Physics 123 - Spring 2013 - Solns to P.S. 4

1. 14-65

We approximate that each spring of the car will effectively support one-fourth of the mass. The rotation of the improperly-balanced car tire will force the spring into oscillation. The shaking will be most prevalent at resonance, where the frequency of the tire matches the frequency of the spring. At resonance, the angular velocity of the car tire, $\omega = \frac{v}{r}$, will be the same as the angular frequency of the spring system, $\omega = \sqrt{\frac{k}{m}}$.

$$\omega = \frac{v}{r} = \sqrt{\frac{k}{m}} \rightarrow v = r\sqrt{\frac{k}{m}} = (0.42\,\mathrm{m})\sqrt{\frac{16{,}000\,\mathrm{N/m}}{\frac{1}{4}(1150\,\mathrm{kg})}} = \boxed{3.1\,\mathrm{m/s}}$$

2.
15-15

From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 3 \rightarrow A_2/A_1 = \sqrt{3} = \boxed{1.73}$$

The more energetic wave has the larger amplitude.

3.
15-17

We assume that all of the wave motion is outward along the surface of the water – no waves are propagated downwards. Consider two concentric circles on the surface of the water, centered on the place where the circular waves are generated. If there is no damping, then the power (energy per unit time) being transferred across the boundary of each of those circles must be the same. Or, the power associated with the wave must be the same at each circular boundary. The intensity depends on the amplitude squared, so for the power we have this.

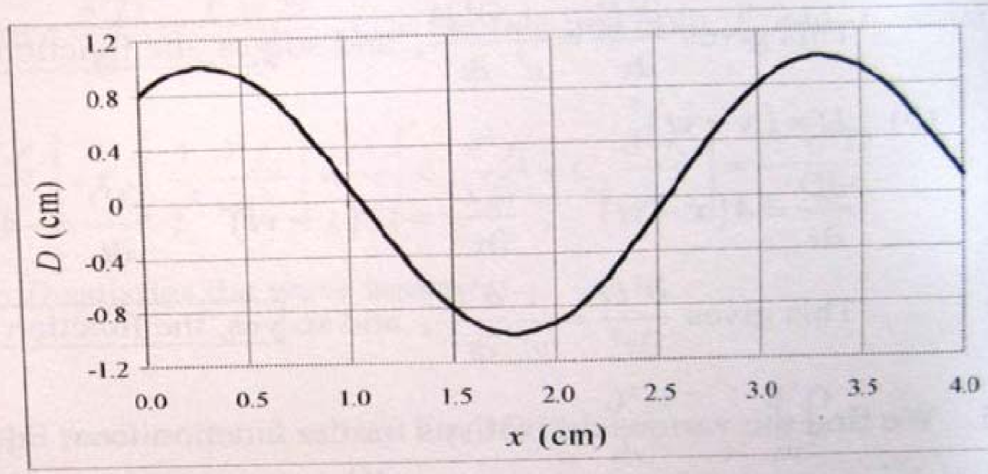
$$P = I(2\pi r) = kA^2 2\pi r = \text{constant} \rightarrow A^2 = \frac{\text{constant}}{2\pi rk} \rightarrow \boxed{A \propto \frac{1}{\sqrt{r}}}$$

4.

15-30

(a)

For the particle of string at $x = 0$, the displacement is not at the full amplitude at $t = 0$. The particle is moving upwards, and so a maximum is approaching from the right. The general form of the wave is given by $D(x,t) = A\sin(kx + \omega t + \phi)$. At $x = 0$ and $t = 0$, $D(0,0) = A\sin\phi$ and so we can find the phase angle.



$$D(0,0) = A\sin\phi \rightarrow 0.80\,\text{cm} = (1.00\,\text{cm})\sin\phi \rightarrow \phi = \sin^{-1}(0.80) = 0.93$$

So we have $D(x,0) = A\sin\left(\dfrac{2\pi}{3.0}x + 0.93\right)$, x in cm.

(b)

We use the given data to write the wave function. Note that the wave is moving to the right, and that the phase angle has already been determined.

$$D(x,t) = A\sin(kx + \omega t + \phi)$$

$$A = 1.00\,\text{cm}\ ;\ k = \frac{2\pi}{3.00\,\text{cm}} = 2.09\,\text{cm}^{-1}\ ;\ \omega = 2\pi f = 2\pi(245\,\text{Hz}) = 1540\,\text{rad/s}$$

$$\boxed{D(x,t) = (1.00\,\text{cm})\sin\left[(2.09\,\text{cm}^{-1})x + (1540\,\text{rad/s})t + 0.93\right],\ x \text{ in cm},\ t \text{ in s}}$$

S.
15.34

Find the various derivatives for the linear combination.

$$D(x,t) = C_1 D_1 + C_2 D_2 = C_1 f_1(x,t) + C_2 f_2(x,t)$$

$$\frac{\partial D}{\partial x} = C_1 \frac{\partial f_1}{\partial x} + C_2 \frac{\partial f_2}{\partial x} \; ; \; \frac{\partial^2 D}{\partial x^2} = C_1 \frac{\partial^2 f_1}{\partial x^2} + C_2 \frac{\partial^2 f_2}{\partial x^2}$$

$$\frac{\partial D}{\partial t} = C_1 \frac{\partial f_1}{\partial t} + C_2 \frac{\partial f_2}{\partial t} \; ; \; \frac{\partial^2 D}{\partial t^2} = C_1 \frac{\partial^2 f_1}{\partial t^2} + C_2 \frac{\partial^2 f_2}{\partial t^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$. Use the fact that both f_1 and f_2 satisfy the wave equation.

$$\frac{\partial^2 D}{\partial x^2} = C_1 \frac{\partial^2 f_1}{\partial x^2} + C_2 \frac{\partial^2 f_2}{\partial x^2} = C_1\left[\frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}\right] + C_2\left[\frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\left[C_1 \frac{\partial^2 f_1}{\partial t^2} + C_2 \frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

Thus we see that $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so D satisfies the wave equation.

6.
15.42

(*a*) The resultant wave is the algebraic sum of the two component waves.

$$D = D_1 + D_2 = A\sin(kx - \omega t) + A\sin(kx - \omega t + \phi) = A[\sin(kx - \omega t) + A\sin(kx - \omega t + \phi)]$$

$$= A\{2\sin\tfrac{1}{2}[(kx - \omega t) + (kx - \omega t + \phi)]\}\{\cos\tfrac{1}{2}[(kx - \omega t) - (kx - \omega t + \phi)]\}$$

$$= 2A\{\sin\tfrac{1}{2}(2kx - 2\omega t + \phi)\}\{\cos\tfrac{1}{2}(\phi)\} = \boxed{\left(2A\cos\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

(*b*) The amplitude is the absolute value of the coefficient of the sine function, $\boxed{\left|2A\cos\frac{\phi}{2}\right|}$. The wave is $\boxed{\text{purely sinusoidal}}$ because the dependence on x and t is $\sin\left(kx - \omega t + \frac{\phi}{2}\right)$.

(*c*) If $\phi = 0, 2\pi, 4\pi, \cdots, 2n\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{2n\pi}{2}\right| = |2A\cos n\pi| = |2A(\pm 1)|$ $= 2A$, which is constructive interference. If $\phi = \pi, 3\pi, 5\pi, \cdots, (2n+1)\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{(2n+1)\pi}{2}\right| = \left|2A\cos\left[\left(n+\tfrac{1}{2}\right)\pi\right]\right| = 0$, which is destructive interference.

(*d*) If $\phi = \frac{\pi}{2}$, then the resultant wave is as follows.

$$D = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right) = \left(2A\cos\frac{\pi}{4}\right)\sin\left(kx - \omega t + \frac{\pi}{4}\right) = \sqrt{2}A\sin\left(kx - \omega t + \frac{\pi}{4}\right)$$

This wave has an amplitude of $\sqrt{2}A$, is traveling in the positive x direction, and is shifted to the left by an eighth of a cycle. This is "halfway" between the two original waves. The displacement is $\frac{1}{2}A$ at the origin at $t = 0$.

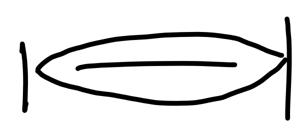
7.
15-43

The fundamental frequency of the full string is given by $f_{\text{unfingered}} = \frac{v}{2\ell} = 441\,\text{Hz}$. If the length is reduced to 2/3 of its current value, and the velocity of waves on the string is not changed, then the new frequency will be as follows.

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}\ell\right)} = \frac{3}{2}\frac{v}{2\ell} = \left(\frac{3}{2}\right) f_{\text{unfingered}} = \left(\frac{3}{2}\right)(441\,\text{Hz}) = \boxed{662\,\text{Hz}}$$

Derived in class
for case

Derive for yourself
for practice



8.
15-52

The string must vibrate in a standing wave pattern to have a certain number of loops. The frequency of the standing waves will all be 120 Hz, the same as the vibrator. That frequency is also expressed by Eq. 15-17b, $f_n = \frac{nv}{2\ell}$. The speed of waves on the string is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The tension in the string will be the same as the weight of the masses hung from the end of the string. $F_T = mg$, ignoring the mass of the string itself. Combining these relationships gives an expression for the masses hung from the end of the string.

(a) $f_n = \frac{nv}{2\ell} = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}} = \frac{n}{2\ell}\sqrt{\frac{mg}{\mu}} \rightarrow m = \frac{4\ell^2 f_n^2 \mu}{n^2 g}$

$$m_1 = \frac{4(1.50\,\mathrm{m})^2(120\,\mathrm{Hz})^2(6.6\times10^{-4}\,\mathrm{kg/m})}{1^2(9.80\,\mathrm{m/s^2})} = 8.728\,\mathrm{kg} \approx \boxed{8.7\,\mathrm{kg}}$$

(b) $m_2 = \frac{m_1}{2^2} = \frac{8.728\,\mathrm{kg}}{4} = \boxed{2.2\,\mathrm{kg}}$

(c) $m_5 = \frac{m_1}{5^2} = \frac{8.728\,\mathrm{kg}}{25} = \boxed{0.35\,\mathrm{kg}}$

9.
16-34

For a vibrating string, the frequency of the fundamental mode is given by $f=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{F_T}{m/L}}$.

$$f=\frac{1}{2L}\sqrt{\frac{F_T}{m/L}} \rightarrow F_T=4Lf^2m=4(0.32\,\text{m})(440\,\text{Hz})^2(3.5\times10^{-4}\,\text{kg})=\boxed{87\,\text{N}}$$

10.
16-35

(*a*) If the pipe is closed at one end, only the odd harmonic frequencies are present, and are given by

$$f_n = \frac{nv}{4L} = nf_1, n = 1, 3, 5 \cdots .$$

convince yourself

Derive

$$f_1 = \frac{v}{4L} = \frac{343\,\text{m/s}}{4(1.24\,\text{m})} = \boxed{69.2\,\text{Hz}}$$

$$f_3 = 3f_1 = \boxed{207\,\text{Hz}} \quad f_5 = 5f_1 = \boxed{346\,\text{Hz}} \quad f_7 = 7f_1 = \boxed{484\,\text{Hz}}$$

(*b*) If the pipe is open at both ends, all the harmonic frequencies are present, and are given by

$$f_n = \frac{nv}{2\ell} = nf_1 .$$

$$f_1 = \frac{v}{2\ell} = \frac{343\,\text{m/s}}{2(1.24\,\text{m})} = 138.3\,\text{Hz} \approx \boxed{138\,\text{Hz}}$$

$$f_2 = 2f_1 = \frac{v}{\ell} = \boxed{277\,\text{Hz}} \quad f_3 = 3f_1 = \frac{3v}{2\ell} = \boxed{415\,\text{Hz}} \quad f_4 = 4f_1 = \frac{2v}{\ell} = \boxed{553\,\text{Hz}}$$

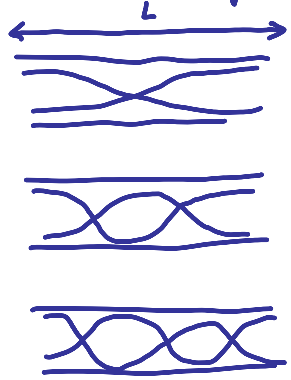
11.
16-47

The difference in frequency for two successive harmonics is 40 Hz. For an open pipe, two successive harmonics differ by the fundamental, so the fundamental could be 40 Hz, with 240 Hz being the 6th harmonic and 280 Hz being the 7th harmonic. For a closed pipe, two successive harmonics differ by twice the fundamental, so the fundamental could be 20 Hz. But the overtones of a closed pipe are odd multiples of the fundamental, and both overtones are even multiples of 30 Hz. So the pipe must be an $\boxed{\text{open pipe}}$.

$$f = \frac{v}{2\ell} \rightarrow \ell = \frac{v}{2f} = \frac{[331 + 0.60(23.0)]\,\text{m/s}}{2(40\,\text{Hz})} = \boxed{4.3\,\text{m}}$$

Giancoli's soln manual doesn't give a hugely helpful soln here.
Let's do it more explicitly

open pipe



fundamental $L = \frac{1}{2}\lambda$

2ND harmonic $L = \lambda$

3RD harmonic $L = \frac{3}{2}\lambda$

⋮

$L = \frac{m}{2}\lambda = \frac{m}{2}\frac{v}{f}$ $m = 1, 2, 3 \ldots$

$f_m = \frac{m}{2}\frac{v}{L}$

Look at Δf

$$\Delta f_{1-0} = \frac{v}{2L}$$

$$\Delta f_{1-2} = \frac{v}{2L}$$

$$\vdots$$

$$\Delta f = \frac{v}{2L}$$

$$40\ Hz = 280 - 240 = \frac{v}{2L}$$

$$L = \frac{v}{(2)(40)}$$

v sound in air at 23°

Typically I'd give this to you in a problem

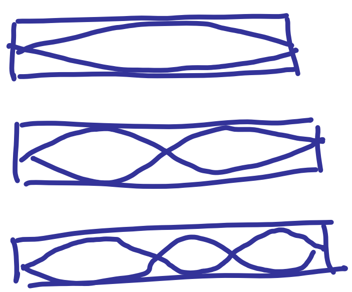
here use Giancoli eqn on p. 425

$$v \approx 331 + 0.6\,T$$

T in $^\circ C$

$$\rightarrow v = 331 + 0.6(23)$$

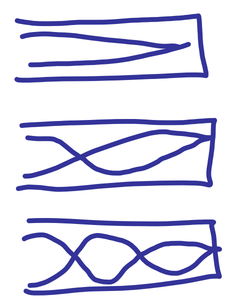
Closed Pipe



$L = \frac{1}{2}\lambda$

$L = \lambda$

$\ldots,\ L = \frac{3}{2}\lambda$



$L = \frac{1}{4}\lambda$

$L = \frac{3}{4}\lambda$

$L = \frac{5}{4}\lambda$

From wording of problem ··· NOT obvious if the closed pipe is  or  closed at both ends or just one end

From Giancoli Soln → They mean closed at one end

$L = \frac{m\lambda}{4}$ $m = 1, 3, 5 \cdots$

$L = \frac{m}{4}\frac{v}{f}$ $f = \frac{mv}{4f}$

$\Delta f = (2)\frac{v}{4} = \frac{v}{2L}$ could work as above

but if we take fundamental to be $\frac{v}{4L}$ so that $\Delta f = 40\,Hz = \frac{2v}{4L}$

then $\frac{v}{4L} = 20\,Hz$

1	×	20	=	20
3	×	20	=	60
5	×	20	=	100
7	×	20	=	140
9	×	20	=	180
11	×	20	=	220
13	×	20	=	260
14	×	20	=	280

NO 240 Hz in this case

so pipe is open at both ends

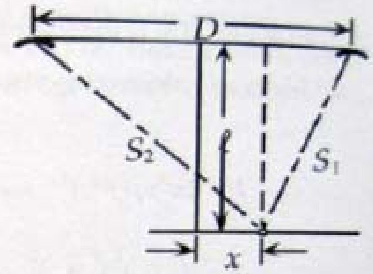
12.

16-55

Since there are 4 beats/s when sounded with the 350 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 354 Hz. Since there are 9 beats/s when sounded with the 355 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 364 Hz. The common value is $\boxed{346\text{ Hz}}$.

13.
16-58

(*a*) The microphone must be moved to the right until the difference in distances from the two sources is half a wavelength. See the diagram. We square the expression, collect terms, isolate the remaining square root, and square again.



$$S_2 - S_1 = \tfrac{1}{2}\lambda \rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}D + x\right)^2 + \ell^2} - \sqrt{\left(\tfrac{1}{2}D - x\right)^2 + \ell^2} = \tfrac{1}{2}\lambda \rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}D + x\right)^2 + \ell^2} = \tfrac{1}{2}\lambda + \sqrt{\left(\tfrac{1}{2}D - x\right)^2 + \ell^2} \rightarrow$$

$$\left(\tfrac{1}{2}D + x\right)^2 + \ell^2 = \tfrac{1}{4}\lambda^2 + 2\left(\tfrac{1}{2}\lambda\right)\sqrt{\left(\tfrac{1}{2}D - x\right)^2 + \ell^2} + \left(\tfrac{1}{2}D - x\right)^2 + \ell^2 \rightarrow$$

$$2Dx - \tfrac{1}{4}\lambda^2 = \lambda\sqrt{\left(\tfrac{1}{2}D - x\right)^2 + \ell^2} \rightarrow 4D^2x^2 - 2(2Dx)\tfrac{1}{4}\lambda^2 + \tfrac{1}{16}\lambda^4 = \lambda^2\left[\left(\tfrac{1}{2}D - x\right)^2 + \ell^2\right]$$

$$4D^2x^2 - Dx\lambda^2 + \tfrac{1}{16}\lambda^4 = \tfrac{1}{4}D^2\lambda^2 - Dx\lambda^2 + x^2\lambda^2 + \lambda^2\ell^2 \rightarrow x = \lambda\sqrt{\frac{\left(\tfrac{1}{4}D^2 + \ell^2 - \tfrac{1}{16}\lambda^2\right)}{\left(4D^2 - \lambda^2\right)}}$$

The values are $D = 3.00\,\text{m}$, $\ell = 3.20\,\text{m}$, and $\lambda = v/f = (343\,\text{m/s})/(494\,\text{Hz}) = 0.694\,\text{m}$.

$$x = (0.694\,\text{m})\sqrt{\frac{\tfrac{1}{4}(3.00\,\text{m})^2 + (3.20\,\text{m})^2 - \tfrac{1}{16}(0.694\,\text{m})^2}{4(3.00\,\text{m})^2 - (0.694\,\text{m})^2}} = \boxed{0.411\,\text{m}}$$

(*b*) When the speakers are exactly out of phase, the maxima and minima will be interchanged. The intensity maxima are 0.411 m to the left or right of the midpoint, and the intensity minimum is at the midpoint.