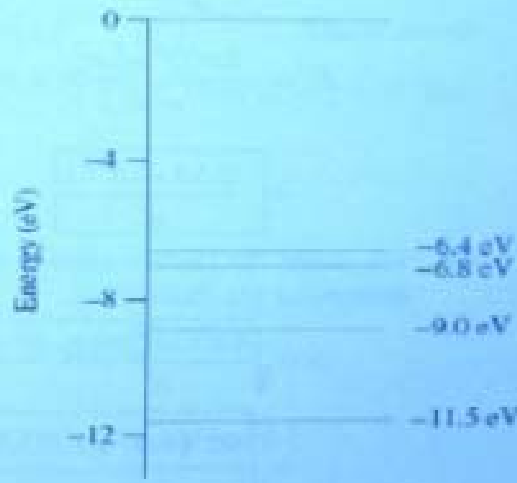
We see that $\Delta r \propto h^2$, and so letting $h \rightarrow 0$ is equivalent to considering $n \rightarrow \infty$.

8.
37-91

(*a*) See the adjacent figure.

(*b*) Absorption of a 5.1 eV photon represents a transition from the **ground state** to the state 5.1 eV above that, the third excited state. Possible photon emission energies are found by considering all the possible downward transitions that might occur as the electron makes its way back to the ground state.

$$-6.4\text{eV}-(-6.8\text{eV})=\boxed{0.4\text{eV}}$$
$$-6.4\text{eV}-(-9.0\text{eV})=\boxed{2.6\text{eV}}$$
$$-6.4\text{eV}-(-11.5\text{eV})=\boxed{5.1\text{eV}}$$
$$-6.8\text{eV}-(-9.0\text{eV})=\boxed{2.2\text{eV}}$$
$$-6.8\text{eV}-(-11.5\text{eV})=\boxed{4.7\text{eV}}$$
$$-9.0\text{eV}-(-11.5\text{eV})=\boxed{2.5\text{eV}}$$



9.
38-5

The uncertainty in position is given. Use Eq. 38-1 to find the uncertainty in the momentum.

$$\Delta p = m\Delta v \geq \frac{\hbar}{\Delta x} \rightarrow \Delta v \geq \frac{\hbar}{m\Delta x} = \frac{(1.055\times10^{-34}\,\text{J}\cdot\text{s})}{(9.11\times10^{-31}\,\text{kg})(2.6\times10^{-8}\,\text{m})} = 4454\,\text{m/s} \approx \boxed{4500\,\text{m/s}}$$

10.
38-7

The uncertainty in the energy is found from the lifetime and the uncertainty principle.

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{(1.055\times10^{-34}\,\text{J}\cdot\text{s})}{(12\times10^{-6}\,\text{s})}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 5.49\times10^{-11}\,\text{eV}$$

$$\frac{\Delta E}{E} = \frac{5.49\times10^{-11}\,\text{eV}}{5500\,\text{eV}} = \boxed{1.0\times10^{-14}}$$

11.
38-13

(a) The minimum uncertainty in the energy is found from Eq. 38-2.

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{(1.055\times10^{-34}\,\text{J}\cdot\text{s})}{(1\times10^{-8}\,\text{s})} = 1.055\times10^{-26}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 6.59\times10^{-8}\,\text{eV} \approx \boxed{10^{-7}\,\text{eV}}$$

(b) The transition energy can be found from Eq. 37-14b. $Z = 1$ for hydrogen.

$$E_n = -(13.6\,\text{eV})\frac{Z^2}{n^2} \rightarrow E_2 - E_1 = \left[-(13.6\,\text{eV})\frac{1^2}{2^2}\right] - \left[-(13.6\,\text{eV})\frac{1^2}{1^2}\right] = 10.2\,\text{eV}$$

$$\frac{\Delta E}{E_2 - E_1} = \frac{6.59\times10^{-8}\,\text{eV}}{10.2\,\text{eV}} = 6.46\times10^{-9} \approx \boxed{10^{-8}}$$

(c) The wavelength is given by Eq. 37-3.

$$E = h\nu = \frac{hc}{\lambda} \rightarrow$$

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^8\,\text{m/s})}{(10.2\,\text{eV})\left(\frac{1.60\times10^{-19}\,\text{J}}{\text{eV}}\right)} = 1.22\times10^{-7}\,\text{m} = 122\,\text{nm} \approx \boxed{100\,\text{nm}}$$

Take the derivative of the above relationship to find $\Delta\lambda$.

$$\lambda = \frac{hc}{E} \rightarrow d\lambda = -\frac{hc}{E^2}dE \rightarrow \Delta\lambda = -\frac{hc}{E^2}\Delta E = -\lambda\frac{\Delta E}{E} \rightarrow$$

$$|\Delta\lambda| = \lambda\frac{\Delta E}{E} = (122\,\text{nm})(6.46\times10^{-9}) = 7.88\times10^{-7}\,\text{nm} \approx \boxed{10^{-6}\,\text{nm}}$$

12.
38-19

The general expression for the wave function of a free particle is given by Eq. 38-3a. The particles are not relativistic.

(a) $k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{mv}{\hbar} = \frac{(9.11\times10^{-31}\,\text{kg})(3.0\times10^{5}\,\text{m/s})}{(1.055\times10^{-34}\,\text{J}\cdot\text{s})} = 2.6\times10^{9}\,\text{m}^{-1}$

$\psi = \boxed{A\sin\left[(2.6\times10^{9}\,\text{m}^{-1})x\right] + B\cos\left[(2.6\times10^{9}\,\text{m}^{-1})x\right]}$

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{mv}{\hbar} = \frac{(1.67\times10^{-27}\,\text{kg})(3.0\times10^{5}\,\text{m/s})}{(1.055\times10^{-34}\,\text{J}\cdot\text{s})} = 4.7\times10^{12}\,\text{m}^{-1}$

$\psi = \boxed{A\sin\left[(4.7\times10^{12}\,\text{m}^{-1})x\right] + B\cos\left[(4.7\times10^{12}\,\text{m}^{-1})x\right]}$

13.
38-21

The minimum speed corresponds to the lowest energy state. The energy is given by Eq. 38-13.

$E_{min} = \frac{h^2}{8m\ell^2} = \frac{1}{2}mv_{min}^2 \rightarrow v_{min} = \frac{h}{2m\ell} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{2(9.11\times10^{-31}\,\text{kg})(0.20\times10^{-9}\,\text{m})} = \boxed{1.8\times10^{6}\,\text{m/s}}$

14.
38-23

(*a*) The longest wavelength photon will be the photon with the lowest frequency, and thus the lowest energy. The difference between energy levels increases with high states, so the lowest energy transition is from $n = 2$ to $n = 1$. The energy levels are given by Eq. 38-13.

$$E_n = n^2 \frac{h^2}{8m\ell^2} = n^2 E_1$$

$$\lambda = \frac{c}{\nu} = \frac{hc}{\Delta E} = \frac{hc}{E_2 - E_1} = \frac{hc}{4E_1 - E_1} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})}{3(9.0\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = \boxed{4.6\times10^{-8}\,\text{m}}$$

(*b*) We use the ground state energy and Eq. 38-13.

$$E_1 = \frac{h^2}{8m\ell^2} \rightarrow$$

$$\ell = \frac{h}{\sqrt{8mE_1}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{\sqrt{8(9.11\times10^{-31}\,\text{kg})(9.0\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}} = \boxed{2.0\times10^{-10}\,\text{m}}$$

15.
38-29

The energy levels for a particle in a rigid box are given by Eq. 38-13. We substitute the appropriate mass in for each part of the problem.

(*a*) For an electron we have the following:

$$E=\frac{h^2n^2}{8m\ell^2}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)^2}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(2.0\times10^{-14}\,\mathrm{m}\right)^2\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}=\boxed{940\,\mathrm{MeV}}$$

(*b*) For a neutron we have the following:

$$E=\frac{h^2n^2}{8m\ell^2}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)^2}{8\left(1.675\times10^{-27}\,\mathrm{kg}\right)\left(2.0\times10^{-14}\,\mathrm{m}\right)^2\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}=\boxed{0.51\,\mathrm{MeV}}$$

(*c*) For a proton we have the following:

$$E=\frac{h^2n^2}{8m\ell^2}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)^2}{8\left(1.673\times10^{-27}\,\mathrm{kg}\right)\left(2.0\times10^{-14}\,\mathrm{m}\right)^2\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}=\boxed{0.51\,\mathrm{MeV}}$$

16.
38-34

We choose the zero of potential energy to be at the bottom of the well. Thus in free space, outside the well, the potential is $U_0 = 56\text{eV}$. Thus the total energy of the electron is $E = K + U_0 = 236\ \text{eV}$.

(*a*) In free space, the kinetic energy of the particle is 180 eV. Use that to find the momentum and then the wavelength.

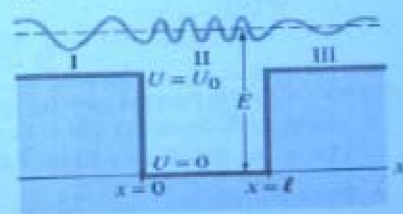
$$K = \frac{p^2}{2m} \rightarrow p = \sqrt{2mK} = \frac{h}{\lambda} \rightarrow$$

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{\left[2(9.11\times10^{-31}\,\text{kg})(180\text{eV})(1.60\times10^{-19}\,\text{J/eV})\right]^{1/2}} = \boxed{9.15\times10^{-11}\,\text{m}}$$

(*b*) Over the well, the kinetic energy is 236 eV.

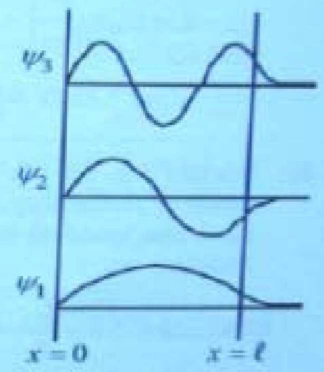
$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{\left[2(9.11\times10^{-31}\,\text{kg})(236\text{eV})(1.60\times10^{-19}\,\text{J/eV})\right]^{1/2}} = \boxed{7.99\times10^{-11}\,\text{m}}$$

(*c*) The diagram is qualitatively the same as Figure 38-14, reproduced here. Notice that the wavelength is longer when the particle is not over the well, and shorter when the particle is over the well.



17.
38-36

(*a*) We assume that the lowest three states are bound in the well, so that $E < U_0$. See the diagrams for the proposed wave functions. Note that, in the well, the wave functions are similar to those for the infinite well. Outside the well, for $x > \ell$, the wave functions are drawn with an exponential decay, similar to the right side of Figure 38-13.



(*b*) In the region $x < 0$, $\boxed{\psi = 0}$.

In the well, with $0 < x < \ell$, the wave function is similar to that of a free particle or a particle in an infinite potential well, since $U = 0$.

Thus $\boxed{\psi = A\sin kx + B\sin kx}$, where $k = \dfrac{\sqrt{2mE}}{\hbar}$.

In the region $x > \ell$, $\boxed{\psi = De^{-Gx}}$, where $G = \dfrac{\sqrt{2m(U_0 - E)}}{\hbar}$.