P142 University of Rochester NAME Solution Key - SLM
S. Manly Fall 2008

Exam 1 (October 7, 2008)

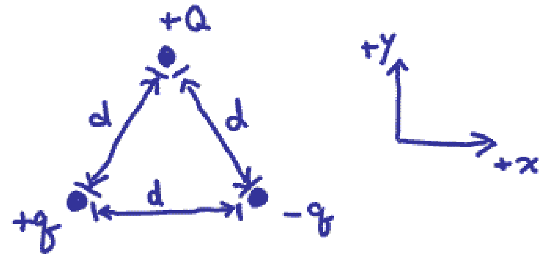
Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (10 pts, ~~justify your answer~~):

Three charges (+q,-q, +Q) are placed at the vertices of an equilateral triangle as shown in the figure. The force on +Q due to the charges +q and –q

a) points in the positive x direction.
b) points in the negative x direction.
c) points in the positive y direction.
d) points in the negative y direction.
e) is zero.

y components cancel



Problem 2 (10 pts, ~~justify your answer~~):

Consider the potential across the plates of a parallel-plate capacitor. As the potential is increased, the following occur:

$Q = CV$ $V = \frac{Q}{C}$

C is geometry, const.

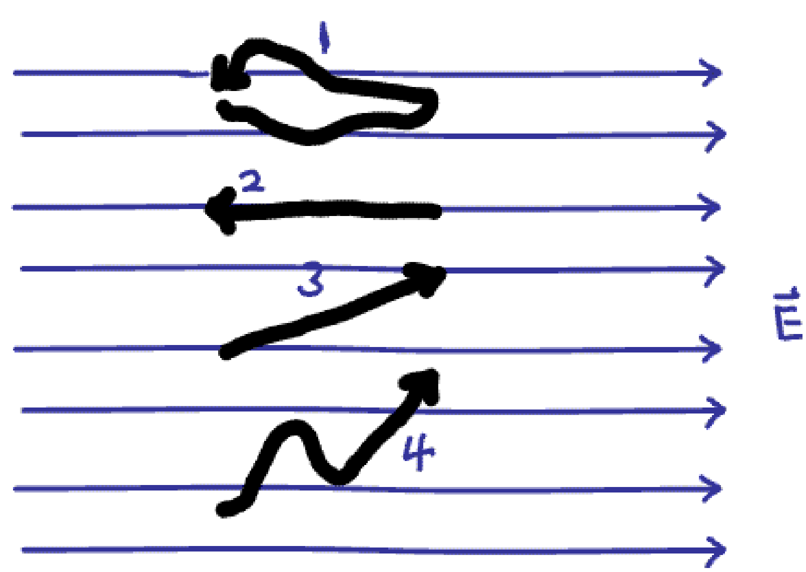
V incr → Q incr

a) The capacitance remains constant and the charge decreases.
b) The capacitance increases and the charge remains constant.
c) The capacitance increases and the charge increases.
d) The capacitance remains constant and the charge increases.

Problem 3 (10 pts, ~~justify your answer~~):

Four identical charges are moved in a uniform electric field on the paths shown in the figure. The dots represent the initial and the arrows the final positions of the charges. The same amount of work was done on charges

a) 1 and 2.
b) 2 and 3.
c) 3 and 4.
d) 2, 3, and 4.
e) 1, 2, 3, and 4.
f) A different amount of work was done on each of the charges.



3 + 4 have same // component of motion with respect to $\vec{E}$

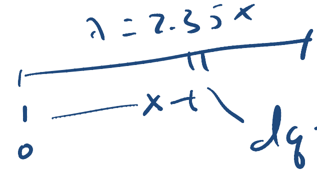
 NAME Solution key - SLM

1)	10/10
2)	10/10
3)	10/10
4)	15/15
5)	15/15
6)	20/20
7)	20/20
tot	100/100

wow! :)

Problem 4 (15 pts, justify your answer):

A wire has a linear charge density λ(x) = 2.35x, where x is in meters and λ is in C/m. If the length of the wire is 3.7 m, calculate the total charge on the wire.



$dq = \lambda dx$

$$Q = \int_0^L dq = \int_0^L \lambda dx$$

$$Q = \int_0^{3.7} 2.35\,x\,dx = 2.35\left[\frac{x^2}{2}\right]_0^{3.7} = \frac{(2.35)(3.7)^2}{2} = 16.1\ C$$

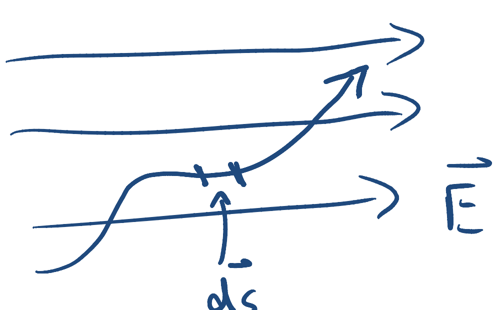
Problem 5 (15 pts, justify your answer):

You graduate and get older. The economy keeps getting worse and soon it becomes obvious even to people on the street that we really need a President that understands electromagnetism. Because of your excellent physics training, a major political party drafts you to run for President of the US. Your political mantra of "It isn't what you can do for Coulomb, but rather what Coulomb can do for you!" resounds throughout the country. During the first Presidential debate shortly before the election, the moderator ask you to "explain why equipotential lines are always perpendicular to electric field lines." Briefly give your answer below. Feel free to use coherent text, sketches, equations, and political slogans as your mood dictates.

An equipotential line represents a path along which a charge can be moved without work. If the motion of a charge includes a component parallel to an electric field line, that motion will require work since

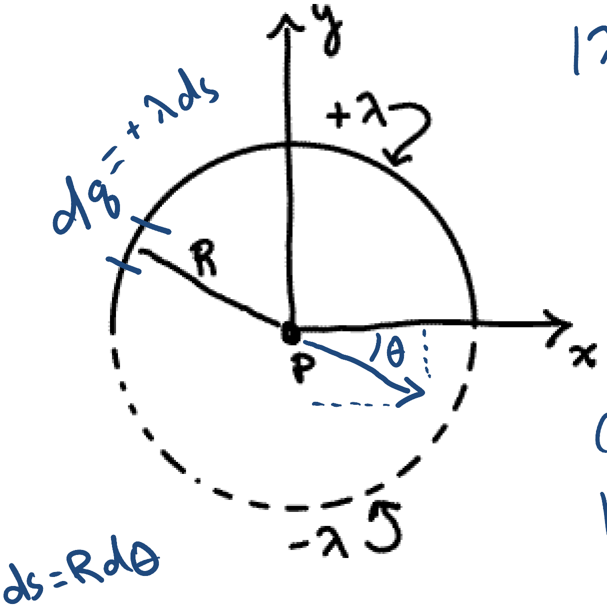
$$\text{work} = \int \vec{F}\cdot d\vec{s} = \int q\vec{E}\cdot d\vec{s}.$$

The component of motion parallel to $\vec{E}$ (via the dot product) is what contributes to this integral.



Problem 6 (20 pts, justify your answer):

Assume a charge of +Q is evenly distributed on the top half of a circle in the x-y plane and a charge of –Q is distributed evenly on the bottom half of the circle. Suppose the circle is centered at the origin and has a radius R. The situation to consider is illustrated below in a rough sketch. Determine the electric field at point P at the center of the circle.



$|\lambda| = \dfrac{Q}{\pi R}$

By symmetry, the x component of the field produced by the left half of the ring cancels out the x component of the E field produced by the right half of the ring.

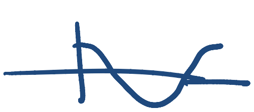
By symmetry, the vertical component of $\vec{E}$ for the top half of the ring is the same as the vertical component of $\vec{E}$ for the bottom half of the ring.

$$\vec{E}_P = \int \frac{k\,dq}{R^2}\hat{r} = \int \frac{k\lambda\,ds}{R^2}\hat{r} = 2\int_0^{\pi} \frac{k|\lambda|R\,d\theta}{R^2}\hat{r} = \frac{2k|\lambda|}{R}\int_0^{\pi} \sin\theta\,d\theta\,(-\hat{y})$$

Vertical component of $\vec{E}$ is $\vec{E}\sin\theta$

$$\int_0^{\pi} \sin\theta\,d\theta = \Big[-\cos\theta\Big]_0^{\pi} = \underbrace{-\cos\pi}_{-1} + \underbrace{\cos 0}_{1} = 2$$

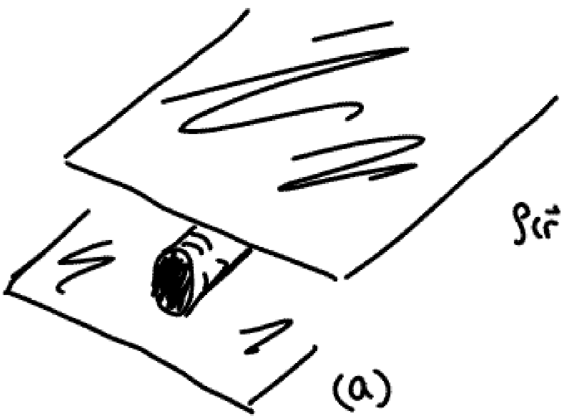
$$\boxed{\vec{E}_P = \frac{4k|\lambda|}{R}(-\hat{y}) = \frac{4kQ}{\pi R^2}(-\hat{y})}$$



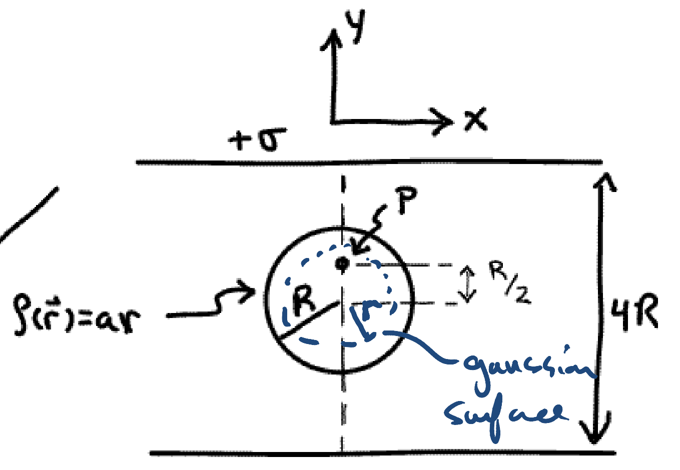
P142 S. Manly — University of Rochester, Fall 2008 — NAME Solution key - SLM

Problem 7 (20 pts, justify your answer):

An infinitely long, cylindrically symmetric, non-conducting cable of radius R is impregnated with positive charge distributed as ρ(R)=ar where "a" is a constant with appropriate units. This cable is placed between two infinite parallel platesseparated by a distance of 4R carrying an area charge density of +σ and -σ, respectively. The situation is pictured from an angle in (a) below and along the axis of the cable in (b) below. Let the x and y axes be defined as shown in the sketch (b) below. Assume the cable is centered between the two parallel plates. Determine the electric field at point P on the y axis a distance R/2 above the center of the cable in terms of the variables ρ, σ, a, and R.

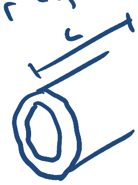


(a)



(b)

$dv = 2\pi r L\,dr$ for cylinder



The principle of superposition tells us that the electric field at point P is a vector sum of the Electric field due to the parallel plates and the Electric field of the cable.

$$\vec{E}_p = \underbrace{\vec{E}_{\parallel plates}}_{P} + \underbrace{\vec{E}_{cable}}_{P}$$

$$\frac{\sigma}{\epsilon_0}(-\hat{y})$$

determine $\vec{E}_{cable}(r)$ using Gauss' Law

Choose cylindrically symetric gaussian surface co-axial with cable, length L, r < R

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow |\vec{E}|2\pi r L = \frac{1}{\epsilon_0}\int_{vol} \rho\, dv$$

$$|\vec{E}|\,2\pi r L = \frac{1}{\epsilon_0}\int_0^r ar\,2\pi r\,dr$$

$$|\vec{E}|\,2\pi r L = \frac{L\,2\pi a}{\epsilon_0}\int_0^r r^2 dr = \frac{L 2\pi a r^3}{3\epsilon_0}$$

$$\vec{E}_{p\,cable} = \frac{a\left(\frac{R}{2}\right)^2}{3\epsilon_0}(+\hat{y})$$

$$\boxed{\vec{E}_p = \left(\frac{aR^2}{12\epsilon_0} - \frac{\sigma}{\epsilon_0}\right)\hat{y}}$$

$\vec{E}(r) = \frac{ar^2}{[illegible]}\hat{r}$

Potentially useful formulas (ask if you need an integral or series expansion)

$$\vec{F} = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$\phi_E = \oint \vec{E}\cdot\vec{dA}$$

$$\oint \vec{E}\cdot\vec{dA} = \frac{Q_{encl}}{\epsilon_0}$$

$$E_S = -dV/ds$$

$$V = W/q$$

$$V_{ptchg} = \frac{kq}{R}$$

$$\vec{E} = \int_{vol} \frac{k\,dQ}{r^2}\,dr\,\hat{r}$$

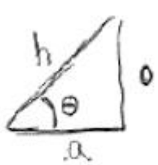
$$V = \int_{vol} \frac{k\,dQ}{r}$$

Sphere: $A = 4\pi r^2$

$V = 4/3\,\pi r^3$

Cylinder: $A = 2\pi r L + 2\pi r^2$

$V = \pi r^2 L$



$\sin\theta = \frac{o}{h}$

$\cos\theta = a/h$

$\tan\theta = o/a$

Const. Accel.

$$\begin{cases} v = v_0 + at \\ x = x_0 + v_0 t + \frac{1}{2}at^2 \\ v^2 = v_0^2 + 2a(x - x_0) \\ x^2 = x_0 + \frac{1}{2}(v_0 + v)\,t \end{cases}$$

$$a_c = \frac{mv^2}{R}$$

$$S = R\Theta$$

$$KE = \frac{1}{2}mv^2$$

$$PE_{spring} = \frac{1}{2}kx^2$$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

$$\int \frac{du}{u} = \ln(u)$$

$$\int e^u du = e^u$$

$$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots \quad -1<x<1$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \cdots \quad x \geq \frac{1}{2}$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots \quad -1<x<1$$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - \cdots \quad -1<x<1$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{x^2}{8} + \cdots \quad -1<x<1$$