Physics 123 - Spring 2013 - Solutions to problem set 12

1.
41-4

Use Eq. 41-1 for both parts.

(*a*) $r = (1.2\times10^{-15}\,\text{m})A^{1/3} = (1.2\times10^{-15}\,\text{m})(112)^{1/3} = \boxed{5.8\times10^{-15}\,\text{m}} = 5.8\,\text{fm}$

(*b*) $r = (1.2\times10^{-15}\,\text{m})A^{1/3} \rightarrow A = \left(\dfrac{r}{1.2\times10^{-15}\,\text{m}}\right)^3 = \left(\dfrac{3.7\times10^{-15}\,\text{m}}{1.2\times10^{-15}\,\text{m}}\right)^3 = 29.3 \approx \boxed{29}$

2.
41-13

From Figure 41–1, we see that the average binding energy per nucleon at A = 63 is about 8.7 MeV. Multiply this by the number of nucleons in the nucleus.

$$(63)(8.7\,\text{MeV}) = 548.1\,\text{MeV} \approx \boxed{550\,\text{MeV}}$$

3.
41-17

$^{23}_{11}\text{Na}$ consists of 11 protons and 12 neutrons. We find the binding energy from the masses:

$$\text{Binding energy} = \left[11m\left({}^{1}_{1}\text{H}\right) + 12m\left({}^{1}_{0}\text{n}\right) - m\left({}^{23}_{11}\text{Na}\right)\right]c^2$$
$$= \left[11(1.007825\,\text{u}) + 12(1.008665\,\text{u}) - (22.989769\,\text{u})\right]c^2\left(931.5\,\text{MeV/uc}^2\right)$$
$$= 186.6\,\text{MeV}$$

$$\frac{\text{Binding energy}}{\text{nucleon}} = \frac{186.6\,\text{MeV}}{23} = \boxed{8.113\,\text{MeV/nucleon}}$$

We do a similar calculation for $^{24}_{11}\text{Na}$, consisting of 11 protons and 13 neutrons.

$$\text{Binding energy} = \left[11m\left({}^{1}_{1}\text{H}\right) + 13m\left({}^{1}_{0}\text{n}\right) - m\left({}^{24}_{11}\text{Na}\right)\right]c^2$$
$$= \left[11(1.007825\,\text{u}) + 13(1.008665\,\text{u}) - (23.990963\,\text{u})\right]c^2\left(931.5\,\text{MeV/uc}^2\right)$$
$$= 193.5\,\text{MeV}$$

$$\frac{\text{Binding energy}}{\text{nucleon}} = \frac{193.5\,\text{MeV}}{24} = \boxed{8.063\,\text{MeV/nucleon}}$$

By this measure, the nucleons in $^{23}_{11}\text{Na}$ are more tightly bound than those in $^{24}_{11}\text{Na}$.

4.
41-20

$$\text{Energy released} = \left[m\left({}^{3}_{1}\text{H}\right) - m\left({}^{3}_{2}\text{He}\right)\right]c^{2}$$
$$= \left[(3.016049\,\text{u}) - (3.016029\,\text{u})\right]c^{2}\left(931.5\,\text{MeV}/\text{u}c^{2}\right) = \boxed{0.019\,\text{MeV}}$$

5.
41-22

For the decay ${}^{11}_{6}\text{C} \rightarrow {}^{10}_{5}\text{B} + {}^{1}_{1}\text{p}$, we find the difference of the initial and the final masses:

$$\Delta m = m\left({}^{11}_{6}\text{C}\right) - m\left({}^{10}_{5}\text{B}\right) - m\left({}^{1}_{1}\text{H}\right)$$
$$= (11.011434\,\text{u}) - (10.012937\,\text{u}) - (1.007825\,\text{u}) = -0.009328\,\text{u}$$
$$= (11.011433\,\text{u}) - (10.012936\,\text{u}) - (1.007825\,\text{u}) = -0.0099318\,\text{u}.$$

Since the final masses are more than the original mass, energy would not be conserved.

6.
41-26

The kinetic energy of the electron will be maximum if the (essentially) massless neutrino has no kinetic energy. We also ignore the recoil energy of the sodium. The maximum kinetic energy of the reaction is then the Q-value of the reaction. Note that the "new" electron mass is accounted for by using atomic masses.

$$K = Q = \left[m\left({}^{23}_{10}\text{Ne}\right) - m\left({}^{23}_{11}\text{Na}\right)\right]c^2 = \left[(22.9945\,\text{u}) - (22.9898\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right)$$

$$= \boxed{4.4\,\text{MeV}}$$

If the neutrino were to have all of the kinetic energy, then the minimum kinetic energy of the electron is $\boxed{0.}$ The sum of the kinetic energy of the electron and the energy of the neutrino must be the Q-value, and so the neutrino energies are $\boxed{0}$ and $\boxed{4.4\text{ MeV}}$, respectively.

7.
41-30

For alpha decay we have ${}^{218}_{84}\text{Po} \rightarrow {}^{214}_{82}\text{Pb} + {}^{4}_{2}\text{He}$. We find the Q value.

$$Q = \left[m\left({}^{218}_{84}\text{Po}\right) - m\left({}^{214}_{82}\text{Pb}\right) - m\left({}^{4}_{2}\text{He}\right)\right]c^2$$

$$= [218.008965\,\text{u} - 213.999805\,\text{u} - 4.002603\,\text{u}]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right)$$

$$= \boxed{6.108\,\text{MeV}}$$

For beta decay we have ${}^{218}_{84}\text{Po} \rightarrow {}^{218}_{85}\text{At} + {}^{0}_{-1}\text{e} + \overline{\nu}$. We assume the neutrino is massless, and find the Q value.

$$Q = \left[m\left({}^{218}_{84}\text{Po}\right) - m\left({}^{218}_{85}\text{At}\right)\right]c^2$$

$$= [218.008965\,\text{u} - 218.008694\,\text{u}]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{0.252\,\text{MeV}}$$

8.
41-39

(*a*) The decay constant can be found from the half-life, using Eq. 41-8.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.5\times10^{9}\,\text{yr}} = \boxed{1.5\times10^{-10}\,\text{yr}^{-1}} = 4.9\times10^{-18}\,\text{s}^{-1}$$

(*b*) The half-life can be found from the decay constant, using Eq. 41-8.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{3.2\times10^{-5}\,\text{s}^{-1}} = 21660\,\text{s} = \boxed{6.0\,\text{h}}$$

9.
41-44

The activity of a sample is given by Eq. 41-7a. There are two different decay constants involved. Note that Appendix F gives half-lives, not activities.

$$\lambda_1 N_1 = \lambda_{\text{Co}} N_{\text{Co}} \rightarrow \lambda_1 N_0 e^{-\lambda_1 t} = \lambda_{\text{Co}} N_0 e^{-\lambda_{\text{Co}} t} \rightarrow \frac{\lambda_1}{\lambda_{\text{Co}}} = e^{(\lambda_1 - \lambda_{\text{Co}})t} \rightarrow$$

$$t = \frac{\ln(\lambda_1/\lambda_{\text{Co}})}{\lambda_1 - \lambda_{\text{Co}}} = \frac{\ln\left(\dfrac{T_{\text{Co}\,1/2}}{T_{1\,1/2}}\right)}{\dfrac{\ln 2}{T_{1\,1/2}} - \dfrac{\ln 2}{T_{\text{Co}\,1/2}}} = \frac{\ln\left[\dfrac{(5.2710\,\text{y})(365.25\,\text{d/y})}{(8.0233\,\text{d})}\right]}{\ln 2\left[\dfrac{1}{(8.0233\,\text{d})} - \dfrac{1}{(5.2710\,\text{y})(365.25\,\text{d/y})}\right]} = \boxed{63.703\,\text{d}}$$

10.
41-49

We find the mass from the initial decay rate and Eq. 41-7b.

$$\left|\frac{dN}{dt}\right|_0 = \lambda N_0 = \lambda m \frac{6.02\times 10^{23}\ \text{nuclei/mole}}{(\text{atomic weight})\,\text{g/mole}} \rightarrow$$

$$m = \left|\frac{dN}{dt}\right|_0 \frac{1}{\lambda}\frac{(\text{atomic weight})}{(6.02\times 10^{23})} = \left|\frac{dN}{dt}\right|_0 \frac{T_{1/2}}{\ln 2}\frac{(\text{atomic weight})}{(6.02\times 10^{23})}$$

$$= (2.0\times 10^{5}\,\text{s}^{-1})\frac{(1.265\times 10^{9}\,\text{yr})(3.156\times 10^{7}\,\text{s/yr})}{(\ln 2)}\frac{(39.963998\,\text{g})}{(6.02\times 10^{23})} = \boxed{0.76\,\text{g}}$$

11.
41-53

We use Eq. 41-7c.

$$R = R_0 e^{-\lambda t} = R_0 e^{-\frac{\ln 2}{T_{1/2}}t} \rightarrow T_{1/2} = -\frac{\ln 2}{\ln \frac{R}{R_0}}t = -\frac{\ln 2}{\ln \frac{1}{4}}(8.6\,\text{min}) = \boxed{4.3\,\text{min}}$$

12.
41-65

Because the tritium in water is being replenished, we assume that the amount is constant until the wine is made, and then it decays. We use Eq. 41-6.

$$N = N_0 e^{-\lambda t} \rightarrow \lambda = -\frac{\ln \frac{N}{N_0}}{t} = \frac{\ln 2}{T_{1/2}} \rightarrow t = -\frac{T_{1/2}}{\ln 2}\ln\frac{N}{N_0} = -\frac{(12.3\,\text{yr})\ln 0.10}{\ln 2} = \boxed{41\,\text{yr}}$$

13.
42-4

The Q-value tells whether the reaction requires or releases energy.

$$Q = m_p c^2 + m_{^{7}_{3}Li} c^2 - m_{^{4}_{2}He} c^2 - m_\alpha c^2 = \left[1.007825\,u + 7.016005\,u - 2(4.002603)\,u\right]\left(931.5 \frac{MeV/c^2}{u}\right)c^2$$

$$= 17.35\,MeV$$

The reaction $\boxed{\text{releases } 17.35\,\text{MeV}}$.

14.
42-9

$$Q = m_\alpha c^2 + m_{^{16}_{8}O} c^2 - m_{^{20}_{10}Ne}$$

$$= \left[4.002603\,u + 15.994915\,u - 19.992440\,u\right]\left(931.5 \frac{MeV/c^2}{u}\right)c^2 = \boxed{4.730\,MeV}$$

15.
42-24

The Q-value gives the energy released in the reaction, assuming the initial kinetic energy of the neutron is very small.

$$Q = m_n c^2 + m_{^{235}_{92}U} c^2 - m_{^{88}_{38}Sr} c^2 - m_{^{136}_{54}Xe} c^2 - 12 m_n c^2$$

$$= \left[1.008665\,u + 235.043930\,u - 87.905612\,u - 135.907219\,u - 12(1.008665\,u)\right]\left(931.5 \frac{MeV/c^2}{u}\right)c^2$$

$$= \boxed{126.5\,MeV}$$

16.
42-29

We assume as stated in problems 26 and 27 that an average of 200 MeV is released per fission of a uranium nucleus. Also, note that the problem asks for the mass of ${}^{238}_{92}\mathrm{U}$, but it is the ${}^{235}_{92}\mathrm{U}$ nucleus that undergoes the fission. Since ${}^{238}_{92}\mathrm{U}$ is almost 100% of the natural abundance, we can use the abundance of ${}^{235}_{92}\mathrm{U}$ from Appendix F as a ratio of ${}^{235}_{92}\mathrm{U}$ to ${}^{238}_{92}\mathrm{U}$.

$$\left(3\times10^{7}\,\mathrm{J}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)\left(\frac{1\,\text{nucleus}\;{}^{235}_{92}\mathrm{U}}{200\,\mathrm{MeV}}\right)\left(\frac{1\,\text{atom}\;{}^{238}_{92}\mathrm{U}}{0.0072\,\text{atoms}\;{}^{235}_{92}\mathrm{U}}\right)\left(\frac{0.238\,\mathrm{kg}}{6.02\times10^{23}\,\text{nuclei}\;{}^{238}_{92}\mathrm{U}}\right)$$

$$=5.15\times10^{-5}\,\mathrm{kg}\;{}^{238}_{92}\mathrm{U}\approx\boxed{5\times10^{-5}\,\mathrm{kg}\;{}^{238}_{92}\mathrm{U}}$$

17.
42-36

The Q-value gives the energy released in the reaction.

$$Q=m_{{}^{2}_{1}\mathrm{H}}c^{2}+m_{{}^{3}_{1}\mathrm{H}}c^{2}-m_{{}^{4}_{2}\mathrm{He}}c^{2}-m_{n}c^{2}$$

$$=\left[2.014082\,\mathrm{u}+3.016049\,\mathrm{u}-4.002603\,\mathrm{u}-1.008665\,\mathrm{u}\right]\left(931.5\,\frac{\mathrm{MeV}/c^{2}}{\mathrm{u}}\right)c^{2}=\boxed{17.57\,\mathrm{MeV}}$$

18.
42-42

(*a*) The reactants have a total of 3 protons and 7 neutrons, and so the products should have the same. After accounting for the helium, there are 3 neutrons and 1 proton in the other product, and so it must be tritium, ${}^{3}_{1}\text{He}$. The reaction is ${}^{6}_{3}\text{Li} + {}^{1}_{0}\text{n} \rightarrow {}^{4}_{2}\text{He} + {}^{3}_{1}\text{H}$.

(*b*) The *Q*-value gives the energy released.

$$Q = m_{{}^{6}_{3}\text{Li}}c^2 + m_{{}^{1}_{0}\text{n}}c^2 - m_{{}^{4}_{2}\text{He}}c^2 - m_{{}^{3}_{1}\text{He}}c^2$$

$$= \left[6.015123\,\text{u} + 1.008665\,\text{u} - 4.002603\,\text{u} - 3.016049\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{4.784\,\text{MeV}}$$

19.
42-48

Because the quality factor of alpha particles is 20 and the quality factor of X-rays is 1, it takes 20 times as many rads of X-rays to cause the same biological damage as compared to alpha particles. Thus the 250 rads of alpha particles is equivalent to $250\,\text{rad} \times 20 = \boxed{5000\,\text{rad}}$ of X-rays.