1. Consider the periodic function \( f(x) = \frac{x}{\lambda} \) for \( 0 \leq x < \lambda \) shown below. Determine the Fourier series that describes \( f(x) \). Determine the first several terms in the and draw them. What does the frequency spectrum look like for the first several terms?

\[
\begin{align*}
\int x \cos ax \, dx &= \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \\
\int x \sin ax \, dx &= -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2}
\end{align*}
\]

Below are problems from past exams that I have given in Phy 113, 114, 121, and 142 which would be fair game for your upcoming exam. Clearly, we have done some things that are not covered here and you should assume that you might see some problems more difficult than some of these. Still, this will give you an idea of what some of my exam problems are like and give you a bit of practice. You can find my past exams at

http://www.pas.rochester.edu/~manly/class/Exams/
Erving Von Humbolt, famed Professor of Pre-Columbian Artifacts has discovered a musical instrument he believes was once used by native peoples in what is now southeast Paraguay for “some serious jammin’, rockin’, and gettin’ down” during adolescent mating rituals. Unfortunately, the instrument he has discovered is broken. He comes to you for help in understanding what sounds the instrument might have made. Please help him out!

The instrument has one string. That string is tied at one end and constrained to move freely up and down a thin rod on the other end (refer to the figure). Please determine the correct expression for the frequency of the \( n \)th harmonic of the string in terms of the length \( L \), tension \( T \), and the mass/length \( \mu \) of the string. You must show your work to get credit.

(a) \( v_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \)

(b) \( v_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \)

(c) \( v_n = \frac{n}{2L} \sqrt{\frac{gT}{\mu}} \)

where \( n=1,3,5 \ldots \)

(d) \( v_n = \frac{n}{4L} \sqrt{\frac{T}{\mu}} \)

(e) \( v_n = \frac{n}{4L} \sqrt{\frac{T}{\mu}} \)

where \( n = 1,3,5, \ldots \)
**P113 2000:**
Thog the Hairy was the world’s first musician. Neanderthal lore says his first instrument was made from a 5 kg rock hanging from a 2 m long mammoth-hair string with a mass per unit length of 1.5 g/cm tied to a tree. What were the frequencies of the lowest 3 harmonics Thog could play on this instrument? (Assume the rock is heavy enough that its transverse motion is negligible.)

**P113 2001:**
One can hold a long, slender aluminum rod near the rod’s midpoint and stroke the rod with the other hand and make the rod "sing", or emit a clear, loud, ringing noise. Determine the resonant frequencies for a 90 cm rod.

*Useful information: the speed of sound in aluminum is 5100 m/s. This is the speed of the waves traveling in the rod that cause the sound.*

**P113 2002:**
A simple pendulum is constructed from a mass of 0.3 kg and a (massless) string of length 1.2 meters hanging from the ceiling of a rocketship sitting on the surface of the earth. The mass is displaced such that the string is displaced by 10 degrees from the vertical.

a) What is the period of the motion of the pendulum? (show your work)

b) If the rocketship is sitting on the surface of the moon what is period of the motion of the pendulum? (show your work)

\[ \text{period} = \frac{2 \pi \sqrt{\text{length}}}{\text{accel. due to gravity}} \]

(Memo: \( G = 6.7 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \))
**P113 2002:**
While traveling in the Australian outback you wake up in the middle of the night and hear a strange instrument being played. You are certain it is an instrument based on a vibrating air column of some sort (like a flute or a horn of some description). You detect the following frequencies being emitted: 100 Hz, 300 Hz, 500 Hz and 700 Hz. From this, what can you conclude about the instrument? Be sure to show how you arrived at your answer.

a) It is closed at both ends.
b) It is open at both ends.
c) It is longer than it is wide.
d) It is open at one end and closed at the other end.
e) You cannot conclude any of the above about the instrument.

**P113 2003:**
The graph below shows a wave traveling to the right with a velocity of 4 m/s. The equation that best represents the wave is

- a) $y(x,t)=2\sin(\pi x/4 - \pi t)$ meters
- b) $y(x,t)=2\sin(16\pi x - 8\pi t)$ meters
- c) $y(x,t)=2\sin(\pi x/4 + \pi t)$ meters
- d) $y(x,t)=4\sin(\pi x/4 - \pi t)$ meters
- e) $y(x,t)=4\sin(16\pi x - 8\pi t)$ meters

**P113 2004:**
Suppose, as you go home for the holidays, in the midst of a big family holiday feast your grandmother announces that she is about to go on tour as the lead guitarist for the Bare Naked Old Ladies. Everyone is thrilled! After dinner, Grandma, knowing you are now a physics god(ess), asks you to explain the basic physical principles behind the operation of the guitar. Briefly explain what you would tell her. Use sketches and equations as you feel appropriate (Grandma can handle it).

At an outdoor bandstand, what would be the difference in the sound intensity level (in dB) received by a listener 5 m from the bandstand and a listener 40 m from the bandstand, assuming there is no sound reflection?

- a) 8 dB
- b) 9 dB
- c) 10 dB
- d) 18 dB
- e) The answer depends on how loudly the band plays.
After exams, Chaz Johnson travels to Jamaica to relax. He finds himself lying on a pier listening to the sound of the water below. For reasons that are best left obscure, one of Chaz’s buddies has placed a tuning fork on a floating log directly beneath where Chaz’s head lies on the pier. The tuning fork emits a frequency of 500 Hz. Chaz knows this and is surprised to find the frequency he hears varies with time. Being an analytical sort of guy, Chaz observes the water waves flowing toward the pier to be harmonic. He determines the water waves satisfy an equation describing the surface height (y) as a function of position (x) and time (t) as \( y(x,t)=A\sin(kx-\omega t)=2\sin(1.5x-5t) \) in meters.

What is the range in frequencies Chaz hears coming from the tuning fork (not that which is emitted, but what Chaz hears)? Please calculate the range and show your work. *For simplicity, assume the tuning fork stays a fixed distance above the surface of the water as the waves pass.*
**P113 2012:**
Consider the waves moving on a string as shown in the sketch below.

At a later time, as the waves pass each other, which of the following shapes (of the 4 shown) will NOT be observed instantaneously at some point during the passage? Circle that shape.

**P113 2012:**
Below is a sketch of a transverse wave on a string at time $t=0$. The wave moves to the left with time from your point of view. The divisions in $x$ and in $y$ are 1 meter apart. The bit of string at point $P$ executes simple harmonic motion in the $y$ direction with a period of 0.5 seconds.

What is the speed of the wave?

Write down a mathematical description of the wave, i.e. $y(x,t) =$?
P114 2004:
A sample of particles are measured to have a lifetime of 1 $\mu$s when at rest. When you go down the street to your local neighborhood particle accelerator, you measure the lifetime of the same type of particle (now accelerated in a beam) to be 4 $\mu$s. How fast are the particles in the beam moving relative to you when you measure them?

a) 0.90c  
b) 0.94c  
c) 0.97c  
d) 1.03c  
e) 0.25c

P114 2010:
A spacecraft travels along a straight line from Earth to the Moon, a distance of 3.84x10$^8$ m. Its speed measured on Earth is 0.50c. (*The speed of light is 3x10$^8$ m/s.*)

a) How long does the trip take, according to a clock on Earth?

t

b) How long does the trip take, according to a clock on the spacecraft?

t

c) Determine the distance between the Earth and the moon if it were measured by a person on the spacecraft?
P121 2001:
A cork bobbing on the surface of a lake has the vertical displacement as a function of time given by the graph below.

(a) What is the amplitude for the harmonic motion of the cork (look carefully!)?

(b) What is the period of the simple harmonic motion of the cork?

(c) What is the frequency of the water wave passing by the cork?

(d) What is the wavelength of the water wave passing by the cork (assume the velocity of the wave is 1 m/s)?

(e) Write a mathematical expression that describes the vertical motion of the cork with time.
P121 2003:
If two identical waves with the same phase are added, the result is

a) a wave with the same frequency but twice the amplitude.
b) a wave with the same amplitude but twice the frequency.
c) a wave with zero amplitude.
d) a wave with zero frequency.
e) This problem cannot be solved without knowing the wavelength.

P121 2003:
Two waves with the same frequency and wavelength but with different amplitudes are added. If \( A_1 = 2A_2 \) and the waves are 180 degrees out of phase, then the amplitude of the resultant wave is

a) zero
b) the same as \( A_1 \)
c) the same as \( A_2 \)
d) equal to \( A_1 + A_2 \)
e) coherent.

P142 2003:
Some say that if Dante were alive today, he might write about a hell where the Devil is a physics laboratory instructor who studies the inelastic collisions of innocent and hard-working undergraduate students. One student gets thrown at high velocity against a brick wall (and survives unharmed to do it again and again, of course) while the rest of the class has to make observations. The particularly sinful students get to do the error analysis!

P142 2003:
Anyway, suppose Biff Jones is thrown by the Devil toward the wall. Biff emits a constant yell at a pure frequency of 400 Hz. While Biff is in flight, the observing students hear a sound that oscillates up and down in intensity with a frequency of 5 Hz. What is the speed with which the Devil throws Biff at the wall? (Ignore air resistance. Assume the velocity of sound is 343 m/s. Consider this as a one-dimensional problem.)

P142 2010:
Biff the spacefarer travels to the Centauri star system 4.5 light years away at a speed of 0.9c. How long does it take Biff to travel to the Centauri star system in Biff’s frame of reference?

P142 2010:
The large radio telescope in Arecibo, Puerto Rico has been used to search for extra-terrestrial intelligence. The radio telescope has a diameter of 1000 feet = 304.8 meters. According to one of the researchers in Arecibo, the telescope can detect a signal that lays down over the surface of the Earth a power of 1 picowatt (1x10^{-12} Watts). If a signal emanating from the center of our galaxy (2.2x10^4 light years distant) were detected, what is the minimum power of the source of the signal (assuming the source radiates equally in all directions)? The radius of the Earth is 6.4x10^6 m and the speed of light is 3x10^8 m/s.
**P142 2008:**
A particle beam is created in a laboratory. A particle in the beam is created and lives for 25 ns in its rest frame before decaying. In the laboratory, that particle appears to travel 10 m in a straight line from the point of creation to the point of decay. Determine the speed of the particle in the laboratory.

**P142 2007:**
An average person in the US lives approximately 72 years. Does this mean that it is impossible for such an average person to travel more than 72 light years from earth (in principle)? Briefly explain your reasoning.