Physics 123 - Spring 2013 - Solutions to problem Set 3

1. 14-7

The maximum velocity is given by Eq. 14-9a.

$$v_{max} = \omega A = \frac{2\pi A}{T} = \frac{2\pi(0.15\,\mathrm{m})}{7.0\,\mathrm{s}} = \boxed{0.13\,\mathrm{m/s}}$$

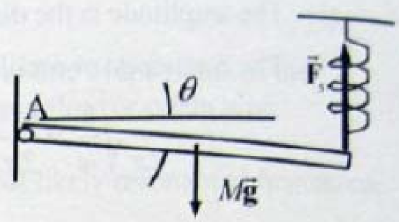
The maximum acceleration is given by Eq. 14-9b.

$$a_{max} = \omega^2 A = \frac{4\pi^2 A}{T^2} = \frac{4\pi^2(0.15\,\mathrm{m})}{(7.0\,\mathrm{s})^2} = 0.1209\,\mathrm{m/s^2} \approx \boxed{0.12\,\mathrm{m/s^2}}$$

$$\frac{a_{max}}{g} = \frac{0.1209\,\mathrm{m/s^2}}{9.80\,\mathrm{m/s^2}} = 1.2\times10^{-2} = \boxed{1.2\%}$$

2. 14-11

We assume that the spring is stretched some distance y_0 while the rod is in equilibrium and horizontal. Calculate the net torque about point A while the object is in equilibrium, with clockwise torques as positive.



$$\sum \tau = Mg\left(\tfrac{1}{2}\ell\right) - F_s\ell = \tfrac{1}{2}Mg\ell - ky_0\ell = 0$$

Now consider the rod being displaced an additional distance y below the horizontal, so that the rod makes a small angle of θ as shown in the free-body diagram. Again write the net torque about point A. If the angle is small, then there has been no appreciable horizontal displacement of the rod.

$$\sum \tau = Mg\left(\tfrac{1}{2}\ell\right) - F_s\ell = \tfrac{1}{2}Mg\ell - k\left(y + y_0\right)\ell = I\alpha = \tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2}$$

Include the equilibrium condition, and the approximation that $y = \ell\sin\theta \approx \ell\theta$.

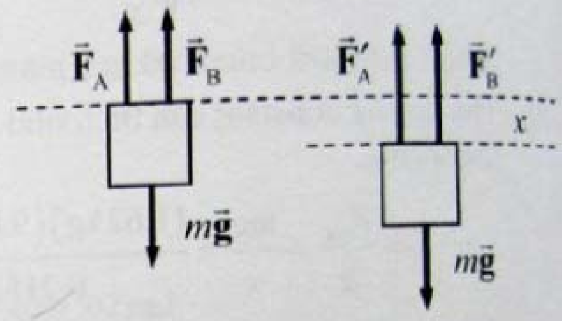
$$\tfrac{1}{2}Mg\ell - ky\ell - ky_0\ell = \tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2} \rightarrow \tfrac{1}{2}Mg\ell - ky\ell - \tfrac{1}{2}Mg\ell = \tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2} \rightarrow$$

$$-k\ell^2\theta = \tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2} \rightarrow \frac{d^2\theta}{dt^2} + \frac{3k}{M}\theta = 0$$

This is the equation for simple harmonic motion, corresponding to Eq. 14-3, with $\omega^2 = \frac{3k}{M}$.

$$\omega^2 = 4\pi^2 f^2 = \frac{3k}{M} \rightarrow f = \boxed{\frac{1}{2\pi}\sqrt{\frac{3k}{M}}}$$

3.
P4-24



Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's second law for vertical forces, with up as positive, gives this.

$$\sum F_y = F_A + F_B - mg = 0 \quad \rightarrow \quad F_A + F_B = mg$$

Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x < 0$. Again write Newton's second law for vertical forces.

$$\sum F_y = F_{net} = F'_A + F'_B - mg = F_A - kx + F_B - kx - mg = -2kx + (F_A + F_B - mg) = -2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows.

$$f = \frac{1}{2\pi}\sqrt{k_{\text{effective}}/m} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

4,
14-25

(*a*) If the block is displaced a distance x to the right in Figure 14-32a, then the length of spring # 1 will be increased by a distance x_1 and the length of spring # 2 will be increased by a distance x_2, where $x = x_1 + x_2$. The force on the block can be written $F = -k_{\text{eff}}x$. Because the springs are massless, they act similar to a rope under tension, and the same force F is exerted by each spring. Thus $F = -k_{\text{eff}}x = -k_1x_1 = -k_2x_2$.

$$x = x_1 + x_2 = -\frac{F}{k_1} - \frac{F}{k_2} = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = -\frac{F}{k_{\text{eff}}} \rightarrow \frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}} = \boxed{2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}}$$

(*b*) The block will be in equilibrium when it is stationary, and so the net force at that location is zero. Then, if the block is displaced a distance x to the right in the diagram, then spring # 1 will exert an additional force of $F_1 = -k_1x$, in the opposite direction to x. Likewise, spring # 2 will exert an additional force $F_2 = -k_2x$, in the same direction as F_1. Thus the net force on the displaced block is $F = F_1 + F_2 = -k_1x - k_2x = -(k_1 + k_2)x$. The effective spring constant is thus $k = k_1 + k_2$, and the period is given by $T = 2\pi\sqrt{\frac{m}{k}} = \boxed{2\pi\sqrt{\frac{m}{k_1 + k_2}}}$.

5. 14-41

The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{L/g} \rightarrow \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Mars}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \rightarrow$$

$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (1.35\,\text{s})\sqrt{\frac{1}{0.37}} = \boxed{2.2\,\text{s}}$$

6. 14-43

We consider this a simple pendulum. Since the motion starts at the amplitude position at $t = 0$, we may describe it by a cosine function with no phase angle, $\theta = \theta_{\max}\cos\omega t$. The angular velocity can be written as a function of the length, $\theta = \theta_{\max}\cos\left(\sqrt{\frac{g}{\ell}}\,t\right)$.

(a) $\theta(t = 0.35\,\text{s}) = 13^\circ\cos\left(\sqrt{\frac{9.80\,\text{m/s}^2}{0.30\,\text{m}}}(0.35\,\text{s})\right) = \boxed{-5.4^\circ}$

(b) $\theta(t = 3.45\,\text{s}) = 13^\circ\cos\left(\sqrt{\frac{9.80\,\text{m/s}^2}{0.30\,\text{m}}}(3.45\,\text{s})\right) = \boxed{8.4^\circ}$

(c) $\theta(t = 6.00\,\text{s}) = 13^\circ\cos\left(\sqrt{\frac{9.80\,\text{m/s}^2}{0.30\,\text{m}}}(6.00\,\text{s})\right) = \boxed{-13^\circ}$

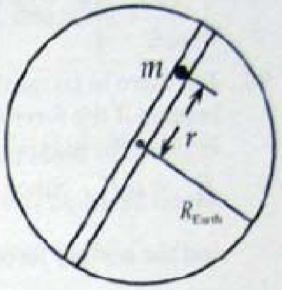
7. 14-60

The amplitude of a damped oscillator decreases according to $A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$. The data can be used to find the damping constant.

$$A = A_0 e^{-\frac{bt}{2m}} \rightarrow b = \frac{2m}{t}\ln\left(\frac{A_0}{A}\right) = \frac{2(0.075\,\text{kg})}{(3.5\,\text{s})}\ln\left(\frac{5.0}{2.0}\right) = \boxed{0.039\,\text{kg/s}}$$

8.
14-87

We quote from the next to last paragraph of Appendix D: "... we see that at points within a solid sphere, say 100 km below the Earth's surface, only the mass up to that radius contributes to the net force. The outer shells beyond the point in question contribute zero net gravitational effect." So when the mass is a distance r from the center of the Earth, there will be a force toward the center, opposite to r, due only to the mass within a sphere of radius r. We call that mass m_r. It is the density of the (assumed uniform) Earth, times the volume within a sphere of radius r.



$$m_r = \rho V_r = \frac{M_{\text{Earth}}}{V_{\text{Earth}}} V_r = \frac{M_{\text{Earth}}}{\frac{4}{3}\pi R_{\text{Earth}}^3} \frac{4}{3}\pi r^3 = M_{\text{Earth}} \frac{r^3}{R_{\text{Earth}}^3}$$

$$F = -\frac{Gmm_r}{r^2} = -\frac{GmM_{\text{Earth}}\frac{r^3}{R_{\text{Earth}}^3}}{r^2} = -\frac{GmM_{\text{Earth}}}{R_{\text{Earth}}^3} r$$

once in form

$$m\frac{d^2s}{dt^2} = -ks$$

$\hookrightarrow$ SHO

ωT

$\omega^2 = k$

The force on the object is opposite to and proportional to the displacement, and so will execute simple harmonic motion, with a "spring constant" of $k = \frac{GmM_{\text{Earth}}}{R_{\text{Earth}}^3}$. The time for the apple to return is the period, found from the "spring constant."

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\frac{GmM_{\text{Earth}}}{R_{\text{Earth}}^3}}} = 2\pi\sqrt{\frac{R_{\text{Earth}}^3}{GM_{\text{Earth}}}} = 2\pi\sqrt{\frac{(6.38\times10^6\,\text{m})^3}{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})}}$$

$$= \boxed{507\,\text{s or } 84.5\,\text{min}}$$

9. 15-6

To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have Eq. 15-2, $v = \sqrt{F_T/\mu}$.

$$v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{\mu}} \rightarrow \Delta t = \frac{\Delta x}{\sqrt{\frac{F_T}{\mu}}} = \frac{8.0\,\text{m}}{\sqrt{\frac{140\,\text{N}}{(0.65\,\text{kg})/(8.0\,\text{m})}}} = \boxed{0.19\,\text{s}}$$

10

15-10

(*a*) Both waves travel the same distance, so $\Delta x = v_1 t_1 = v_2 t_2$. We let the smaller speed be v_1, and the larger speed be v_2. The slower wave will take longer to arrive, and so t_1 is more than t_2.

$$t_1 = t_2 + 1.7\,\text{min} = t_2 + 102\,\text{s} \rightarrow v_1(t_2 + 102\,\text{s}) = v_2 t_2 \rightarrow$$

$$t_2 = \frac{v_1}{v_2 - v_1}(102\,\text{s}) = \frac{5.5\,\text{km/s}}{8.5\,\text{km/s} - 5.5\,\text{km/s}}(102\,\text{s}) = 187\,\text{s}$$

$$\Delta x = v_2 t_2 = (8.5\,\text{km/s})(187\,\text{s}) = \boxed{1600\,\text{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1.9\times10^3\,\text{km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

11.
15-24

The traveling wave is given by $D = 0.22\sin(5.6x + 34t)$.

(*a*) The wavelength is found from the coefficient of *x*.

$$5.6\,\text{m}^{-1} = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{5.6\,\text{m}^{-1}} = 1.122\,\text{m} \approx \boxed{1.1\,\text{m}}$$

(*b*) The frequency is found from the coefficient of *t*.

$$34\,\text{s}^{-1} = 2\pi f \rightarrow f = \frac{34\,\text{s}^{-1}}{2\pi} = 5.411\,\text{Hz} \approx \boxed{5.4\,\text{Hz}}$$

(c) The velocity is the ratio of the coefficients of *t* and *x*.

$$v = \lambda f = \frac{2\pi}{5.6\,\text{m}^{-1}}\frac{34\,\text{s}^{-1}}{2\pi} = 6.071\,\text{m/s} \approx \boxed{6.1\,\text{m/s}}$$

Because both coefficients are positive, the velocity is in the $\boxed{\text{negative } x \text{ direction}}$.

(*d*) The amplitude is the coefficient of the sine function, and so is $\boxed{0.22\,\text{m}}$.

(*e*) The particles on the cord move in simple harmonic motion with the same frequency as the wave. From Chapter 14, $v_{max} = D\omega = 2\pi f D$.

$$v_{max} = 2\pi f D = 2\pi\left(\frac{34\,\text{s}^{-1}}{2\pi}\right)(0.22\,\text{m}) = \boxed{7.5\,\text{m/s}}$$

The minimum speed is when a particle is at a turning point of its motion, at which time the speed is 0.

$$v_{min} = \boxed{0}$$