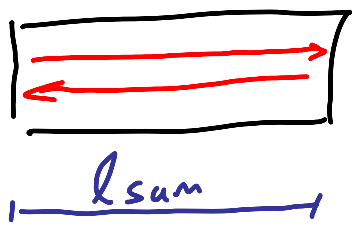
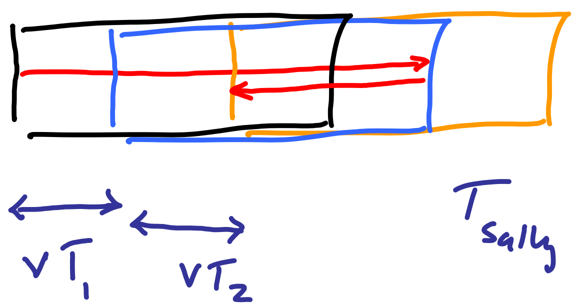
P123 - 2013 - Solns to Prob seT 1

1. length contraction derivation



Sam's point of view

$T_{sam}\, c = l_{sam}$



Sally's point of view

$T_{sally} = T_1 + T_2$

$$c = \frac{l_{sally} + vT_1}{T_1} \rightarrow T_1 c = l_{sally} + vT_1 \rightarrow T_1 = \frac{l_{sally}}{c-v}$$

$$c = \frac{l_{sally} - vT_2}{T_2} \rightarrow T_2 = \frac{l_{sally}}{c+v}$$

$$T_{sally} = T_1 + T_2 = l_{sally}\left(\frac{1}{c-v}\right)\left(\frac{1}{c+v}\right) = l_{sally}\frac{2c}{c^2+v^2}$$

$$T_{sally} = \gamma T_{sam} = \gamma \frac{2 l_{sam}}{c} = \frac{2 l_{sally}}{c\left[1-(v/c)^2\right]}$$

Proper frame

$$\boxed{l_{sam} = l_{sally}\,\gamma}$$

Length contraction

2. 36-1

You measure the contracted length. Find the rest length from Eq. 36-3a.

$$\ell_0 = \frac{\ell}{\sqrt{1-v^2/c^2}} = \frac{38.2\,\text{m}}{\sqrt{1-(0.850)^2}} = \boxed{72.5\,\text{m}}$$

3. 36-2

We find the lifetime at rest from Eq. 36-1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = (4.76\times10^{-6}\,\text{s})\sqrt{1-\left(\frac{2.70\times10^8\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2} = \boxed{2.07\times10^{-6}\,\text{s}}$$

You are measuring the dilated time - Transform to proper frame

4. 36-4

The measured distance is the contracted length. Use Eq. 36-3a.

$$\ell = \ell_0\sqrt{1-v^2/c^2} = (135\,\text{ly})\sqrt{1-\left(\frac{2.80\times10^8\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2} = \boxed{48.5\,\text{ly}}$$

5. 36-13

(*a*) In the Earth frame, the clock on the *Enterprise* will run slower. Use Eq. 36-1a.

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = (5.0\,\text{yr})\sqrt{1-(0.74)^2} = \boxed{3.4\,\text{yr}}$$

(*b*) Now we assume the 5.0 years is the time as measured on the *Enterprise*. Again use Eq. 36-1a.

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} \;\rightarrow\; \Delta t = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \frac{(5.0\,\text{yr})}{\sqrt{1-(0.74)^2}} = \boxed{7.4\,\text{yr}}$$

in this case the 5 yrs is in the proper frame

6. 36-16

The dimension along the direction of motion is contracted, and the other two dimensions are unchanged. Use Eq. 36-3a to find the contracted length.

$$\ell = \ell_0\sqrt{1 - v^2/c^2}\;;\; V = \ell(\ell_0)^2 = (\ell_0)^3\sqrt{1-v^2/c^2} = (2.0\,\text{m})^3\sqrt{1-(0.80)^2} = \boxed{4.8\,\text{m}^3}$$

7. 36-17

The vertical dimensions of the ship will not change, but the horizontal dimensions will be contracted according to Eq. 36-3a. The base will be contracted as follows.

$$\ell_{\text{base}} = \ell\sqrt{1-v^2/c^2} = \ell\sqrt{1-(0.95)^2} = \boxed{0.31\ell}$$

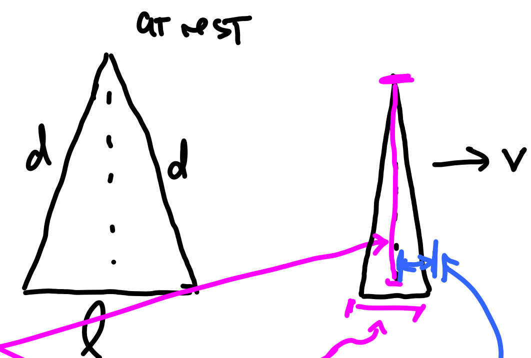
When at rest, the angle of the sides with respect to the base is given by $\theta = \cos^{-1}\dfrac{0.50\ell}{2.0\ell} = 75.52°$.

The vertical component of $\ell_{\text{vert}} = 2\ell\sin\theta = 2\ell\sin 75.52° = 1.936\ell$ is unchanged. The horizontal component, which is $2\ell\cos\theta = 2\ell\left(\tfrac{1}{4}\right) = 0.50\ell$ at rest, will be contracted in the same way as the base.

$$\ell_{\text{horizontal}} = 0.50\ell\sqrt{1-v^2/c^2} = 0.50\ell\sqrt{1-(0.95)^2} = 0.156\ell$$

Use the Pythagorean theorem to find the length of the leg.

$$\ell_{\text{leg}} = \sqrt{\ell^2_{\text{horizontal}} + \ell^2_{\text{vert}}} = \sqrt{(0.156\ell)^2 + (1.936\ell)^2} = 1.942\ell \approx \boxed{1.94\ell}$$



8.
36-18

In the Earth frame, the average lifetime of the pion will be dilated according to Eq. 36-1a. The speed of the pion will be the distance moved in the Earth frame times the dilated time.

$$v=\frac{d}{\Delta t}=\frac{d}{\Delta t_0}\sqrt{1-v^2/c^2} \rightarrow$$

$$v=c\sqrt{\frac{1}{1+\left(\frac{c\Delta t_0}{d}\right)^2}}=c\sqrt{\frac{1}{1+\left(\frac{(3.00\times10^8\text{ m/s})(2.6\times10^{-8}\text{ s})}{(25\text{m})}\right)^2}}=\boxed{0.95c}$$

9.
36-19

We take the positive direction in the direction of the *Enterprise*. Consider the alien vessel as reference frame S, and the Earth as reference frame S′. The velocity of the Earth relative to the alien vessel is $v=-0.60c$. The velocity of the *Enterprise* relative to the Earth is $u'_x=0.90c$. Solve for the velocity of the *Enterprise* relative to the alien vessel, u_x, using Eq. 36-7a.

$$u_x=\frac{(u'_x+v)}{\left(1+\frac{vu'_x}{c^2}\right)}=\frac{(0.90c-0.60c)}{[1+(-0.60)(0.90)]}=\boxed{0.65c}$$

We could also have made the *Enterprise* as reference frame S, with $v=-0.90c$, and the velocity of the alien vessel relative to the Earth as $u'_x=0.60c$. The same answer would result.

Choosing the two spacecraft as the two reference frames would also work. Let the alien vessel be reference frame S, and the *Enterprise* be reference frame S′. Then we have the velocity of the Earth relative to the alien vessel as $u_x=-0.60c$, and the velocity of the Earth relative to the *Enterprise* as $u'_x=-0.90c$. We solve for v, the velocity of the *Enterprise* relative to the alien vessel.

$$u_x=\frac{(u'_x+v)}{\left(1+\frac{vu'_x}{c^2}\right)} \rightarrow v=\frac{u_x-u'_x}{\left(1-\frac{u'_xu_x}{c^2}\right)}=\frac{(-.60c)-(-0.90c)}{\left(1-\frac{(-0.90c)(-.60c)}{c^2}\right)}=\boxed{0.65c}$$

10.
36-30

(*a*) We choose the train as frame S′ and the Earth as frame S. Since the guns fire simultaneously in S′, we set these times equal to zero, that is $t'_A = t'_B = 0$. To simplify the problem we also set the location of gunman A equal to zero in frame S′ when the guns were fired, $x'_A = 0$. This places gunman B at $x'_B = 55.0\,\text{m}$. Use Eq. 36-6 to determine the time that each gunman fired his weapon in frame S.

$$t_A = \gamma\left(t'_A + \frac{vx'_A}{c^2}\right) = \gamma\left(0 + \frac{v \times 0}{c^2}\right) = 0$$

$$t_B = \gamma\left(t'_B + \frac{vx'_B}{c^2}\right) = \frac{1}{\sqrt{1-\left(35.0\,\text{m/s}/3.00\times10^8\,\text{m/s}\right)^2}}\left(0 + \frac{(35\,\text{m/s})(55.0\,\text{m})}{\left(3.00\times10^8\,\text{m/s}\right)^2}\right) = 2.14\times10^{-14}\,\text{s}$$

Therefore, in Frame S, $\boxed{\text{A fired first}}$.

(*b*) As found in part (*a*), the difference in time is $\boxed{2.14\times10^{-14}\,\text{s}}$.

(*c*) In the Earth frame of reference, since A fired first, $\boxed{\text{B was struck first}}$. In the train frame, A is moving away from the bullet fired toward him, and B is moving toward the bullet fired toward him. Thus B will be struck first in this frame as well.

True ... but you can calculate the times + show which is first.