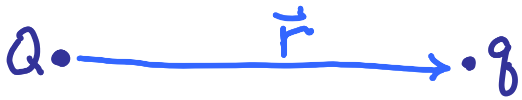
Physics 142 - September 9, 2010

- Technical issues w/ Slides or audio?
- Workshops begin Monday, Sept. 13
- Please place PS1 in Box outside My office (B+L 203E) before Morning of Sept. 10
- Email with questions

Last time

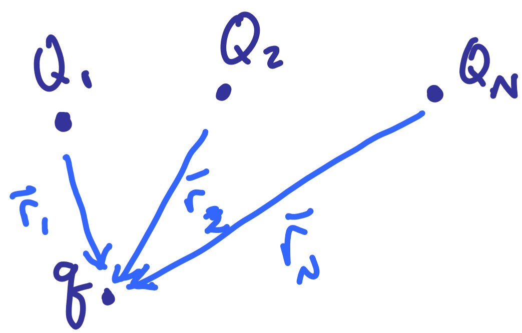
Single charge



Force of Q on q

$$\vec{F} = \frac{kQq}{r^2}\hat{r}$$

Multiple discrete charges

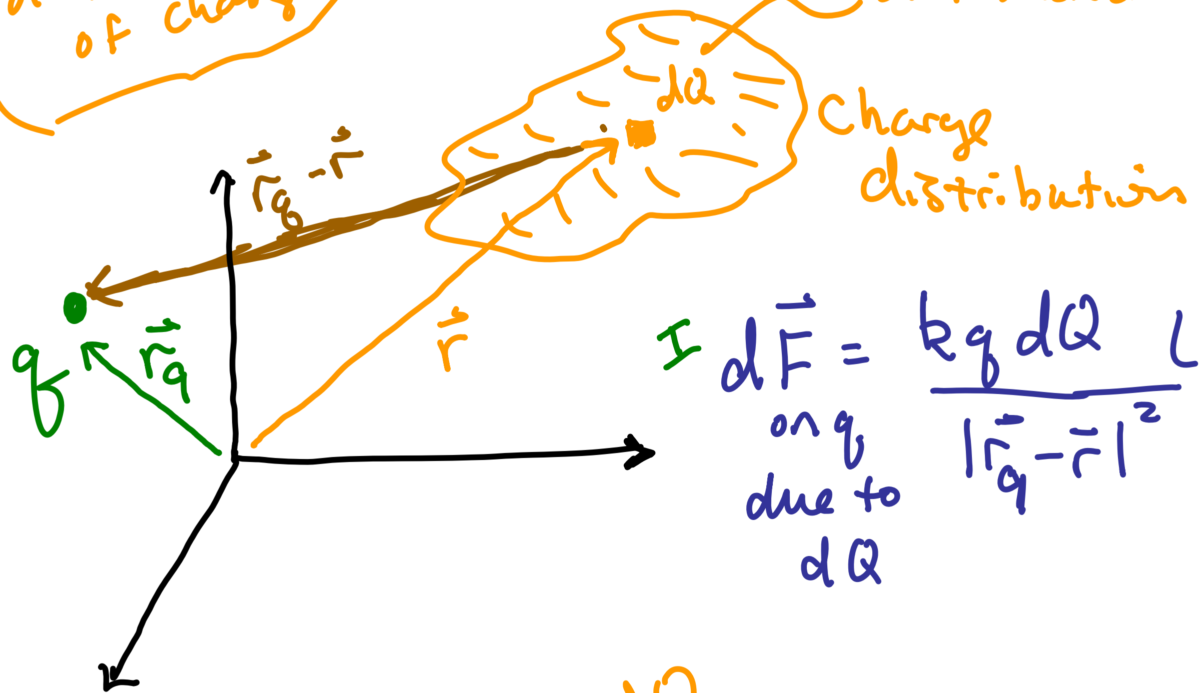


vector superposition

Force of $Q_1, Q_2, \cdots Q_N$ on q

$$\vec{F} = \sum_i \frac{kQ_i q}{r_i^2}\hat{r}_i$$

Continuous distribution of charge



I $$d\vec F_{\text{on } q \text{ due to } dQ} = \frac{k q\, dQ}{|\vec r_q - \vec r|^2}\,\widehat{(\vec r_q - \vec r)}$$

II $$\vec F_q = \int_{\text{vol of charge}} \frac{k q\, \overbrace{\rho(\vec r)\, dv}^{dQ}}{|\vec r_q - \vec r|^2}\,\widehat{(\vec r_q - \vec r)}$$

coulomb's law

$$dQ = \underbrace{\rho(\vec r)}_{\text{volume charge density}}\, dv$$

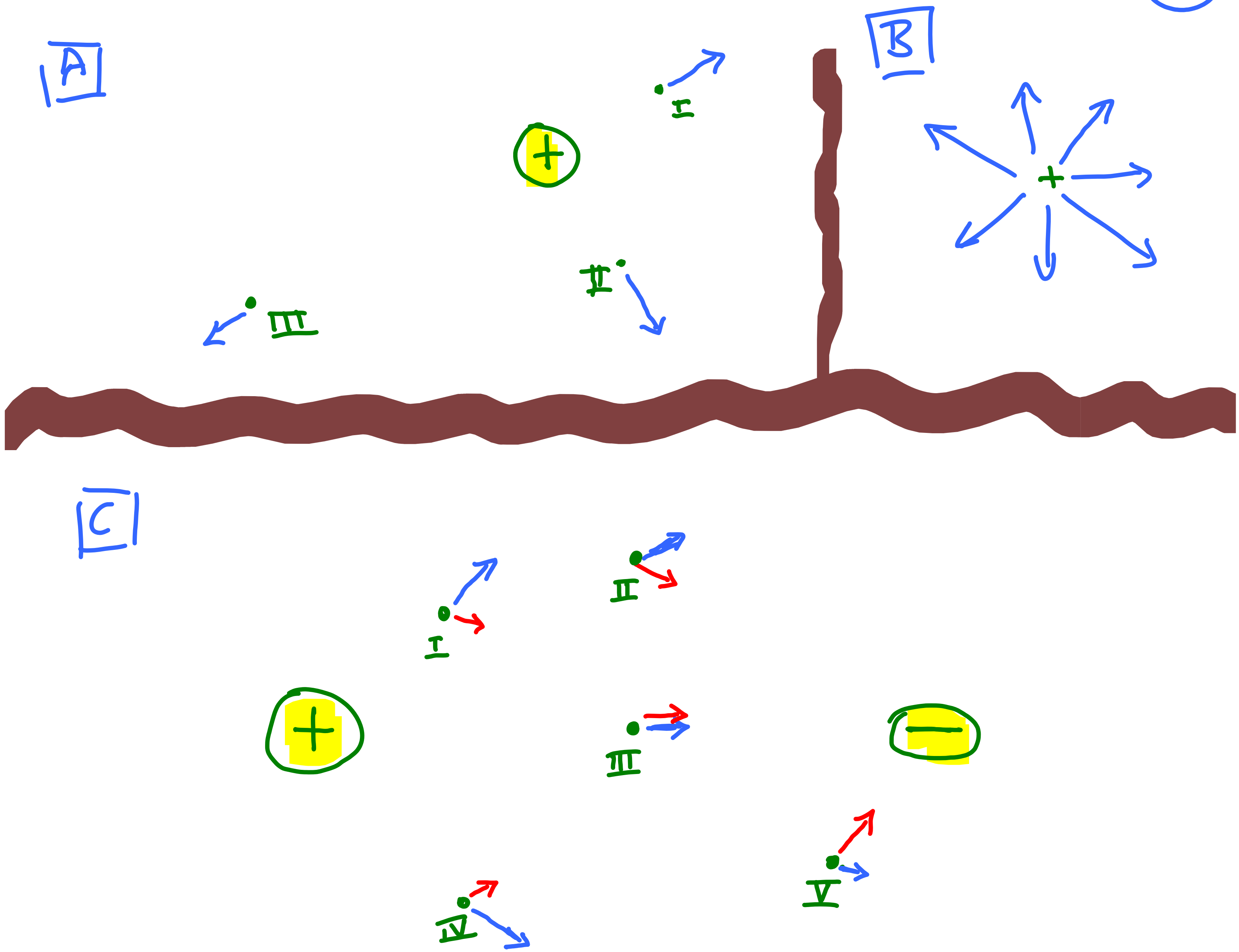
Electric Field

Q

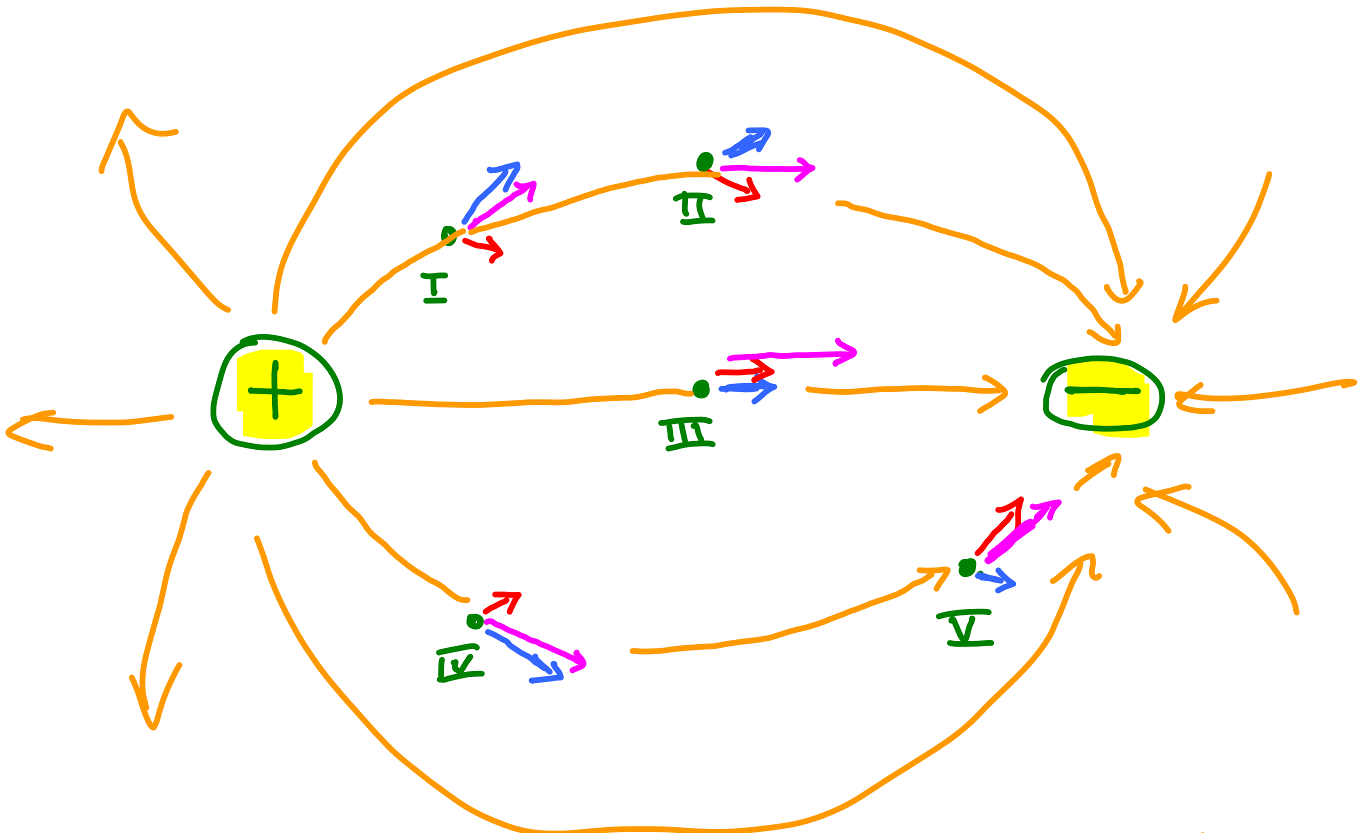
$$\vec{E}_p = \frac{\vec{F}_q}{q}$$

Imagine placing positive test charge q at point p $\rightarrow$ $\vec{F}_{on\,q} = q\vec{E}_p$

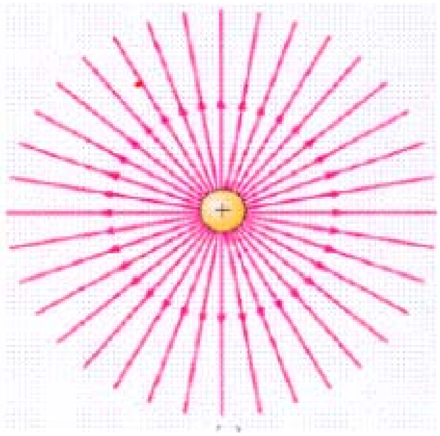
use this to visualize $\vec{E}$

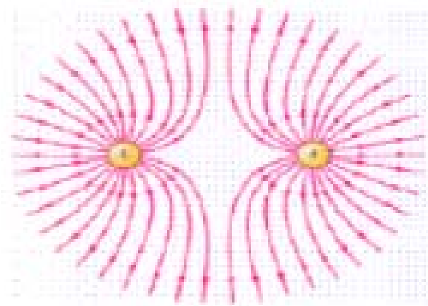

A
B
C
+
I
II
III
IV
V
−

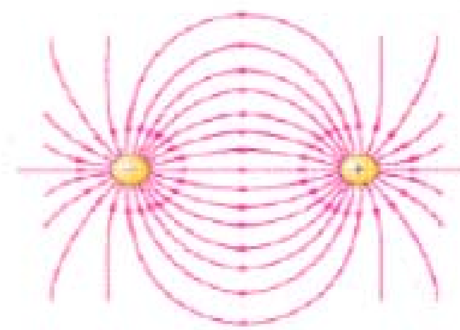
5a



Electric Lines of Force





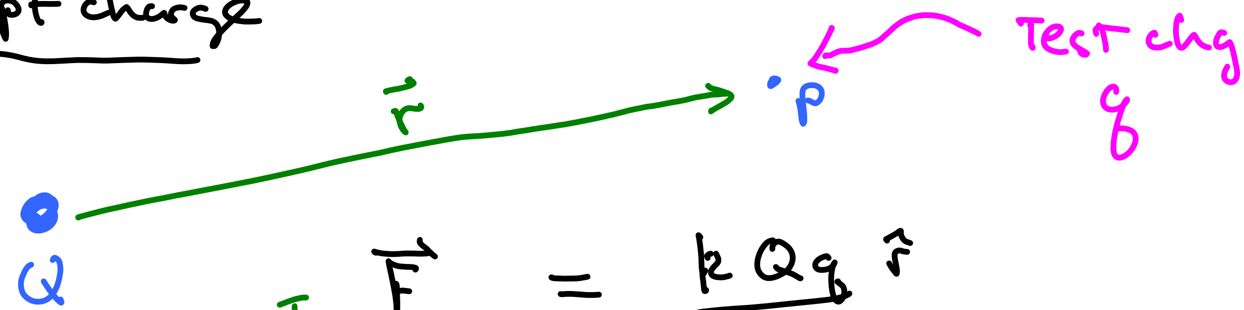


use "lines of force" to visualize the electric field

- go from ⊕ to ⊖ or have an endpoint at ∞
- lines never cross
- density of lines $\propto |\vec{E}|$
- $\vec{F}, \vec{E}$ always tangent to line of force

see electric field applet http://web.pas.rochester.edu/~manly/class/P142_2010/Lectures/EField/index.html

E field of pt charge

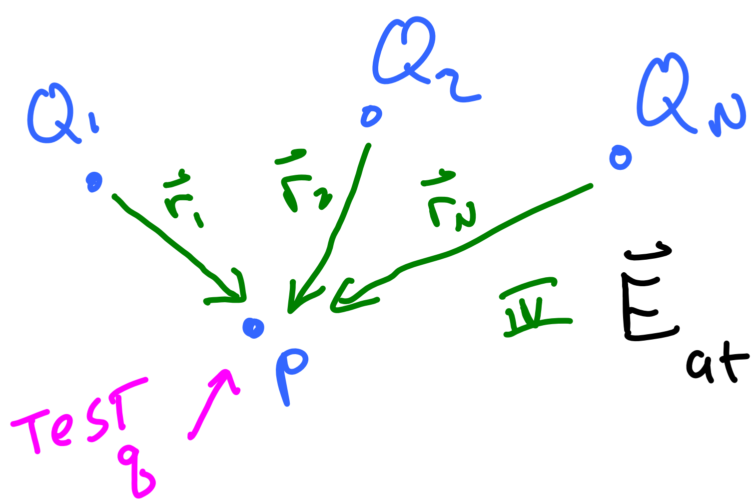


I $\quad \vec F_{Q\,on\,q} = \dfrac{kQq}{|\vec r|^2}\,\hat r$

II $\quad \vec E_{\text{due to } Q} = \dfrac{\vec F}{q} = \dfrac{kQ}{|\vec r|^2}\,\hat r$

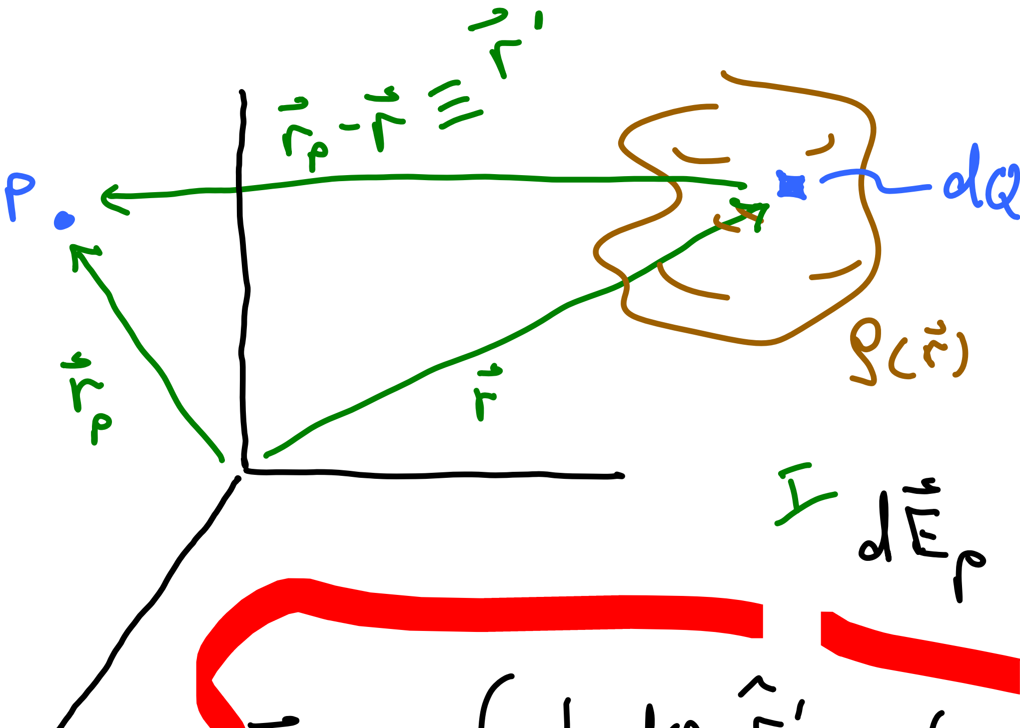
discrete charge distr.

III $\quad \vec F_q = \sum \dfrac{kQ_i q}{|\vec r_i|^2}\,\hat r_i$



IV $\quad \vec E_{\text{at } p} = \sum \dfrac{kQ_i}{|\vec r_i|^2}\,\hat r_i$

Electric Field for general chg. distribution

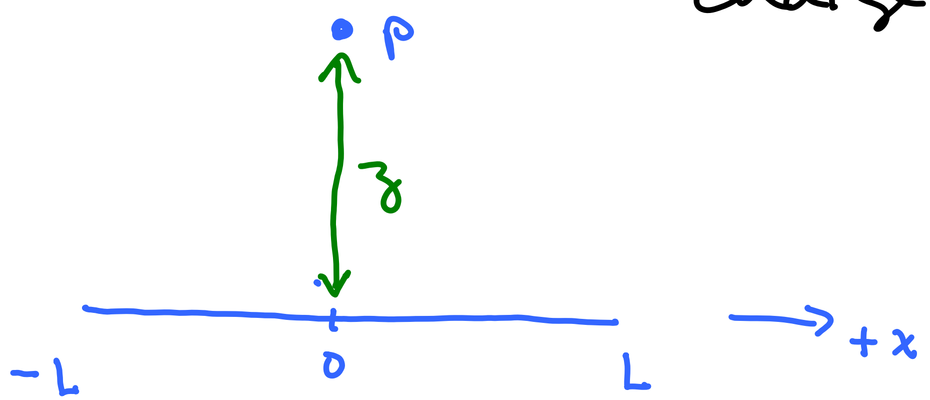


I $\quad d\vec{E}_p = \dfrac{k\,dQ}{|\vec{r}\,'|^2}\,\hat{r}\,'$

$$\boxed{\vec{E}_p = \int_{\substack{\text{chg}\\ \text{dist}}} \frac{k\,dQ}{|\vec{r}\,'|^2}\,\hat{r}\,' = \int \frac{k\,\rho(\vec{r})\,dv}{|\vec{r}\,'|^2}\,\hat{r}\,'}$$

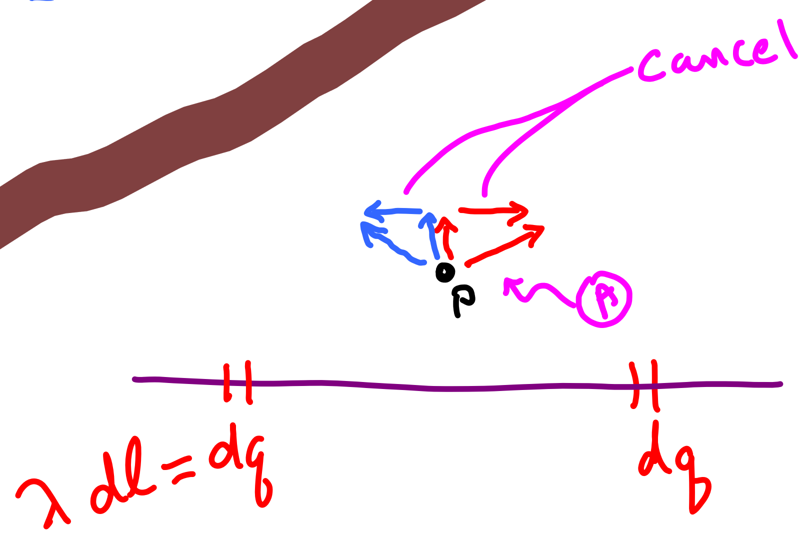
Example

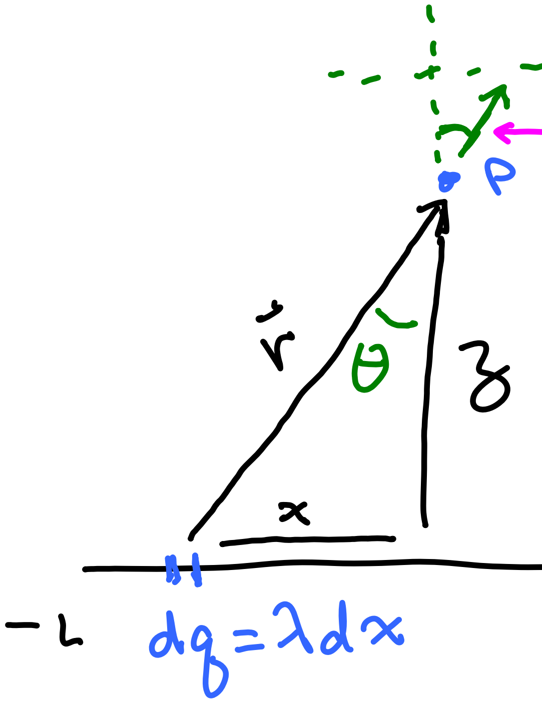
Find $\vec{E}$ at dist z above the midpoint of a line segment of length $2L$ which carries a uniform line charge of λ



A

B





$d\vec{E}_p$ due to $dq=\lambda dx$

impt part

$dE_p \cos\theta \, \hat{z}$

$dE_p \dfrac{z}{|\vec{r}|} \hat{z}$

I $$\vec{E}_p = \int \frac{k\,dq}{r^2}\,\hat{r}$$

II $$\vec{E}_p = 2\int_0^L \frac{k\lambda\,dx\cos\theta}{r^2}\,\hat{z}$$

III $$|\vec{r}| = (x^2+z^2)^{1/2}$$

IV $$\cos\theta = \frac{z}{\sqrt{x^2+z^2}}$$

V $$\vec{E}_p = 2\int_0^L \frac{k\lambda z\,dx}{(x^2+z^2)^{1/2}(x^2+z^2)}\,\hat{z}$$

I $\quad \vec{E}_p = 2k\lambda z \int_0^L \frac{dx}{(x^2+z^2)^{3/2}} \hat{z}$

II $\quad \vec{E}_p = 2k\lambda z \left[\frac{x}{z^2(x^2+z^2)^{1/2}} \right]_0^L \hat{z}$

III $\quad \vec{E}_p = \frac{2k\lambda z L}{z^2(L^2+z^2)^{1/2}} \hat{z} = \underbrace{\frac{2k\lambda L}{z(L^2+z^2)^{1/2}}}_{\frac{NT\,\cancel{m^2}}{\cancel{C^2}} \frac{\cancel{C}}{\cancel{m}} \cancel{m} \frac{1}{\cancel{m^2}}} \hat{z}$ ✓

How do we know we are right?

- dimensional analysis

$E \to NT/C$

$F \sim k \frac{qq}{r^2} \qquad \frac{NT \cdot m^2}{C^2} \; k$

- Limiting Cases

I $z \to \infty$ $\quad E \sim \frac{Q}{z^2}$ $(2\lambda L)$ $\quad$ L'Hopital

This is the Field of a point charge at distance z

II $L \to \infty$ $\quad E \to \frac{2\lambda k}{z} \hat{z}$

As we'll see, this is the field around an infinite uniform line charge