Physics 142 – Fall 2008 - Solutions to Problem Set 2

1) Ohanian 23-5

Weight = mg, where $m = \rho V$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0 \times 10^{-4}\ \text{cm})^3 = 4.2 \times 10^{-12}\ \text{cm}^3$

$\rho = 0.8\ g/\text{cm}^3$

Therefore,

$m = 4.2 \times 10^{-12}\ \text{cm}^3\ (0.8\ g/\text{cm}^3) = 4.2 \times 10^{-12}\ \text{g}$

$= 3.35 \times 10^{-15}\ \text{kg}$

Force of gravity $mg = 3.35 \times 10^{-15} \times 9.8 = 3.28 \times 10^{-14}\ \text{N}$

Therefore,

$E = F/q = 3.28 \times 10^{-14}\ \text{N}/1.6 \times 10^{-19}\ C \quad \underline{= 2.1 \times 10^5\ \text{N/C}}$

2) Ohanian 23-57

From $\frac{1}{2}\left(v^2 - v_0^2\right) = a\ (x - x_0)$;

$$a = \frac{1}{2}\frac{\left(v^2 - v_0^2\right)}{(x - x_0)} = \frac{1}{2}\frac{(3.3 \times 10^7\ \text{m/s})^2}{0.01\ \text{m}} = 5.45 \times 10^{16}\ \text{m/s}^2$$

Therefore,

$E = F/q = \frac{m_e a}{q}$ (m_e = mass of electron)

$E = (9.1 \times 10^{-31}\ \text{kg} \times 5.45 \times 10^{16}\ \text{m/s}^2)/(1.6 \times 10^{-19}\ \text{C})$

$\underline{= 3.1 \times 10^5\ \text{N/C}}$

3) Ohanian 23-9

$\frac{1}{2}mv^2 = K$ = kinetic energy. Therefore, $v = \sqrt{2K/m}$ = velocity at a given K.

Therefore, velocity of electron = v

$= \sqrt{(2 \times 3 \times 10^{-19})/(9.1 \times 10^{-31})}\ \text{m/s} = 8.1 \times 10^5\ \text{m/s}$

Acceleration of electron in that field = $F/m_e = qE/m_e$

$= (1.6 \times 10^{-19} \times 3 \times 10^6)/(9.1 \times 10^{-31}) = 5.3 \times 10^{17}\ \text{m/s}$

$\frac{1}{2}\left(v^2 - v_0^2\right) = a(x - x_0)$. Therefore, $(x - x_0)$ = distance moved

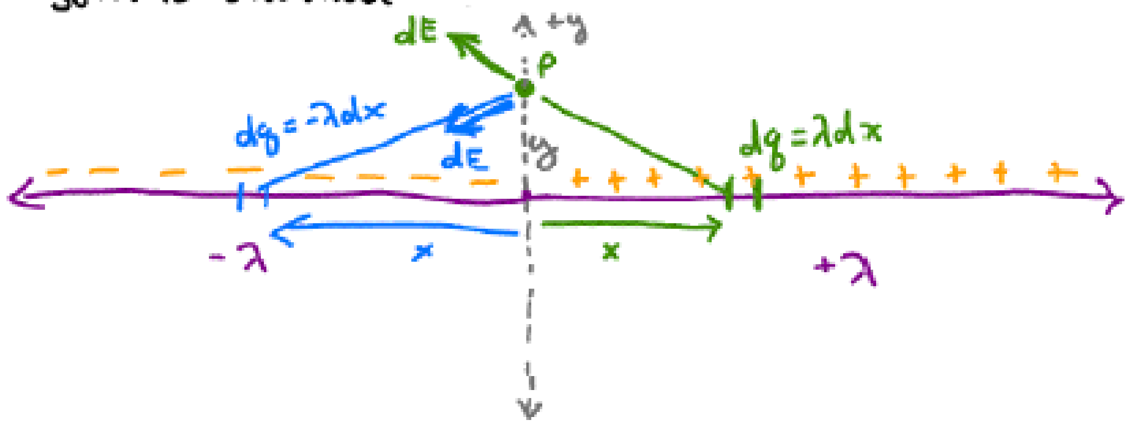
$= \left(v^2 - v_0^2\right)/2a = (8.1 \times 10^5\ \text{m/s})^2/(2 \times 5.3 \times 10^{17})\ \text{m/s}^2 \quad \underline{= 6.2 \times 10^{-7}\ \text{m}}$

4) Ohanian 23-11

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 9\times10^9\times(1.6\times10^{-19})/(0.53\times10^{-10})^2 \; \underline{= 5.1\times10^{11}\ \mathrm{N/C}}$$

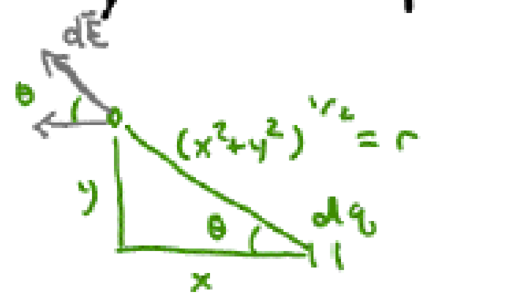
5) Ohanian 23-34

Soln to Ohanian **23-34** 5



From symmetry $\vec{E}$ will be in $-\hat{x}$ direction
y components of $\vec{E}$ all cancel (Left side cancels the right side since for every element of dq at +x there is an element of −dq at −x ... mirror image about x=0 plane).

Let P be at (0,y)



$\cos\theta = \dfrac{|d\vec{E}_y|}{|d\vec{E}|}$ $\qquad$ $\cos\theta = \dfrac{x}{r}$

$$\vec{E}_p = 2k\int_0^{\infty} \frac{\lambda\, dx}{(x^2+y^2)}\,\frac{x}{(x^2+y^2)^{1/2}}\,(-\hat{x})$$

factor of 2
Right side
and
left side

Coulomb
$\lambda dx = dq$
$x^2+y^2 = r^2$

Factor of $\cos\theta$
to project out
x component
of $\underline{E}$ needed
by symmetry

$$\vec{E}_p = 2k\lambda\int_0^{\infty} \frac{x\, dx}{(x^2+y^2)^{3/2}}\,(-\hat{x})$$

From Table $\displaystyle\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{(x^2+a^2)^{1/2}}$

$$\vec{E}_p = 2k\lambda\left[\frac{-1}{(x^2+y^2)^{1/2}}\right]_0^{\infty}(-\hat{x}) = \boxed{\frac{2k\lambda}{y}(-\hat{x}) = \vec{E}_p}$$

6) Ohanian 23-39

The electric field at the given charge distribution will be equivalent to the field obtained by superimposing the field of a large sheet of charge without the hole and that of a disc of radius R of charge density, $-\sigma$.

$$E = \frac{\sigma}{2\varepsilon_0} + \underbrace{\frac{(-\sigma)\, Z}{2\varepsilon_0}\left(\frac{1}{Z} - \frac{1}{(Z^2+R^2)^{1/2}}\right)}$$

E- field due to the disc (for details, see Example 23-7

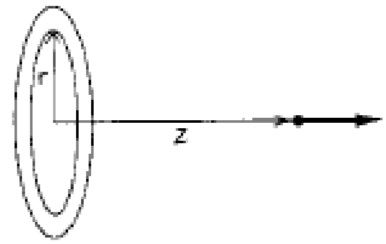
$$= \frac{\sigma Z}{2\varepsilon_0\,(Z^2+R^2)^{1/2}}$$

Above is the solution given by Ohanian. You can see example 6 in Chapter 23 for details of finding the E-field of a plane of charge by integrating rings of charge. I think a simpler way of doing this one would be to set up the integral determined in example 6 of Chapter 23 but change the limits of integration from 0 to infinity to R to infinity.

7) Ohanian 23-79

As shown in Example 6 electric field due to a ring of charge is

$$\frac{1}{4\pi\varepsilon_0}\frac{qz}{(z^2+a^2)^{3/2}}.$$



Let the disc be made of many ring elements, of radius r, thickness dr. The surface charge density $\sigma = Q/\pi R^2$.Therefore, charge dQ on ring radius r, thickness dr is

$$(2\pi r\,dr)\times\sigma = \frac{Q}{\pi R^2}2\pi r\,dr = \frac{2Qr}{R^2}dr$$

Therefore,

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{2Qrz\,dr}{R^2(z^2+r^2)^{3/2}}$$ from this ring element.

Therefore, $E =$

$$\int dE = \frac{2Qz}{4\pi\varepsilon_0 R^2}\int_{r=0}^{R} r\,(z^2+r^2)^{-3/2}\,dr$$

$$= \frac{Qz}{2\pi\varepsilon_0 R^2}\left[-(z^2+r^2)^{-1/2}\right]_0^R = \frac{Qz}{2\pi\varepsilon_0 R^2}\left(\frac{1}{z}-\frac{1}{\sqrt{z^2+R^2}}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2}\left\{1-\left(1+\frac{R^2}{z^2}\right)^{-1/2}\right\}$$

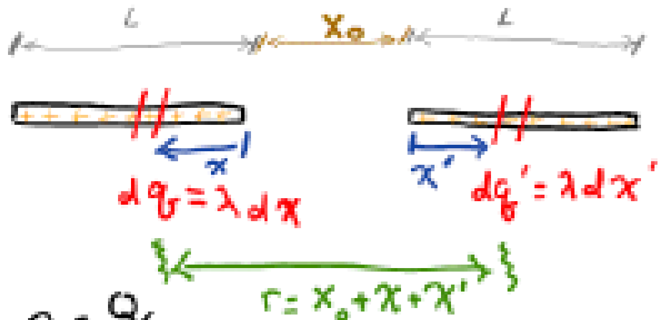
For $z \gg R$, $R^2/z^2 \ll 1$. Therefore, $\left(1+\frac{R^2}{z^2}\right)^{-1/2} \approx 1-\frac{1}{2}\frac{R^2}{z^2}$

Therefore,

$$E \approx \frac{Qz}{2\pi\varepsilon_0 R^2}\;1-1+\frac{1}{2}\frac{R^2}{z^2} = \frac{Q}{4\pi\varepsilon_0 z^2}$$ (as in point charge)

8)

⑧



Q uniformly dist over each rod

$\lambda = Q/L$

Strategy

1) look at Force of dq on dq'

2) get Force of dq on right rod by integrating over $dq' = \lambda dx'$

3) Now integrate over $dq = \lambda dx$ to see full Force between rods.

$$F = \int_{\substack{\text{LT}\\\text{Rod}}} \int_{\substack{\text{RT}\\\text{Rod}}} \frac{k\, dq'\, dq}{r^2} = k\lambda^2 \int_0^L dx \int_0^L dx' \frac{1}{(x_0 + x + x')^2}$$

direction is repulsive along x-AXIS by symmetry

1st do interior integral over x' holding x const.

$$\int_0^L \frac{dx'}{(x_0+x+x')^2}$$

of form

$$\int \frac{dx'}{(\alpha+x')^2}$$

$$= \Big[-(x_0+x+x')^{-1}\Big]_0^L$$

$$= -\frac{1}{(x_0+x+L)} + \frac{1}{(x_0+x)}$$

This multiplied by $k\lambda^2$ Represents the force of dq at position x on left Rod on the Right Rod (total)

I will not simplify further for now because I am guessing integration over x is simpler that way.

$$F = k\lambda^2 \int_0^L dx \left[-\frac{1}{(x_0+x+L)} + \frac{1}{(x_0+x)}\right]$$

After integration over x'

$$F = k\lambda^2 \int_0^L \frac{-dx}{(x_0 + x + L)} + k\lambda^2 \int_0^L \frac{dx}{(x_0 + x)}$$

$$\int_0^L \frac{-dx}{(x_0 + x + L)} = \Big[-\ln(x_0 + x + L) \Big]_0^L$$

$$= -\ln(x_0 + 2L) + \ln(x_0 + L)$$

$$= \ln\left(\frac{x_0 + L}{x_0 + 2L}\right)$$

$$\int_0^L \frac{dx}{(x_0 + x)} = \Big[\ln(x_0 + x) \Big]_0^L$$

$$= \ln(x_0 + L) - \ln x_0 = \ln\left(\frac{x_0 + L}{x_0}\right)$$

$$F = k\lambda^2 \ln\left(\frac{x_0 + L}{x_0 + 2L}\right) + \ln\left(\frac{x_0 + L}{x_0}\right)$$

$$F = k\lambda^2 \ln\left[\left(\frac{x_0+L}{x_0+2L}\right)\left(\frac{x_0+L}{x_0}\right)\right]$$

$$F = k\lambda^2 \ln\left[\frac{(x_0+L)^2}{x_0(x_0+2L)}\right]$$ repulsive

9) Ohanian 23-68

$$|\tau| = pE \sin\theta = (3.4\times10^{-30}\ \mathrm{Cm})(2.0\times10^{6}\ \mathrm{N/C})\ \sin 45^\circ$$

$$\underline{= 4.8\times10^{-24}\ \mathrm{Nm}}$$

10) Ohanian 24-53

The cubical surface 0.2 m × 0.2 m × 0.2 m is *inside* the safe.

$$\Phi = Q/\varepsilon_0 = \frac{1\times10^{-6}}{8.85\times10^{-12}} \ \underline{= 1.1\times10^{5}\ \mathrm{Nm^2/C}}$$

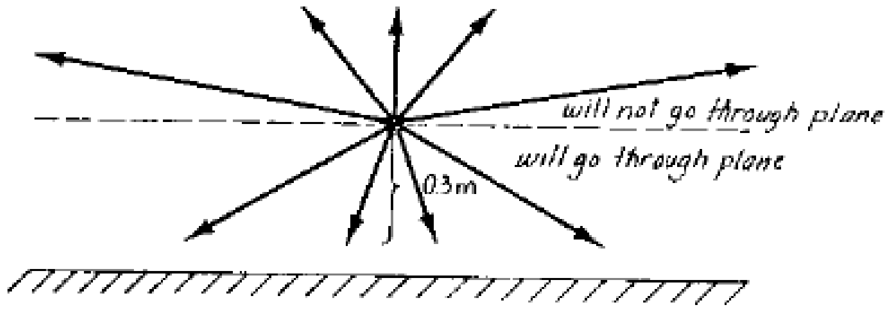
The cubical surface 0.35 m × 0.35 m × 0.35 m is entirely within the steel walls of the safe, and $\vec{E} = 0$ inside the steel. Therefore,

$$\underline{\Phi = \int \vec{E}\cdot d\vec{s} = 0}$$

The cubical surface 0.5 m × 0.5 m × 0.5 m is outside the safe.

$$\Phi = Q/\varepsilon_0 \ \underline{= 1.1\times10^{5}}\ \ \mathrm{Nm^2/C}$$

11)



Any flux line that appears from the charge that is in direction above horizontal will not be intercepted by plane. However, any that are below horizontal, (no matter how small the angle) will somewhere meet the plane. Therefore, half of flux lines are intercepted by plane.

Flux $= \frac{1}{2}\, Q/\varepsilon_0 = \frac{1}{2}\ \frac{6.0 \times 10^{-8}}{8.85 \times 10^{-12}}\,\mathrm{Nm}^2/\mathrm{C}$ $\underline{= 3.4 \times 10^3\ \mathrm{Nm}^2/C}$

12) Ohanian 24-24 soln only makes sense for charge limited to finite X

Take a mathematical, rectangular box, and align its sides with x-axis. Let side parallel to x-axis have length ℓ. Then, flux through cube is $A5\,(x_0 + \ell) - A\,x_0$, where A = area of cuboid perpendicular to field lines. Net flux = $5A\ell$. But $A\ell$ = volume of box V. Therefore, net flux

$= 5V = Q/\varepsilon_0 \Rightarrow Q/V = \rho(\text{charge density}) = 5\varepsilon_0$

Since size and position of box are arbitrary, $\Rightarrow \rho$ for all space

$= 5\varepsilon_0 = 4.4 \times 10^{-11}\ \mathrm{C/m^3}$