Physics 123 - 2013 - Solutions to Prob. Set 7

1.
33-4

To form a real image from a real object requires a $\boxed{\text{converging lens}}$. We find the focal length of the lens from Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_o d_i}{d_o+d_i}=\frac{(1.85\,\text{m})(0.483\,\text{m})}{1.85\,\text{m}+0.483\,\text{m}}=\boxed{0.383\,\text{m}}$$

Because $d_i>0$, the image is $\boxed{\text{real.}}$

2.
33-5

(*a*) We find the image distance from Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_o f}{d_o-f}=\frac{(10.0\,\text{m})(0.105\,\text{m})}{10.0\,\text{m}-0.105\,\text{m}}=0.106\,\text{m}=\boxed{106\,\text{mm}}$$

(*b*) Use the same general calculation.

$$d_i=\frac{d_o f}{d_o-f}=\frac{(3.0\,\text{m})(0.105\,\text{m})}{3.0\,\text{m}-0.105\,\text{m}}=0.109\,\text{m}=\boxed{109\,\text{mm}}$$

(*c*) Use the same general calculation.

$$d_i=\frac{d_o f}{d_o-f}=\frac{(1.0\,\text{m})(0.105\,\text{m})}{1.0\,\text{m}-0.105\,\text{m}}=0.117\,\text{m}=\boxed{117\,\text{mm}}$$

(*d*) We find the smallest object distance from the maximum image distance.

$$\frac{1}{d_{o_{\text{min}}}}+\frac{1}{d_{i_{\text{max}}}}=\frac{1}{f} \rightarrow d_{o_{\text{min}}}=\cancel{\frac{d_{i_{\text{max}}}-f}{d_{i_{\text{max}}}f}}=\frac{(132\,\text{mm})(105\,\text{mm})}{132\,\text{mm}-105\,\text{mm}}=513\,\text{mm}=\boxed{0.513\,\text{m}}$$

$$d_{o_{\text{min}}}=\frac{d_{i_{\text{max}}}f}{d_{i_{\text{max}}}-f}$$

3.
33-8

Use Eqs. 33-1 and 33-2 to find the image distance, and Eq. 33-3 to find the image height.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_i=\frac{d_o}{Pd_o-1}=\frac{(0.125\,\text{m})}{(-8.00\,\text{D})(0.125\,\text{m})-1}=-0.0625\,\text{m}=\boxed{-6.25\,\text{cm}}$$

Since the image distance is negative, the image is $\boxed{\text{virtual and behind the lens}}$. same side as ant

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow h_i=-\frac{d_i}{d_o}h_o=-\frac{(-6.25\,\text{cm})}{12.5\,\text{cm}}(1.00\,\text{mm})=\boxed{0.500\,\text{mm (upright)}}$$

4.
33-18

(a) Use Eq. 33-2 with $d_o+d_i=d_T \rightarrow d_i=d_T-d_o$.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{(d_T-d_o)}=\frac{1}{f} \rightarrow d_o^2-d_Td_o+fd_T=0 \rightarrow d_o=\frac{d_T\pm\sqrt{d_T^2-4fd_T}}{2}$$

There are only real solutions for d_o if $d_T^2-4fd_T>0 \rightarrow d_T>4f$. If that condition is met, then there will be two locations for the lens, at distances $d_o=\frac{1}{2}\left(d_T\pm\sqrt{d_T^2-4fd_T}\right)$ from the object, that will form sharp images on the screen.

(b) If $d_T<4f$, then Eq. 33-2 cannot be solved for real values of d_o or d_i.

(c) If $d_T>4f$, the lens locations relative to the object are given by $d_{o1}=\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)$ and $d_{o2}=\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)$.

$$\Delta d=d_{o1}-d_{o2}=\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)-\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)=\boxed{\sqrt{d_T^2-4fd_T}}$$

Find the ratio of image sizes using Eq. 33-3.

$$\frac{h_{i2}}{h_{i1}}=\frac{-h_o\frac{d_{i2}}{d_{o2}}}{-h_o\frac{d_{i1}}{d_{o1}}}=\frac{d_{i2}}{d_{o2}}\frac{d_{o1}}{d_{i1}}=\frac{d_T-d_{o2}}{d_{o2}}\frac{d_{o1}}{d_T-d_{o1}}$$

$$=\left[\frac{d_T-\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)}{\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)}\right]\left[\frac{\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)}{d_T-\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)}\right]=\boxed{\left(\frac{d_T+\sqrt{d_T^2-4fd_T}}{d_T-\sqrt{d_T^2-4fd_T}}\right)^2}$$

5.
33-20

The first lens is the converging lens. An object at infinity will form an image at the focal point of the converging lens, by Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1}=\frac{1}{\infty}+\frac{1}{d_{i1}} \rightarrow d_{i1}=f_1=20.0\,\mathrm{cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, and so $d_{o2}=-6.0\,\mathrm{cm}$. Again use Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-6.0\,\mathrm{cm})(-33.5\,\mathrm{cm})}{(-6.0\,\mathrm{cm})-(-33.5\,\mathrm{cm})}=7.3\,\mathrm{cm}$$

Thus the final image is real, $\boxed{\text{7.3 cm beyond the second lens.}}$

6.
33-21

Find the image formed by the first lens, using Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(35.0\,\mathrm{cm})(25.0\,\mathrm{cm})}{(35.0\,\mathrm{cm})-(25.0\,\mathrm{cm})}=87.5\,\mathrm{cm}$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance.

$$d_{o2}=16.5\,\mathrm{cm}-87.5\,\mathrm{cm}=-71.0\,\mathrm{cm}$$

Find the image formed by the second lens, again using Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-71.0\,\mathrm{cm})(25.0\,\mathrm{cm})}{(-71.0\,\mathrm{cm})-(25.0\,\mathrm{cm})}=18.5\,\mathrm{cm}$$

Thus the final image is real, $\boxed{\text{18.5 cm beyond second lens.}}$

The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$=\frac{(+87.5\,\mathrm{cm})(+18.5\,\mathrm{cm})}{(+35.0\,\mathrm{cm})(-71.0\,\mathrm{cm})}=\boxed{-0.651\times(\text{inverted})}$$

7.
33-25

(*a*) The first lens is the converging lens. Find the image formed by the first lens.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(60.0\,\text{cm})(20.0\,\text{cm})}{(60.0\,\text{cm})-(20.0\,\text{cm})}=30.0\,\text{cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, and so $d_{o2}=25.0\,\text{cm}-30.0\,\text{cm}=-5.0\,\text{cm}$. Use Eq. 33-2.

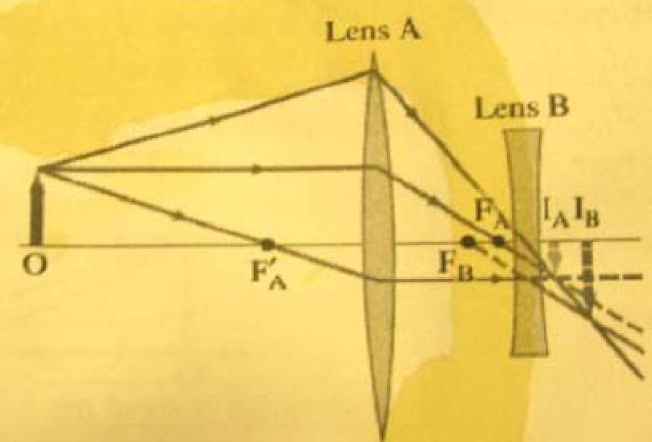
$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-5.0\,\text{cm})(-10.0\,\text{cm})}{(-5.0\,\text{cm})-(-10.0\,\text{cm})}=10\,\text{cm}$$

Thus the final image is real, $\boxed{\text{10 cm beyond the second lens}}$. The distance has two significant figures.

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}=\frac{(30.0\,\text{cm})(10.0\,\text{cm})}{(60.0\,\text{cm})(-5.0\,\text{cm})}=\boxed{-1.0\times}$$

(*c*) See the diagram here.



8.
33-34

The exposure is proportional to the product of the lens opening area and the exposure time, with the square of the f-stop number inversely proportional to the lens opening area. Setting the exposures equal for both exposure times we solve for the needed f-stop number.

$$t_1(f\text{-stop}_1)^{-2} = t_2(f\text{-stop}_2)^{-2} \rightarrow f\text{-stop}_2 = f\text{-stop}_1\sqrt{\frac{t_2}{t_1}} = 16\sqrt{\frac{1/1000\text{ s}}{1/120\text{s}}} = 5.54 \text{ or } \boxed{\frac{f}{5.6}}$$

9.
33-35

We find the f-number from $f\text{-stop} = f/D$.

$$f\text{-stop} = \frac{f}{D} = \frac{(17\text{cm})}{(6.0\text{cm})} = \boxed{\frac{f}{2.8}}$$

10.
33-41

The actual near point of the person is 55 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 55 cm from the eye, or 53 cm from the lens. We find the power of the lens from Eqs. 33-1 and 33-3.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.23\text{m}} + \frac{1}{-0.53\text{m}} = \boxed{2.5\text{D}}$$

11.
33-42

The screen placed 55 cm from the eye, or 53.2 cm from the lens, is to produce a virtual image 105 cm from the eye, or 103.2 cm from the lens. Find the power of the lens from Eqs. 33-1 and 33-2.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.532\text{m}} + \frac{1}{-1.032\text{m}} = \boxed{0.91\text{D}}$$

12.
33-45

(a) The lens should put the image of an object at infinity at the person's far point of 78 cm. Note that the image is still in front of the eye, so the image distance is negative. Use Eqs. 33-2 and 33-1.

$$P=\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{\infty}+\frac{1}{(-0.78\,\mathrm{m})}=-1.282\,\mathrm{D}\approx\boxed{-1.3\,\mathrm{D}}$$

(b) To find the near point with the lens in place, we find the object distance to form an image 25 cm in front of the eye.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_o=\frac{d_i}{Pd_i-1}=\frac{(-0.25\,\mathrm{m})}{(-1.282\,\mathrm{D})(-0.25\,\mathrm{m})-1}=0.37\,\mathrm{m}=\boxed{37\,\mathrm{cm}}$$

13.
33-51

We find the focal length from Eq. 33-6

$$M=\frac{N}{f} \rightarrow f=\frac{N}{M}=\frac{25\,\mathrm{cm}}{3.8}=\boxed{6.6\,\mathrm{cm}}$$

14.
33-54

Maximum magnification is obtained with the image at the near point (which is negative). We find the object distance from Eq. 33-2, and the magnification from Eq. 33-6b.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{d_i f}{d_i - f}=\frac{(-25.0\text{cm})(8.80\text{cm})}{(-25.0\text{cm})-(8.80\text{cm})}=\boxed{6.51\text{cm}}$$

$$M=1+\frac{N}{f}=1+\frac{25.0\text{cm}}{8.80\text{cm}}=\boxed{3.84\times}$$

15.
33-64

For a distant object and a relaxed eye (which means the image is at infinity), the separation of the eyepiece and objective lenses is the sum of their focal lengths. Use Eq. 33-7 to find the magnification.

$$\ell = f_o + f_e \ ; \ M=-\frac{f_o}{f_e}=-\frac{f_o}{\ell - f_o}=-\frac{75.5\text{cm}}{78.0\text{cm}-75.5\text{cm}}=\boxed{-30\times}$$

16.
33-69

We use Eq. 33-6a and the magnification of the eyepiece to calculate the focal length of the eyepiece. We set the sum of the focal lengths equal to the length of the telescope to calculate the focal length of the objective. Then using both focal lengths in Eq. 33-7 we calculate the maximum magnification.

$$f_e = \frac{N}{M} = \frac{25\text{cm}}{5} = 5\text{cm} \;\; ; \;\; \ell = f_e + f_o \;\rightarrow\; f_o = \ell - f_e = 50\text{cm} - 5\text{cm} = 45\text{cm}$$

$$M = -\frac{f_o}{f_e} = -\frac{45\text{cm}}{5\text{cm}} = \boxed{-9\times}$$

16.
33-95

We use Eq. 33-3 to write the image distance in terms of the object distance, image height, and object height. Then using Eq. 33-2 we solve for the object distance, which is the distance between the photographer and the subject.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \;\rightarrow\; \frac{1}{d_i} = -\frac{h_o}{h_i}\frac{1}{d_o}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \left(-\frac{h_o}{h_i}\frac{1}{d_o}\right) = \left(1 - \frac{h_o}{h_i}\right)\frac{1}{d_o} \;\rightarrow$$

$$d_o = \left(1 - \frac{h_o}{h_i}\right) f = \left(1 - \frac{1750\,\text{mm}}{-8.25\,\text{mm}}\right)(220\,\text{mm}) = 46{,}900\,\text{mm} \approx \boxed{47\,\text{m}}$$