Physics 123 - 2013 - Solutions to Prob. Set 8

1\.
34-3

For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. Apply this to the third order.

$$d\sin\theta = m\lambda \rightarrow d = \frac{m\lambda}{\sin\theta} = \frac{3\left(610\times10^{-9}\,\text{m}\right)}{\sin 28^\circ} = \boxed{3.9\times10^{-6}\,\text{m}}$$

2\.
34-6

The slit spacing and the distance from the slits to the screen is the same in both cases. The distance between bright fringes can be taken as the position of the first bright fringe (m = 1) relative to the central fringe. We indicate the lab laser with subscript 1, and the laser pointer with subscript 2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1\ell}{d} \;;\; x_2 = \frac{\lambda_2\ell}{d} \rightarrow$$

$$\lambda_2 = \frac{d}{\ell}x_2 = \frac{\lambda_1}{x_1}x_2 = (632.8\,\text{nm})\frac{5.14\,\text{mm}}{5.00\,\text{mm}} = 650.52\,\text{nm} \approx \boxed{651\,\text{nm}}$$

3.
34-11

The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda + d\sin\theta_{max} = m\lambda,\ m = 0, 1, 2, \cdots \rightarrow d\sin\theta_{max} = \left(m - \tfrac{1}{2}\right)\lambda,\ m = 1, 2, \cdots$$

We could express the result as $d\sin\theta_{max} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \cdots$.

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda + d\sin\theta_{min} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \cdots \rightarrow d\sin\theta_{min} = m\lambda,\ m = 0, 1, 2, \cdots$$

Thus the pattern is just the **reverse of the usual double-slit pattern**. There will be a dark central line. Every place there was a bright fringe will now have a dark line, and vice versa.

4.
34-15

The presence of the water changes the wavelength according to Eq. 34-1, and so we must change λ to $\lambda_n = \lambda/n$. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda_n \rightarrow d\frac{x}{\ell} = m\lambda_n \rightarrow x = \frac{\lambda_n m\ell}{d}\ ;\ x_1 = \frac{\lambda m_1\ell}{d}\ ;\ x_2 = \frac{\lambda(m+1)\ell}{d} \rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{\lambda_n(m+1)\ell}{d} - \frac{\lambda_n m\ell}{d} = \frac{\lambda_n\ell}{d} = \frac{\lambda\ell}{nd} = \frac{(470\times10^{-9}\,\text{m})(0.500\,\text{m})}{(1.33)(6.00\times10^{-5}\,\text{m})} = \boxed{2.94\times10^{-3}\,\text{m}}$$

5.
34-24

Between the 25 dark lines there are 24 intervals. When we add the half-interval at the wire end, we have 24.5 intervals over the length of the plates.

$$\frac{28.5\text{cm}}{24.5\text{intervals}} = \boxed{1.16\text{cm}}$$

6.
34-27

(*a*) When illuminated from above at A, a light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil-water interface undergoes no phase shift. If th oil thickness at A is negligible compared to the wavelength of the light, then there is no significant shift in phase due to a path distance traveled by a ray in the oil. Thus the light reflected from the two surfaces will destructively interfere for all visible wavelengths, and the oil will appear black when viewed from above.

(*b*) From the discussion in part (*a*), the ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray that reflects from the oil-water interface has no phase change due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi$. For constructive interference, the net phase change must be a multiple of 2π.

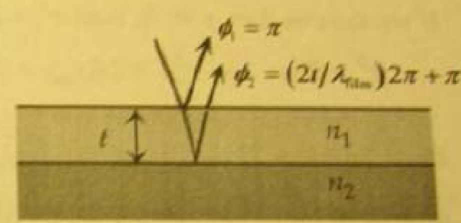
$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \;\rightarrow\; t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_o}$$

From the diagram, we see that point B is the second thickness that yields constructive interference for 580 nm, and so we use $m = 1$. (The first location that yields constructive interference would be for $m = 0$.)

$$t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_o} = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right)\frac{580\,\text{nm}}{1.50} = \boxed{290\,\text{nm}}$$

7.
34-32

With respect to the incident wave, the wave that reflects from the top surface of the alcohol has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the alcohol has a phase change due to both the additional path length and a phase change of π on reflection, so



$\phi_2 = \left(\frac{2t}{\lambda_{film}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of π.

$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{1film}}\right)2\pi + \pi\right] - \pi = m_1 2\pi \;\rightarrow\; t = \tfrac{1}{2}\lambda_{1film} m_1 = \tfrac{1}{2}\frac{\lambda_1}{n_{film}} m_1,\; m_1 = 1, 2, 3, \ldots .$$

For destructive interference, the net phase change must be an odd-integer multiple π.

$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{2film}}\right)2\pi + \pi\right] - \pi = (2m_2 + 1)\pi \;\rightarrow\; t = \tfrac{1}{4}\frac{\lambda_2}{n_{film}}(2m_2 + 1),\; m_2 = 0, 1, 2, \ldots .$$

Set the two expressions for the thickness equal to each other.

$$\tfrac{1}{2}\frac{\lambda_1}{n_{film}} m_1 = \tfrac{1}{4}\frac{\lambda_2}{n_{film}}(2m_2 + 1) \;\rightarrow\; \frac{2m_2 + 1}{2m_1} = \frac{\lambda_1}{\lambda_2} = \frac{(635\,\text{nm})}{(512\,\text{nm})} = 1.24 \approx 1.25 = \frac{5}{4}$$

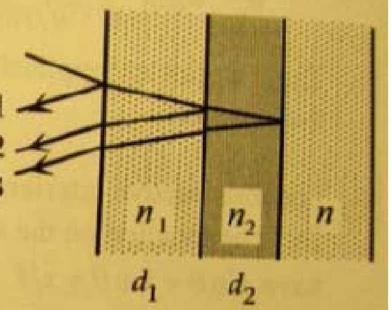
Thus we see that $m_1 = m_2 = 2$, and the thickness of the film is

$$t = \tfrac{1}{2}\frac{\lambda_1}{n_{film}} m_1 = \tfrac{1}{2}\left(\frac{635\,\text{nm}}{1.36}\right)(2) = \boxed{467\,\text{nm}} \text{ or } t = \tfrac{1}{4}\frac{\lambda_2}{n_{film}}(2m_2 + 1) = \tfrac{1}{4}\left(\frac{512\,\text{nm}}{1.36}\right)(5) = \boxed{471\,\text{nm}}$$

With 2 sig.fig., the thickness is 470 nm. The range of answers is due to rounding λ_1/λ_2.

8.
34-52

To maximize reflection, the three rays shown in the figure should be in phase. We first compare rays 2 and 3. Ray 2 reflects from $n_2 > n_1$, and so has a phase shift of $\phi_2 = \pi$. Ray 3 will have a phase change due to the additional path length in material 2, and a phase shift of π because of reflecting from $n > n_2$. Thus $\phi_3 = \left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi$. For constructive interference the net phase change for rays 2 and 3 must be a non-zero integer multiple of 2π.



$$\Delta\phi_{2\text{-}3} = \phi_3 - \phi_2 = \left[\left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi\right] - \pi = 2m\pi \rightarrow d_2 = \tfrac{1}{2}m\lambda_2,\ m = 1, 2, 3\cdots$$

9.
34-65

In order for the two reflected halves of the beam to be 180° out of phase with each other, the minimum path difference ($2t$) should be $\frac{1}{2}\lambda$ in the plastic. Notice that there is no net phase difference between the two halves of the beam due to reflection, because both halves reflect from the same material.

$$2t = \tfrac{1}{2}\frac{\lambda}{n} \rightarrow t = \frac{\lambda}{4n} = \frac{780\,\text{nm}}{4(1.55)} = \boxed{126\,\text{nm}}$$

10.
35-3

The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 35-2 to calculate the angular distance to the first and second minima. Then we average these to values to determine the approximate location of the first maximum. Finally, using trigonometry, we set the linear distance equal to the distance to the screen multiplied by the tangent of the angle.

$$D\sin\theta_m = m\lambda \rightarrow \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times580\times10^{-9}\text{ m}}{3.8\times10^{-6}\text{ m}}\right) = 8.678^\circ \qquad \theta_2 = \sin^{-1}\left(\frac{2\times580\times10^{-9}\text{ m}}{3.8\times10^{-6}\text{ m}}\right) = 17.774^\circ$$

$$\theta = \frac{\theta_1+\theta_2}{2} = \frac{8.678^\circ+17.774^\circ}{2} = 13.23^\circ$$

$$y = \ell\tan\theta_1 = (10.0\text{ m})\tan(13.23^\circ) = \boxed{2.35\text{ m}}$$

11.
35-9

We set the angle to the first minimum equal to half of the separation angle between the dark bands. We insert this angle into Eq. 35-1 to solve for the slit width.

$$\theta = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(55.0^\circ) = 27.5^\circ$$

$$\sin\theta = \frac{\lambda}{D} \rightarrow D = \frac{\lambda}{\sin\theta} = \frac{440\,\text{nm}}{\sin 27.5^\circ} = \boxed{953\,\text{nm}}$$

12.
35-25

The angular resolution is given by Eq. 35-10. The distance between the stars is the angular resolution times the distance to the stars from the Earth.

$$\theta = 1.22\frac{\lambda}{D} \;;\; \ell = r\theta = 1.22\frac{r\lambda}{D} = 1.22\frac{(16\,\text{ly})\left(\frac{9.46\times10^{15}\,\text{m}}{1\,\text{ly}}\right)(550\times10^{-9}\,\text{m})}{(0.66\,\text{m})} = \boxed{1.5\times10^{11}\,\text{m}}$$

13.
35-35

Since the same diffraction grating is being used for both wavelengths of light, the slit separation will be the same. We solve Eq. 35-13 for the slit separation for both wavelengths and set the two equations equal. The resulting equation is then solved for the unknown wavelength.

$$d\sin\theta = m\lambda \;\Rightarrow\; d = \frac{m_1\lambda_1}{\sin\theta_1} = \frac{m_2\lambda_2}{\sin\theta_2} \;\Rightarrow\; \lambda_2 = \frac{m_1}{m_2}\frac{\sin\theta_2}{\sin\theta_1}\lambda_1 = \frac{2}{1}\frac{\sin 20.6^\circ}{\sin 53.2^\circ}(632.8\,\text{nm}) = \boxed{556\,\text{nm}}$$

14.
35-37

We find the second order angles for the maximum and minimum wavelengths using Eq. 35-13, where the slit separation distance is the inverse of the number of lines per centimeter. Subtracting these two angles gives the angular width.

$$d\sin\theta = m\lambda \rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N)$$

$$\theta_1 = \sin^{-1}\left[2\left(4.5\times10^{-7}\,\text{m}\right)\left(6.0\times10^{5}\,/\text{m}\right)\right] = 32.7^\circ$$

$$\theta_2 = \sin^{-1}\left[2\left(7.0\times10^{-7}\,\text{m}\right)\left(6.0\times10^{5}\,/\text{m}\right)\right] = 57.1^\circ$$

$$\Delta\theta = \theta_2 - \theta_1 = 57.1^\circ - 32.7^\circ = \boxed{24^\circ}$$

15.
35-46

Using Eq. 35-13 we calculate the maximum order possible for this diffraction grating, by setting the angle equal to 90°. Then we set the resolving power equal to the product of the number of grating lines and the order, where the resolving power is the wavelength divided by the minimum separation in wavelengths (Eq. 35-19) and solve for the separation.

$$\sin\theta = \frac{m\lambda}{d} \rightarrow m = \frac{d\sin\theta}{\lambda} = \frac{(0.01\,\text{m}/6500\,\text{lines})\sin 90^\circ}{624\times 10^{-9}\,\text{m}} = 2.47 \approx 2$$

$$\frac{\lambda}{\Delta\lambda} = Nm \Rightarrow \Delta\lambda = \frac{\lambda}{Nm} = \frac{624\text{ nm}}{(6500\text{ lines/cm})(3.18\text{ cm})(2)} = \boxed{0.015\text{ nm}}$$

The resolution is best for the **second order**, since it is more spread out than the first order.

(a) The resolving

16.
35-65

The lines act like a grating. We assume that we see the first diffractive order, so $m = 1$. Use Eq. 35-13.

$$d\sin\theta = m\lambda \rightarrow d = \frac{m\lambda}{\sin\theta} = \frac{(1)(480\,\text{nm})}{\sin 56^\circ} = \boxed{580\,\text{nm}}$$

17.
35-71

(*a*) This is very similar to Example 35-6. We use the same notation as in that Example, and solve for the distance ℓ.

$$s = \ell\theta = \ell\frac{1.22\lambda}{D} \rightarrow \ell = \frac{Ds}{1.22\lambda} = \frac{(6.0\times10^{-3}\,\text{m})(2.0\,\text{m})}{1.22(560\times10^{-9}\,\text{m})} = \boxed{1.8\times10^{4}\,\text{m}} = 18\,\text{km}$$

(*b*) We use the same data for the eye and the wavelength.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(560\times10^{-9}\,\text{m})}{(6.0\times10^{-3}\,\text{m})} = 1.139\times10^{-4}\,\text{rad}\left(\frac{180^\circ}{\pi\,\text{rad}}\right)\left(\frac{3600''}{1^\circ}\right) = \boxed{23''}$$

Our answer is less than the real resolution, because of atmospheric effects and aberrations in the eye.

18.
35-72

We first find the angular half-width for the first order, using Eq. 35-1, $\sin\theta = \frac{\lambda}{D}$. Since this angle is small, we may use the approximation that $\sin\theta \approx \tan\theta$. The width from the central maximum to the first minimum is given by $y = L\tan\theta$. That width is then doubled to find the width of the beam, from the first diffraction minimum on one side to the first diffraction minimum on the other side.

$$y = L\tan\theta = L\sin\theta$$

$$\Delta y = 2y = 2L\sin\theta = 2L\frac{\lambda}{D} = \frac{2(3.8\times10^{8}\,\text{m})(633\times10^{-9}\,\text{m})}{0.010\,\text{m}} = \boxed{4.8\times10^{4}\,\text{m}}$$