Physics 142 - Sept. 14, 2010

Note Title 9/11/2008

Last Time

Electrostatic Force — Coulomb's Law

I discrete

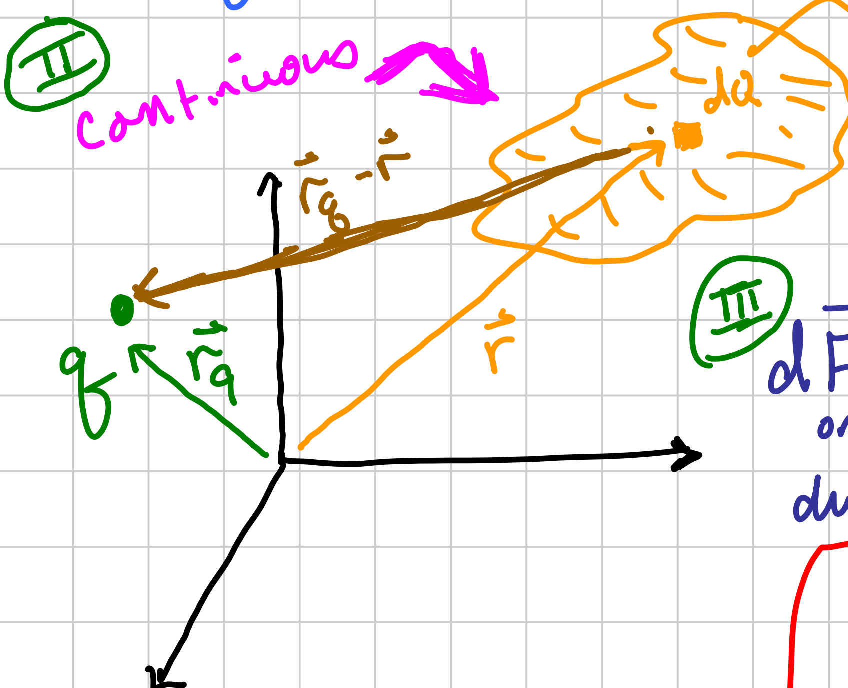
Force of Q on q

$$Q \bullet \xrightarrow{\vec{r}} \bullet q \qquad \vec{F} = \frac{kQq}{r^2}\hat{r}$$

II continuous

differential dQ of charge

Charge distribution



III
$$d\vec{F}_{\text{on } q \text{ due to}} = \frac{kq\,dQ}{|\vec{r}_q - \vec{r}|^2}\,\widehat{(\vec{r}_q - \vec{r})}$$

$\equiv dQ$

IV
$$\vec{F}_q = \int_{\text{vol of charge}} \frac{kq\,\rho(\vec{r})\,dV}{|\vec{r}_q - \vec{r}|^2}\,\widehat{(\vec{r}_q - \vec{r})}$$

Electric Field

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Test chg q

I

discrete

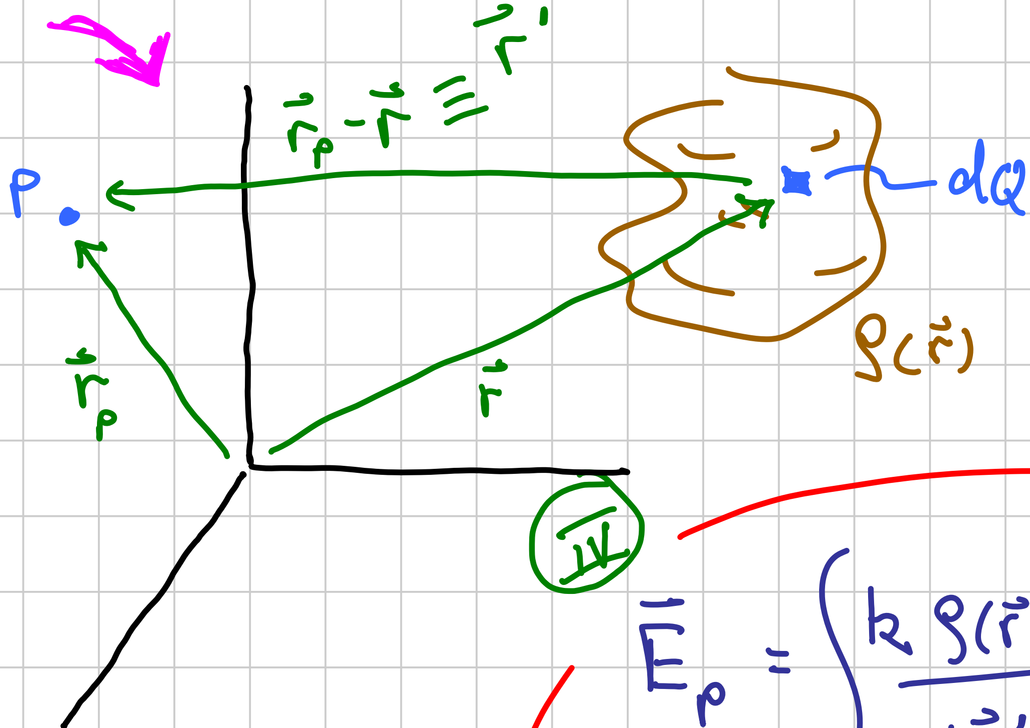
III

$$\bar{E}_{\text{due to } Q} = \frac{\bar{F}}{q} = \frac{kQ}{|\vec{r}|^2}\hat{r}$$

will use often

continuous

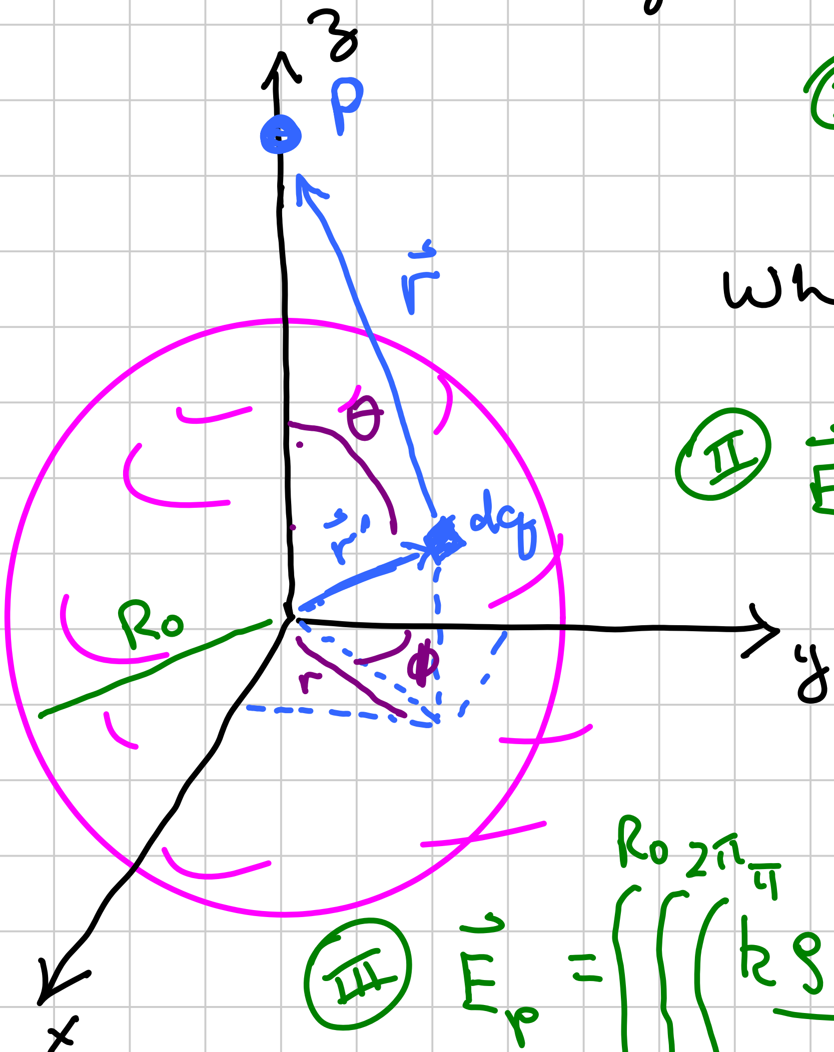
II



IV

$$\bar{E}_p = \int_{\substack{\text{vol} \\ \text{of charge}}} \frac{k\,\rho(\vec{r})\,\hat{r}'\,dv}{|\vec{r}\,'|^2}$$

Charge Q is dist evenly in sphere



I $\rho(\vec{r}') = \dfrac{Q}{\frac{4}{3}\pi R_0^3} = \text{const}$

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What is $\vec{E}_p$?

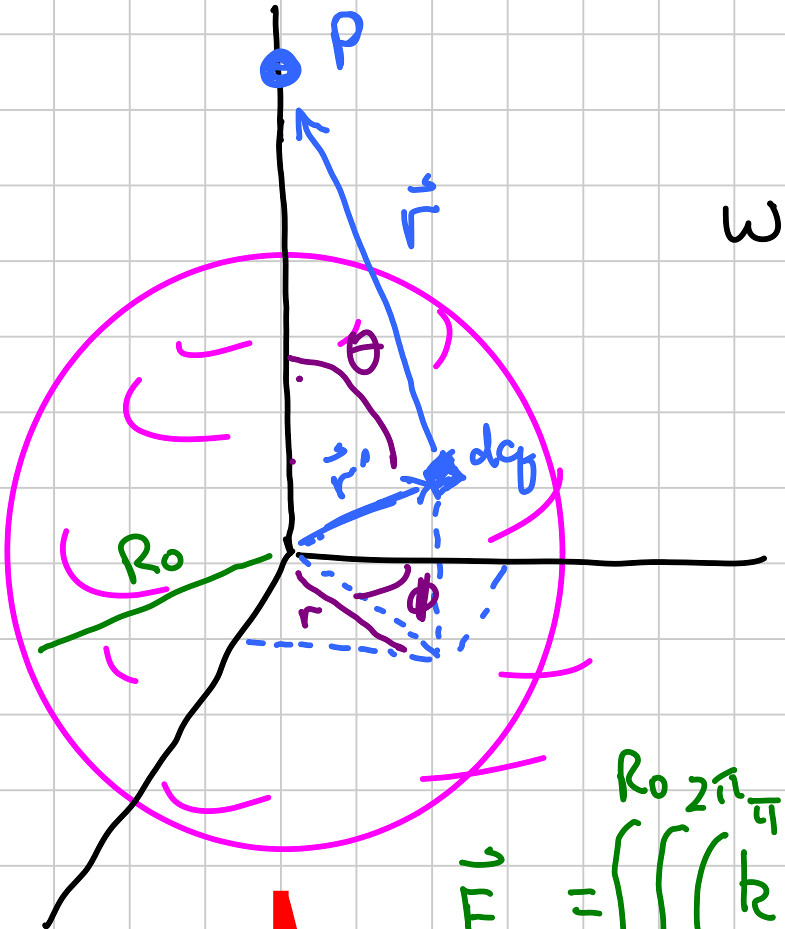
II $\vec{E}_p = \displaystyle\int_{\text{chg dist}} \frac{k \rho \, dv \, \hat{\vec{r}}}{|\vec{r}|^2}$

III $\vec{E}_p = \displaystyle\int_0^{R_0}\int_0^{2\pi}\int_0^{\pi} \frac{k \rho r'^2 \sin\theta \, d\theta \, d\phi \, dr \, \hat{r}}{r^2}$

charge Q is dist evenly in sphere

$$\rho(\vec{r}\,') = \frac{Q}{\frac{4}{3}\pi R_0^3} = \text{const}$$

3a



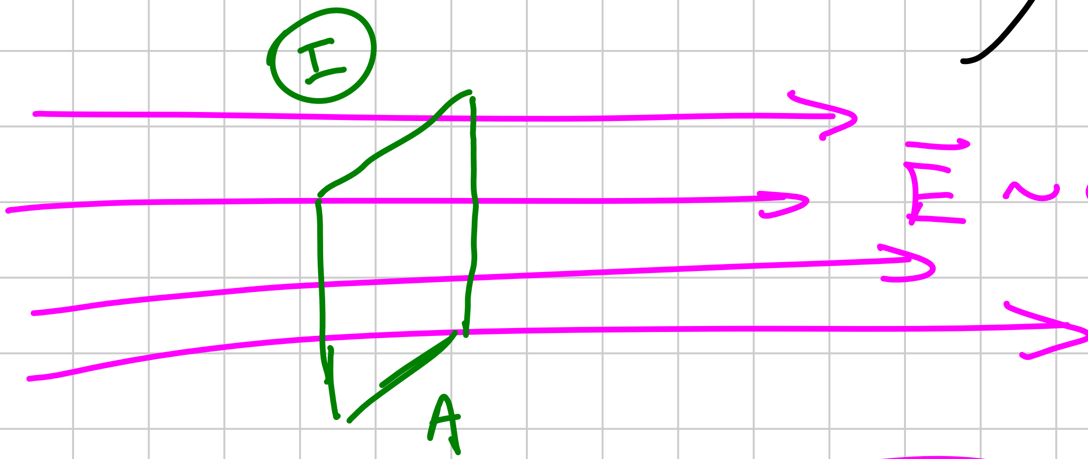
What is $\vec{E}_p$?

$$\vec{E}_p = \int_{\substack{\text{chg}\\ \text{dist}}} \frac{k \rho \, dv \, \hat{\vec{r}}}{|\vec{r}|^2}$$

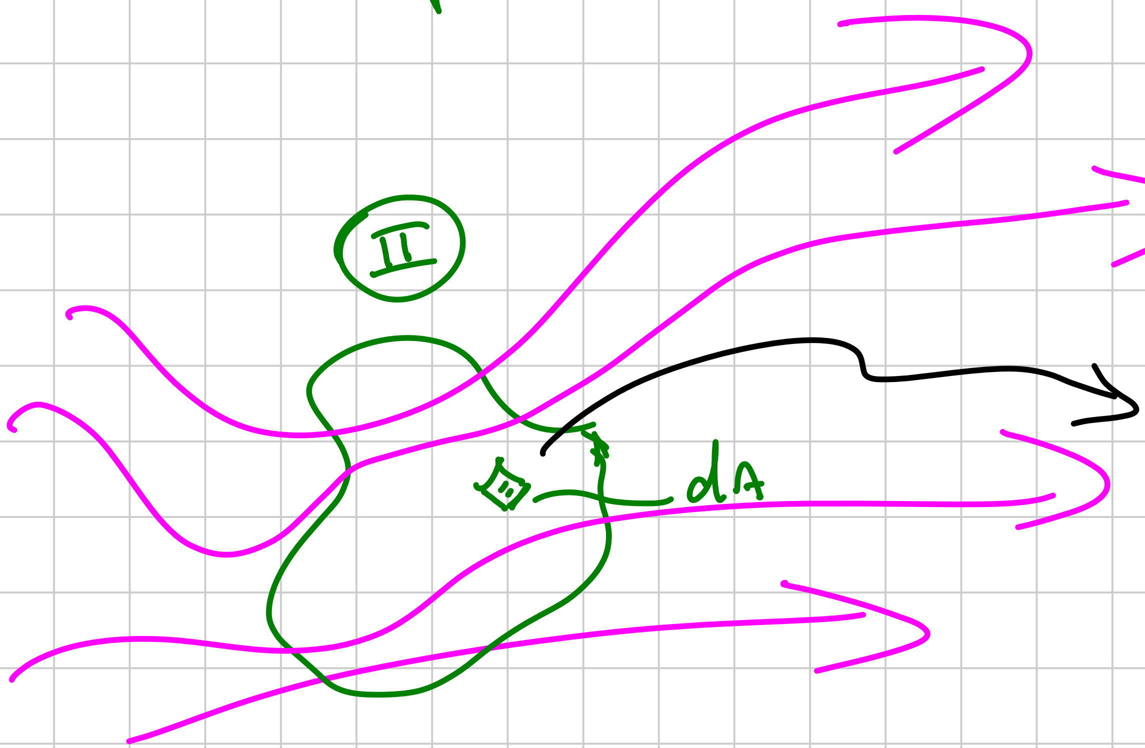
$$\vec{E}_p = \int_0^{R_0}\int_0^{2\pi}\int_0^{\pi} \frac{k \rho \, r'^2 \sin\theta \, d\theta \, d\phi \, dr \, \hat{r}}{r^2}$$

Holy Crap!

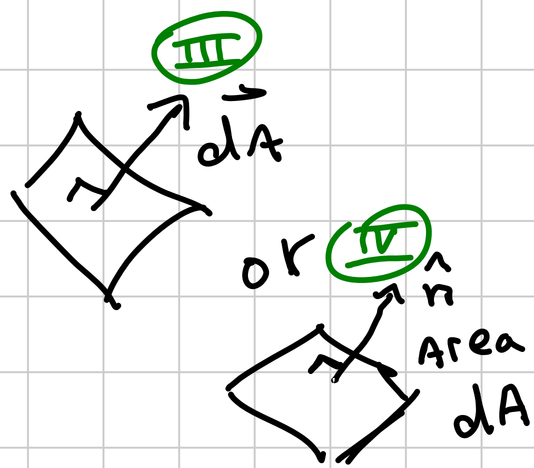
Electric Flux $\equiv \phi = A|\vec{E}|$



$\vec{E} \sim$ const



$\vec{E}$ (not const)



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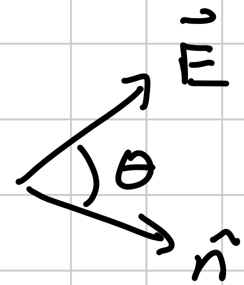
I

$$\boxed{\phi = \int_{\text{surface}} \vec{E}\cdot\hat{n}\, dA}$$

General

II

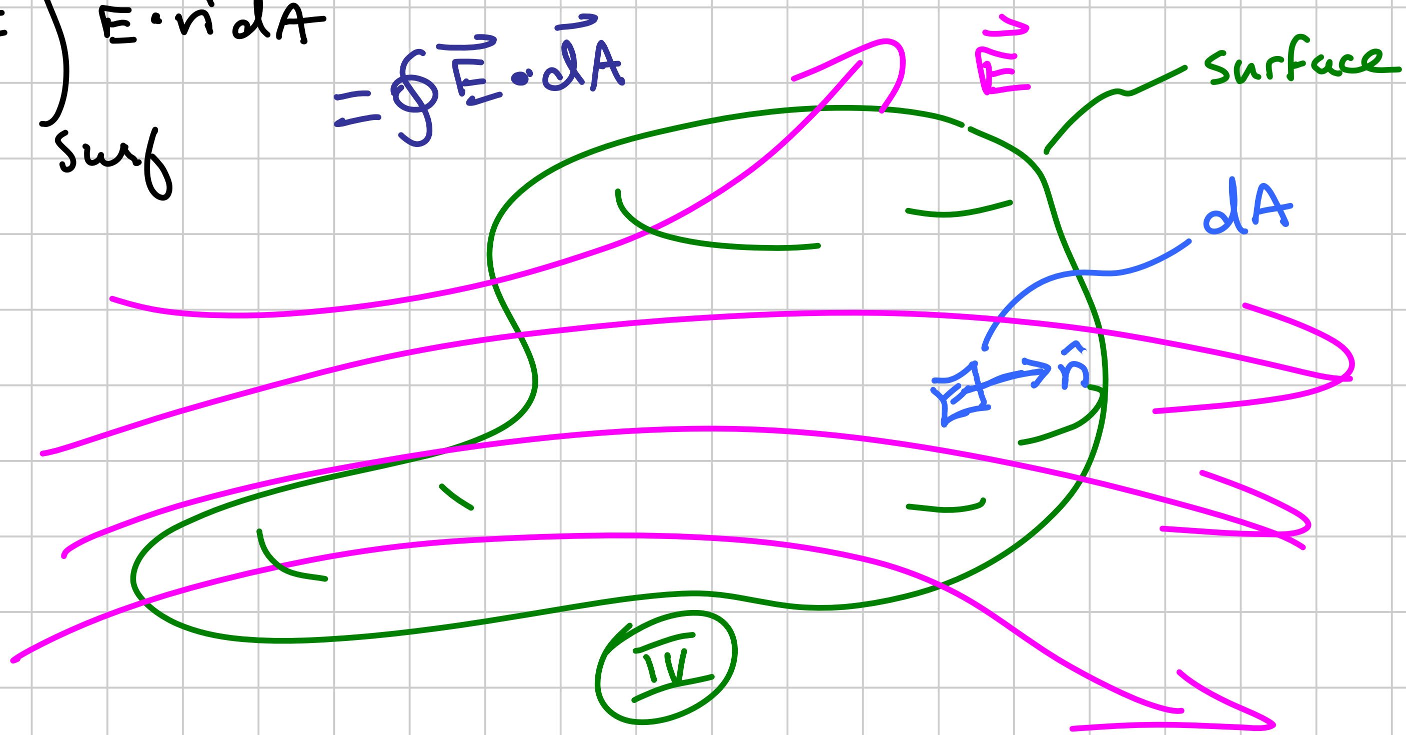


$$\vec{E}\cdot\hat{n} = |\vec{E}||\hat{n}|\cos\theta$$

III

$$\phi = \int_{\text{surf}} \vec{E}\cdot\hat{n}\, dA = \oint \vec{E}\cdot\hat{n}\, dA$$

$$= \oint \vec{E}\cdot d\vec{A}$$



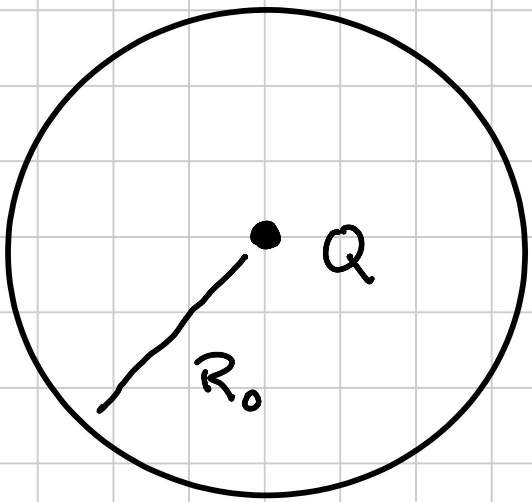
See electric flux applet on class Web site

5a

http://web.pas.rochester.edu/~manly/class/P142_2010/Lectures/Flux/index.html

- Toggle $\vec{E}$ on
- See how $\vec{E}$ penetrates the surface
- Follow the flux calculation
- Vary the angle between the surface and $\vec{E}$ and see how the flux changes.

Example



Pt. chg at origin of spherical surface

→ what is ϕ thru surface?

I

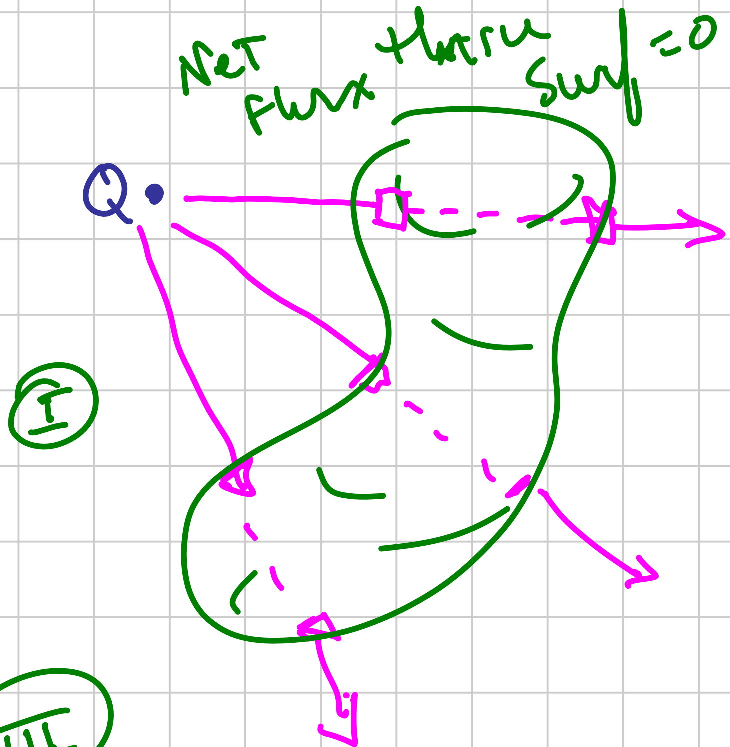
$$\phi_{surf} = \oint \vec{E}\cdot \underbrace{d\vec{A}}_{\equiv \hat{n}\, dA} = |\vec{E}(R_0)| \oint dA = |\vec{E}|\, 4\pi R_0^2$$

II

$$\phi_{surf} = |\vec{E}|\, 4\pi R_0^2 = \frac{kQ}{R_0^2}\, 4\pi R_0^2 = \frac{Q}{\epsilon_0}$$

$$k \equiv \frac{1}{4\pi\epsilon_0}$$

is enclosed



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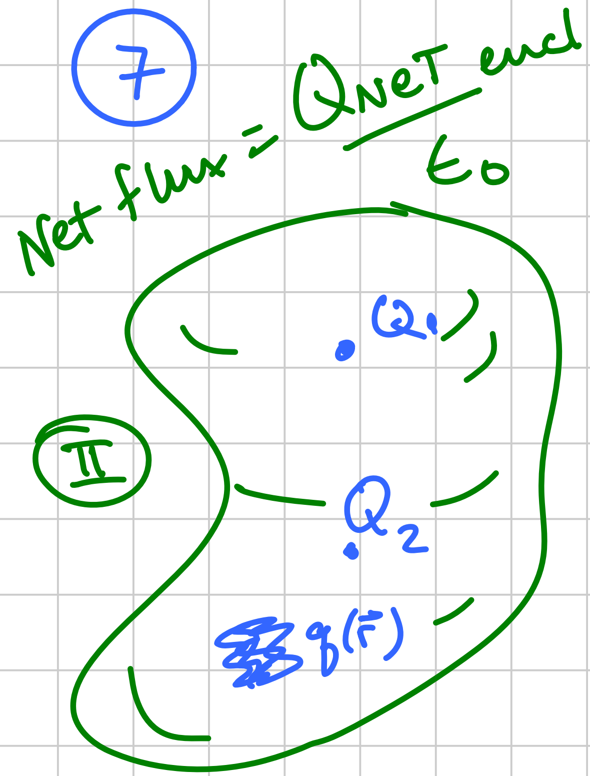


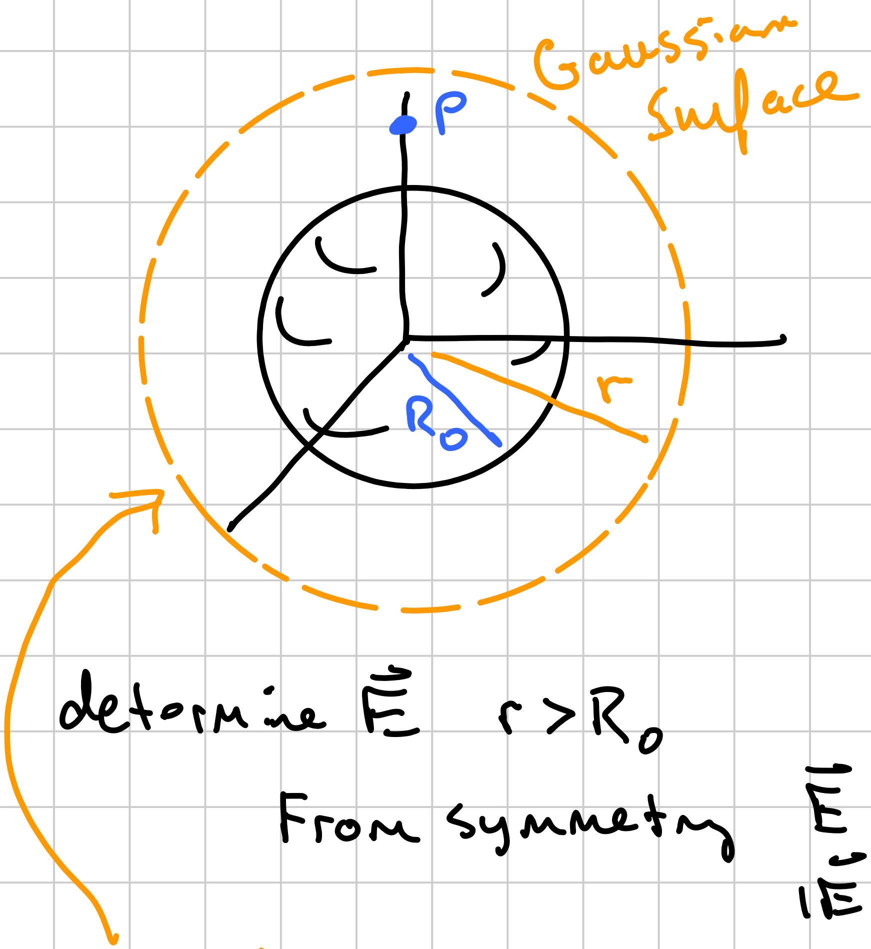
III

$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}}$$

Gauss' Law

very important

True in general – particularly useful if problem has certain symmetries



Spherical dist

const ρ

Total chg Q

What is $\vec{E}_P$?

What is $\vec{E}$ everywhere?

determine $\vec{E}$ $r > R_0$

From symmetry $\vec{E}$ radial

$|\vec{E}|$ on spherical surf centered at origin is const

evaluate Gauss' law

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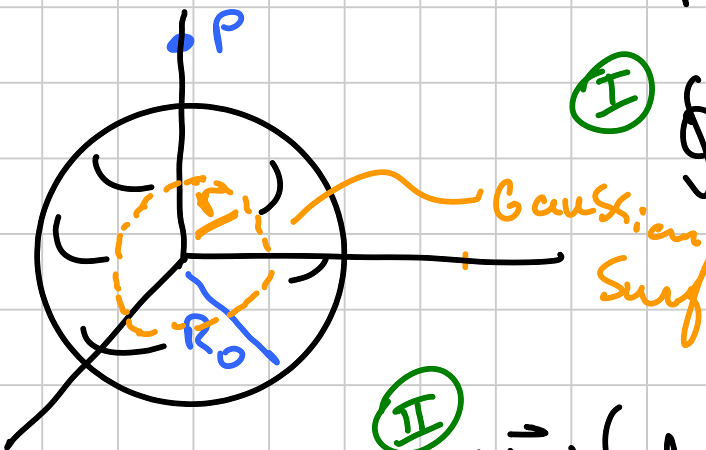
I $\Phi = \oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{encl}}{\epsilon_0}$

II $|\vec{E}| \int dA = \dfrac{Q_{tot}}{\epsilon_0}$

III $|\vec{E}|_{r>R_0} = \dfrac{Q_{TOT}}{4\pi\epsilon_0 r^2}$



$r < R_0$

I $$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

II $$|\vec{E}| \oint dA = \frac{Q_{encl}}{\epsilon_0}$$

III $$|\vec{E}| 4\pi r^2 = \frac{\oint \rho \, dv}{\epsilon_0} = \oint \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3} \frac{1}{\epsilon_0} \, dv$$

IV $$= \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3 \, \epsilon_0} \oint dv$$

← small sphere

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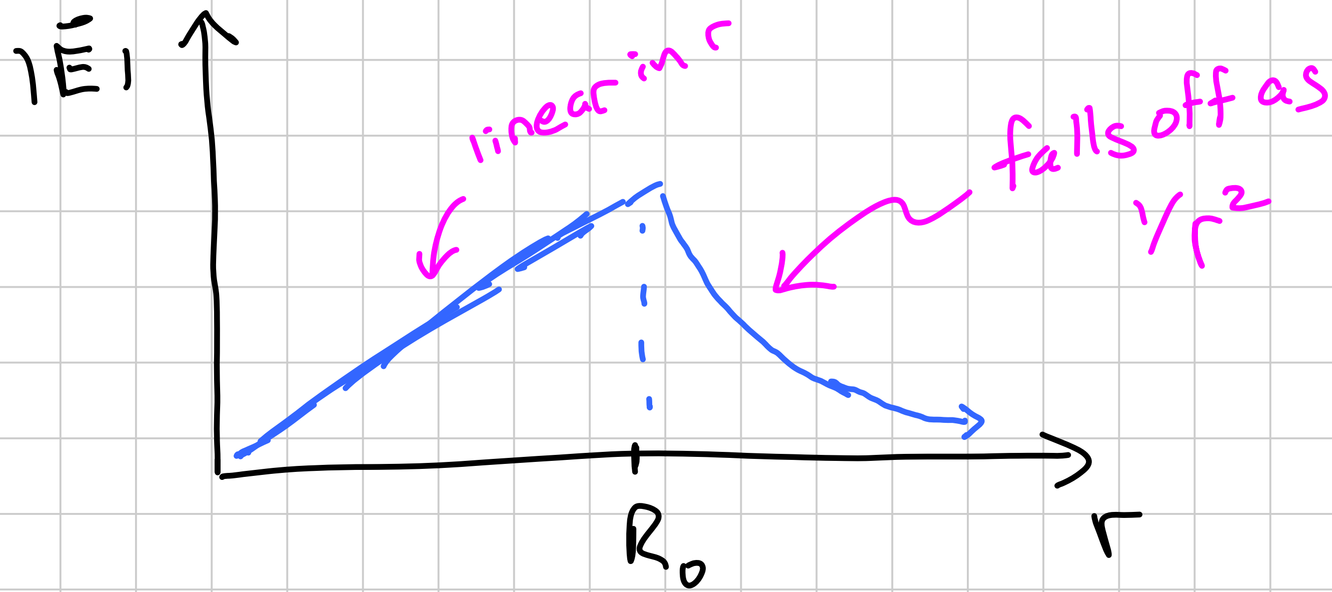
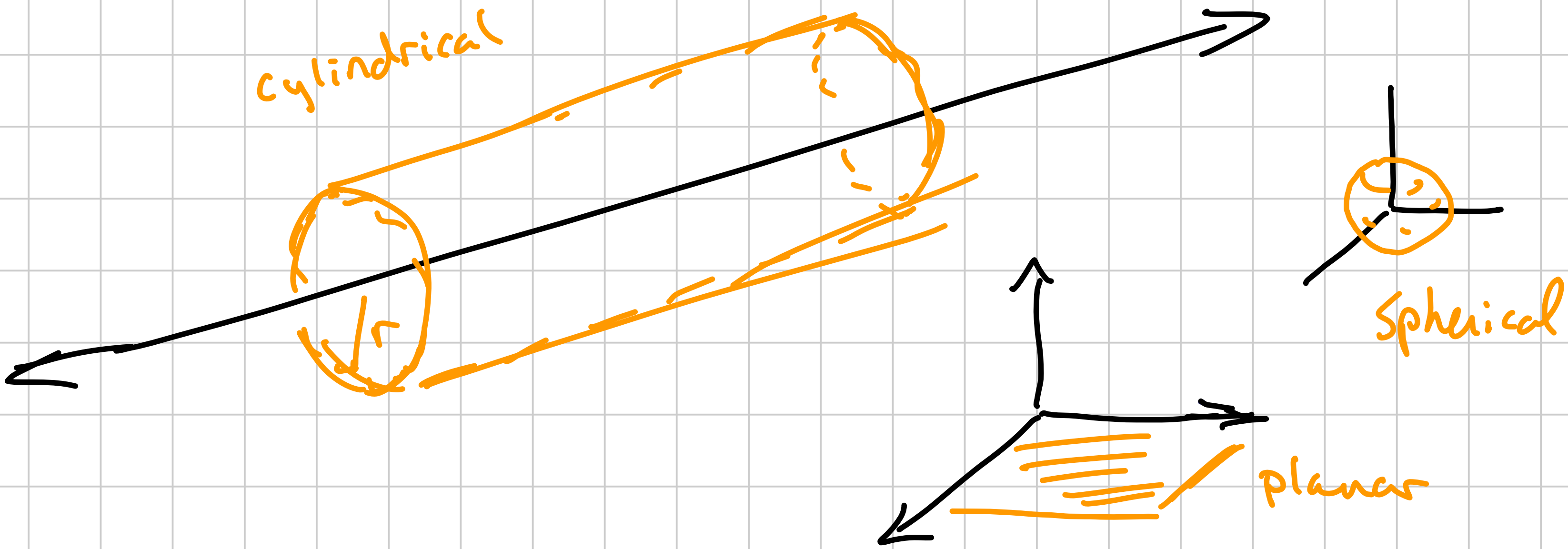
I $$|\vec{E}|\,4\pi r^2 = \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3\,\epsilon_0}\,\frac{4}{3}\pi r^3$$

II $$|\vec{E}|_{r<R_0} = \frac{Q_{TOT}}{4\pi\epsilon_0}\,\frac{r}{R_0^3}$$



Very useful symmetries



Gauss' Law useful when you have a physical situation such that $\vec{E}\cdot\vec{dA}$ is simple to evaluate.

Often most useful when (by symmetry)

$$\vec{E}\cdot\vec{dA} = 0 \quad \text{or} \quad \vec{E}\cdot\vec{dA} = |\vec{E}|\,dA$$

And $|\vec{E}|$ = CONSTANT on Gaussian surface so that

$$\int \vec{E}\cdot\vec{dA} \longrightarrow |\vec{E}| \int dA$$