Physics 123 - Spring 2013 - Solutions to Prob. Set 6

1\. 31-10

The frequency of the two fields must be the same: $\boxed{80.0\,\text{kHz}}$. The rms strength of the electric field can be found from Eq. 31-11 with $v = c$.

$$E_{rms} = cB_{rms} = (3.00 \times 10^8\,\text{m/s})(7.75 \times 10^{-9}\,\text{T}) = \boxed{2.33\,\text{V/m}}$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the $\boxed{\text{horizontal north-south line.}}$

2: 31-11

(*a*) If we write the argument of the cosine function as $kz + \omega t = k(z + ct)$, we see that the wave is traveling in the $\boxed{-z \text{ direction}}$, or $\boxed{-\hat{\mathbf{k}}}$.

(*b*) $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and to the direction of propagation. At the origin, the electric field is pointing in the positive x direction. Since $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ must point in the negative z direction, $\vec{\mathbf{B}}$ must point in the the $\boxed{-y \text{ direction}}$, or $\boxed{-\hat{\mathbf{j}}}$. The magnitude of the magnetic field is found from Eq. 31-11 as $B_0 = \boxed{E_0/c}$.

3
31-18

The length of the pulse is $\Delta d = c\Delta t$. Use this to find the number of wavelengths in a pulse.

$$N = \frac{(c\Delta t)}{\lambda} = \frac{(3.00\times10^8\,\mathrm{m/s})(38\times10^{-12}\,\mathrm{s})}{(1062\times10^{-9}\,\mathrm{m})} = 10734 \approx \boxed{11{,}000 \text{ wavelengths}}$$

If the pulse is to be only one wavelength long, then its time duration is the period of the wave, which is the reciprocal of the wavelength.

$$T = \frac{1}{f} = \frac{\lambda}{c} = \frac{(1062\times10^{-9}\,\mathrm{m})}{(3.00\times10^8\,\mathrm{m/s})} = \boxed{3.54\times10^{-15}\,\mathrm{s}}$$

4.
31-25

The intensity is the power per unit area, and also is the time averaged value of the Poynting vector. The area is the surface area of a sphere, since the wave is spreading spherically.

$$\bar{S} = \frac{P}{A} = \frac{(1500\,\mathrm{W})}{\left[4\pi(5.0\,\mathrm{m})^2\right]} = 4.775\,\mathrm{W/m^2} \approx \boxed{4.8\,\mathrm{W/m^2}}$$

$$\bar{S} = c\varepsilon_0 E_{\mathrm{rms}}^2 \rightarrow E_{\mathrm{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{4.775\,\mathrm{W/m^2}}{(3.00\times10^8\,\mathrm{m/s})(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}} = \boxed{42\,\mathrm{V/m}}$$

5.
31-29

The radiation from the Sun has the same intensity in all directions, so the rate at which it reaches the Earth is the rate at which it passes through a sphere centered at the Sun with a radius equal to the Earth's orbit radius. The $1350\,\mathrm{W/m^2}$ is the intensity, or the magnitude of the Poynting vector.

$$S=\frac{P}{A} \rightarrow P=SA=4\pi R^2 S=4\pi\left(1.496\times10^{11}\,\mathrm{m}\right)^2\left(1350\,\mathrm{W/m^2}\right)=\boxed{3.80\times10^{26}\,\mathrm{W}}$$

6.
31-31

In each case, the required area is the power requirement of the device divided by 10% of the intensity of the sunlight.

(a) $A=\frac{P}{I}=\frac{50\times10^{-3}\,\mathrm{W}}{100\,\mathrm{W/m^2}}=5\times10^{-4}\,\mathrm{m^2}=\boxed{5\,\mathrm{cm^2}}$

A typical calculator is about 17 cm x 8 cm, which is about $140\,\mathrm{cm^2}$. So $\boxed{\text{yes}}$, the solar panel can be mounted directly on the calculator.

(b) $A=\frac{P}{I}=\frac{1500\,\mathrm{W}}{100\,\mathrm{W/m^2}}=15\,\mathrm{m^2}\approx\boxed{20\,\mathrm{m^2}}$ (to one sig. fig.)

A house of floor area $1000\,\mathrm{ft^2}$ would have on the order of $100\,\mathrm{m^2}$ of roof area. So $\boxed{\text{yes}}$, a solar panel on the roof should be able to power the hair dryer.

(c) $A=\frac{P}{I}=\frac{20\,\mathrm{hp}\left(746\,\mathrm{W/hp}\right)}{100\,\mathrm{W/m^2}}=149\,\mathrm{m^2}\approx\boxed{100\,\mathrm{m^2}}$ (to one sig. fig.)

This would require a square panel of side length about 12 m. So $\boxed{\text{no}}$, this panel could not be mounted on a car and used for real-time power.

7.
31-35

The acceleration of the cylindrical particle will be the force on it (due to radiation pressure) divided by its mass. The light is delivering electromagnetic energy to an area A at a rate of $\frac{dU}{dt} = 1.0\,\text{W}$. That power is related to the average magnitude of the Poynting vector by $\bar{S} = \frac{\frac{dU}{dt}}{A}$.

From Eq. 31-21a, that causes a pressure on the particle of $P = \frac{\bar{S}}{c}$, and the force due to that pressure is $F_{\text{laser}} = PA$. Combine these relationships with Newton's second law to calculate the acceleration. The mass of the particle is its volume times the density of water.

$$F_{\text{laser}} = PA = \frac{\bar{S}}{c}A = \frac{1}{c}\frac{dU}{dt} = ma = \rho_{H_2O}\pi r^2 ra \rightarrow$$

$$a = \frac{dU/dt}{c\rho_{H_2O}\pi r^3} = \frac{(1.0\text{W})}{(3.00\times10^8\,\text{m/s})(1000\,\text{kg/m}^3)\pi(5\times10^{-7}\,\text{m})^3} = \boxed{8\times10^6\,\text{m/s}^2}$$

8.
31-57

The power output of the antenna would be the intensity at a given distance from the antenna, times the area of a sphere surrounding the antenna. The intensity is the magnitude of the Poynting vector.

$$S = \tfrac{1}{2} c\varepsilon_0 E_0^2$$

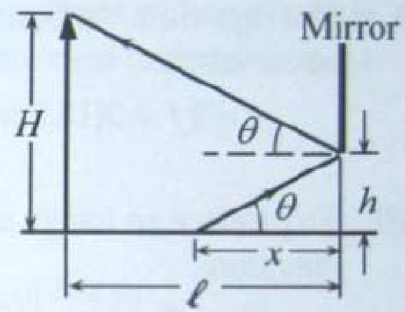
$$P = 4\pi r^2 S = 2\pi r^2 c\varepsilon_0 E_0^2 = 2\pi(0.50\,\text{m})^2 (3.00\times10^8\,\text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(3\times10^6\,\text{V/m})^2$$

$$\approx \boxed{4\times10^{10}\,\text{W}}$$

This is many orders of magnitude higher than the power output of commercial radio stations, which are no higher than the 10's of kilowatts.

9.
32-4

The angle of incidence is the angle of reflection. See the diagram for the appropriate lengths.



$$\tan\theta = \frac{(H-h)}{\ell} = \frac{h}{x} \rightarrow$$

$$\frac{(1.64\,\text{m} - 0.38\,\text{m})}{(2.30\,\text{m})} = \frac{(0.38\,\text{m})}{x} \rightarrow x = \boxed{0.69\,\text{m}}$$

10.

32-40

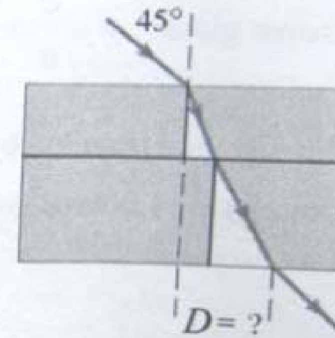
We find the incident angle in the air (relative to the normal) from Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 33.0^\circ\right) = 46.4^\circ$$

Since this is the angle relative to the horizontal, the angle as measured from the horizon is $90.0^\circ - 46.4^\circ = \boxed{43.6^\circ}$.

11.
32-43

The beam forms the hypotenuse of two right triangles as it passes through the plastic and then the glass. The upper angle of the triangle is the angle of refraction in that medium. Note that the sum of the opposite sides is equal to the displacement D. First, we calculate the angles of refraction in each medium using Snell's Law (Eq. 32-5).



$$\sin 45 = n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 = \sin^{-1}\left(\frac{\sin 45}{n_1}\right) = \sin^{-1}\left(\frac{\sin 45}{1.62}\right) = 25.88^\circ$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 45}{n_2}\right) = \sin^{-1}\left(\frac{\sin 45}{1.47}\right) = 28.75^\circ$$

We then use the trigonometric identity for tangent to calculate the two opposite sides, and sum to get the displacement.

$$D = D_1 + D_2 = h_1 \tan\theta_1 + h_1 \tan\theta_1 = (2.0\text{cm})\tan 25.88^\circ + (3.0\text{cm})\tan 28.75^\circ = \boxed{2.6\text{cm}}$$

12.
32-44

(*a*) We use Eq. 32-5 to calculate the refracted angle as the light enters the glass (n=1.56) from the air (n=1.00).

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.56}\sin\ 43.5^\circ\right] = 26.18^\circ \approx \boxed{26.2^\circ}$$

(*b*) We again use Eq. 32-5 using the refracted angle in the glass and the indices of refraction of the glass and water.

$$\theta_3 = \sin^{-1}\left[\frac{n_2}{n_3}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.56}{1.33}\sin\ 26.18^\circ\right] = 31.17^\circ \approx \boxed{31.2^\circ}$$

(c) We repeat the same calculation as in part (a), but using the index of refraction of water.

$$\theta_3 = \sin^{-1}\left[\frac{n_1}{n_3}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.33}\sin\ 43.5^\circ\right] = 31.17^\circ \approx \boxed{31.2^\circ}$$

As expected the refracted angle in the water is the same whether the light beam first passes through the glass, or passes directly into the water.

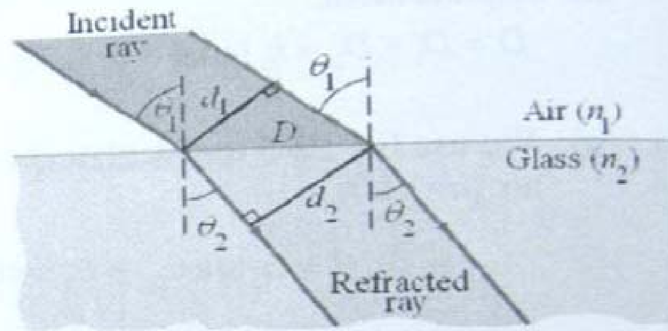
13.

32-47

As the light ray passes from air into glass with an angle of incidence of 25°, the beam will refract. Determine the angle of refraction by applying Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow$$

$$\theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.5}\sin\ 25°\right] = 16.36°$$



We now consider the two right triangles created by the diameters of the incident and refracted beams with the air–glass interface, as shown in the figure. The diameters form right angles with the ray direction and using complementary angles we see that the angle between the diameter and the interface is equal to the incident and refracted angles. Since the air–glass interface creates the hypotenuse for both triangles we use the definition of the cosine to solve for this length in each triangle and set the lengths equal. The resulting equation is solved for the diameter of the refracted ray.

$$D = \frac{d_1}{\cos\theta_1} = \frac{d_2}{\cos\theta_2} \quad \rightarrow \quad d_2 = d_1\frac{\cos\theta_2}{\cos\theta_1} = (3.0\,\text{mm})\frac{\cos 16.36°}{\cos 25°} = \boxed{3.2\,\text{mm}}$$

14.
32-52

We find the speed of light from the speed of light in a vacuum divided by the index of refraction. Examining the graph we estimate that the index of refraction of 450 nm light in silicate flint glas 1.643 and of 680 nm light is 1.613. There will be some variation in the answers due to estimatio from the graph.

$$\frac{v_{\text{red}} - v_{\text{blue}}}{v_{\text{red}}} = \frac{c/n_{680} - c/n_{450}}{c/n_{680}} = \frac{1/1.613 - 1/1.643}{1/1.613} = 0.01826 \approx \boxed{1.8\%}$$

15.
32-57

When the light in the material with a higher index is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

$$n_{\text{diamond}} \sin\theta_1 = n_{\text{water}} \sin\theta_2 \quad \rightarrow \quad \theta_1 = \sin^{-1}\left(\frac{n_{\text{water}}}{n_{\text{diamond}}}\right) = \sin^{-1}\frac{1.33}{2.42} = \boxed{33.3°}$$

Because diamond has the higher index, the light must start in $\boxed{\text{diamond.}}$

16.
32-59

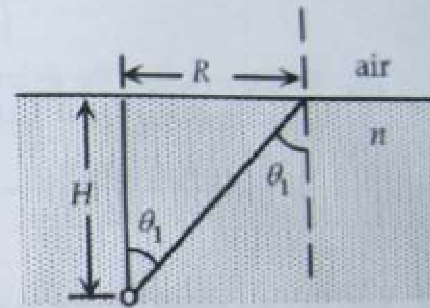
We find the critical angle for light leaving the water:

$$n_{\text{water}} \sin\theta_1 = n_{\text{air}} \sin\theta_2 \rightarrow$$

$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\frac{1.00}{1.33} = 48.75^\circ$$

If the light is incident at a greater angle than this, it will totally reflect. Find R from the diagram.

$$R > H\tan\theta_1 = (72.0\,\text{cm})\tan 48.75^\circ = \boxed{82.1\,\text{cm}}$$



17.
32-63

(*a*) We calculate the critical angle using Eq. 32-7. We calculate the time for each ray to pass through the fiber by dividing the length the ray travels by the speed of the ray in the fiber. The length for ray A is the horizontal length of the fiber. The length for ray B is equal to the length of the fiber divided by the critical angle, since ray B is always traveling along a diagonal line at the critical angle relative to the horizontal. The speed of light in the fiber is the speed of light in a vacuum divided by the index of refraction in the fiber.

$$\sin\theta_c = \frac{n_2}{n_1} \;;\; \Delta t = t_B - t_A = \frac{\ell_B}{v} - \frac{\ell_A}{v} = \frac{\ell_A}{v\sin\theta_c} - \frac{\ell_A}{v} = \frac{\ell_A}{c/n_1}\left(\frac{n_1}{n_2} - 1\right)$$

$$= \frac{(1.0\,\text{km})(1.465)}{(3.00\times10^5\,\text{km/s})}\left(\frac{1.465}{1.000} - 1\right) = \boxed{2.3\times10^{-6}\,\text{s}}$$

(*b*) We now replace the index of refraction of air ($n = 1.000$) with the index of refraction of the glass "cladding" ($n = 1.460$).

$$\Delta t = \frac{\ell_A n_1}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{(1.0\,\text{km})(1.465)}{3.00\times10^5\,\text{km/s}}\left(\frac{1.465}{1.460} - 1\right) = \boxed{1.7\times10^{-8}\,\text{s}}$$