

Physics 1412 - September 16, 2010

①

LAST TIME

Electric Flux

$$\Phi_{\text{surf}} = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

$\hat{n} dA$



Gauss' Law

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{encl}}}{\epsilon_0} \quad (2)$$

integral over volume inside
gaussian surface
(NOT necessarily all charge)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

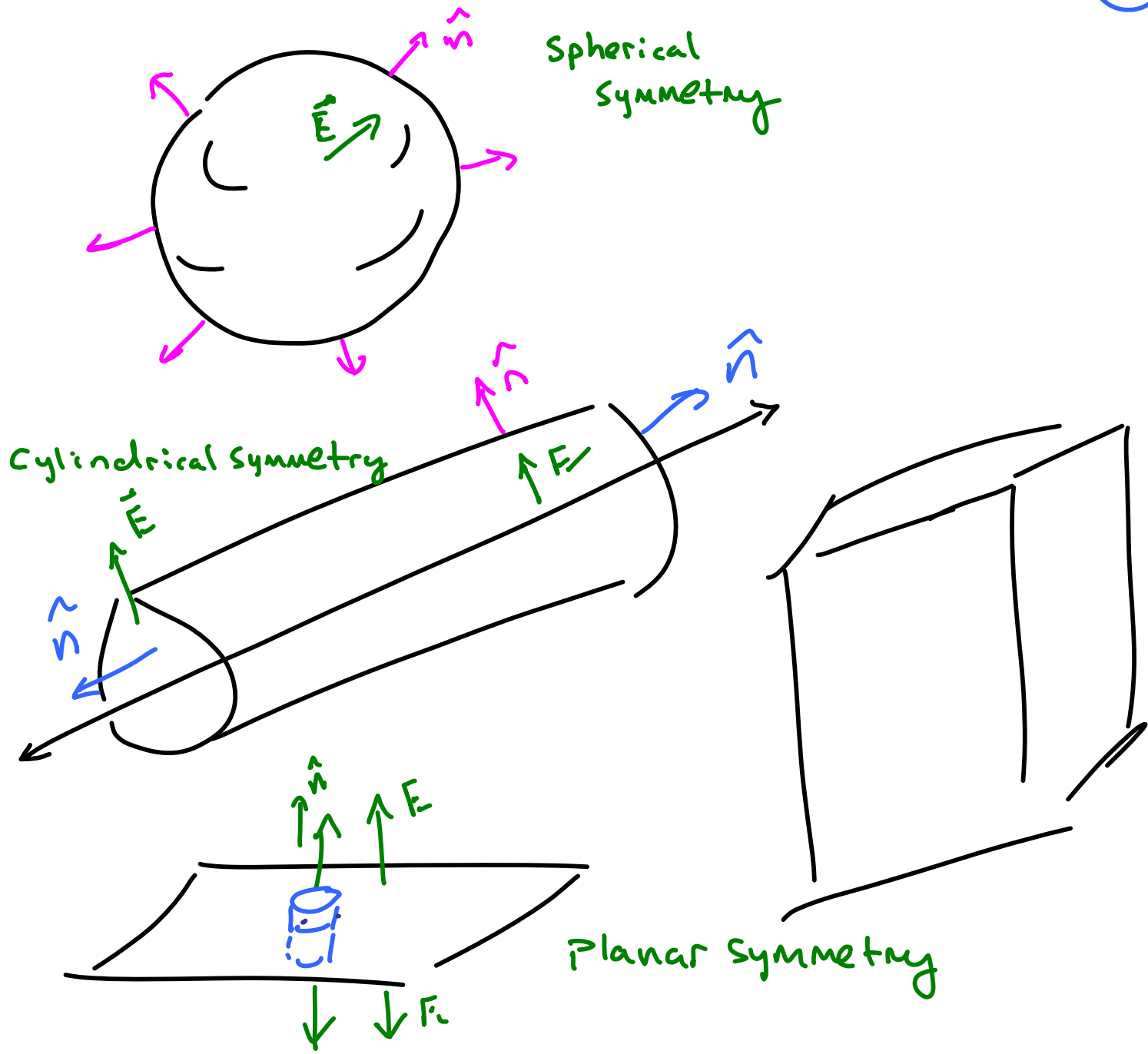
$$\int_V \rho \, dv$$

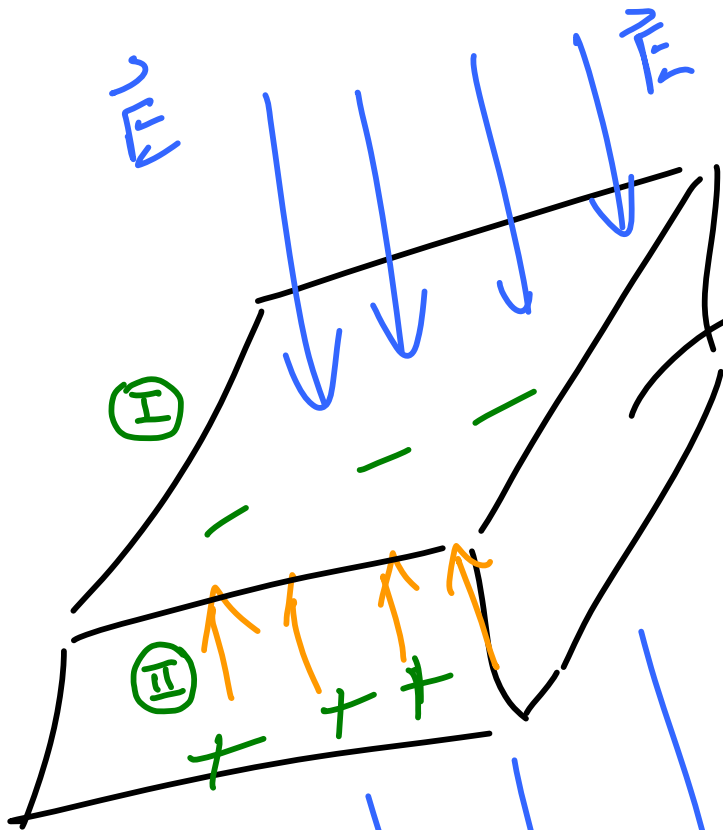
Easy if $|\vec{E}|$ is constant
on surface -
can pull out
of integral

Easy if
 $\vec{E} \perp d\vec{A}$ or $\vec{E} \parallel d\vec{A}$

integral
over
Gaussian
surface

True
in
general

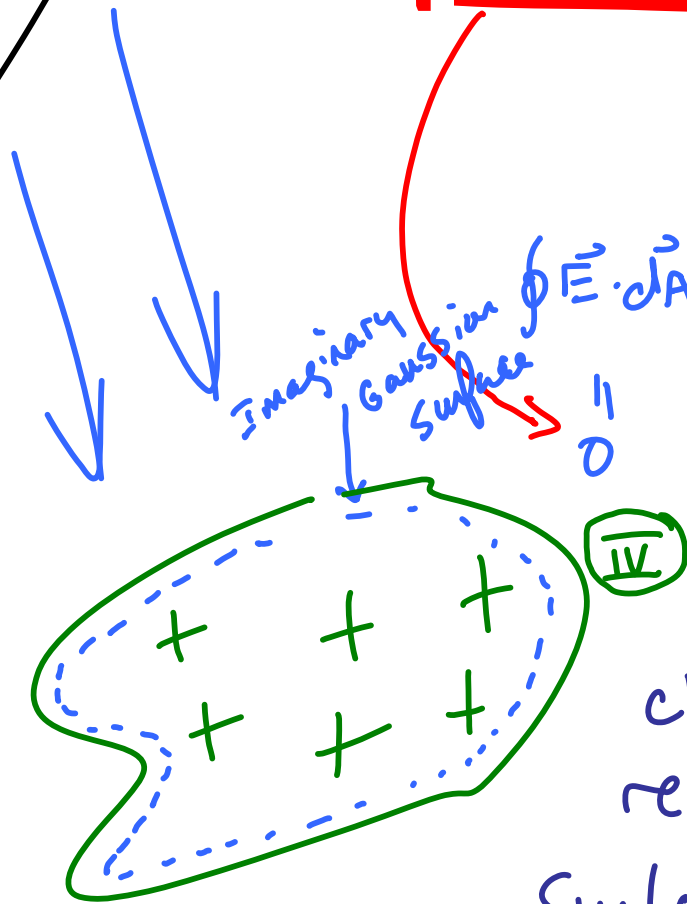
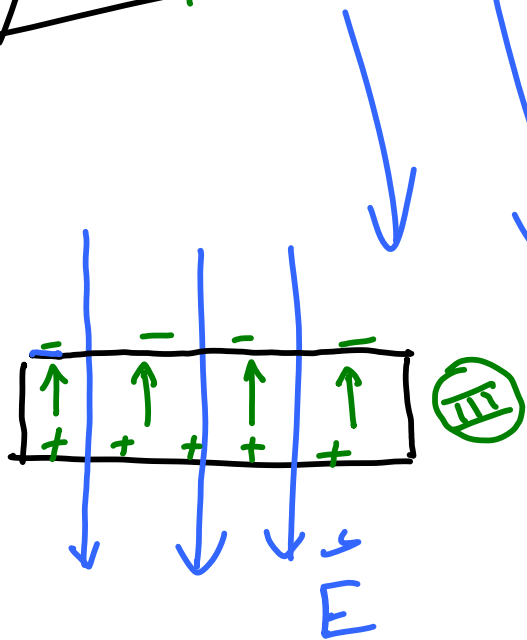




④

$$\vec{E}_{\text{inside}} = 0$$

inside
conductor



Imaginary Gaussian surface

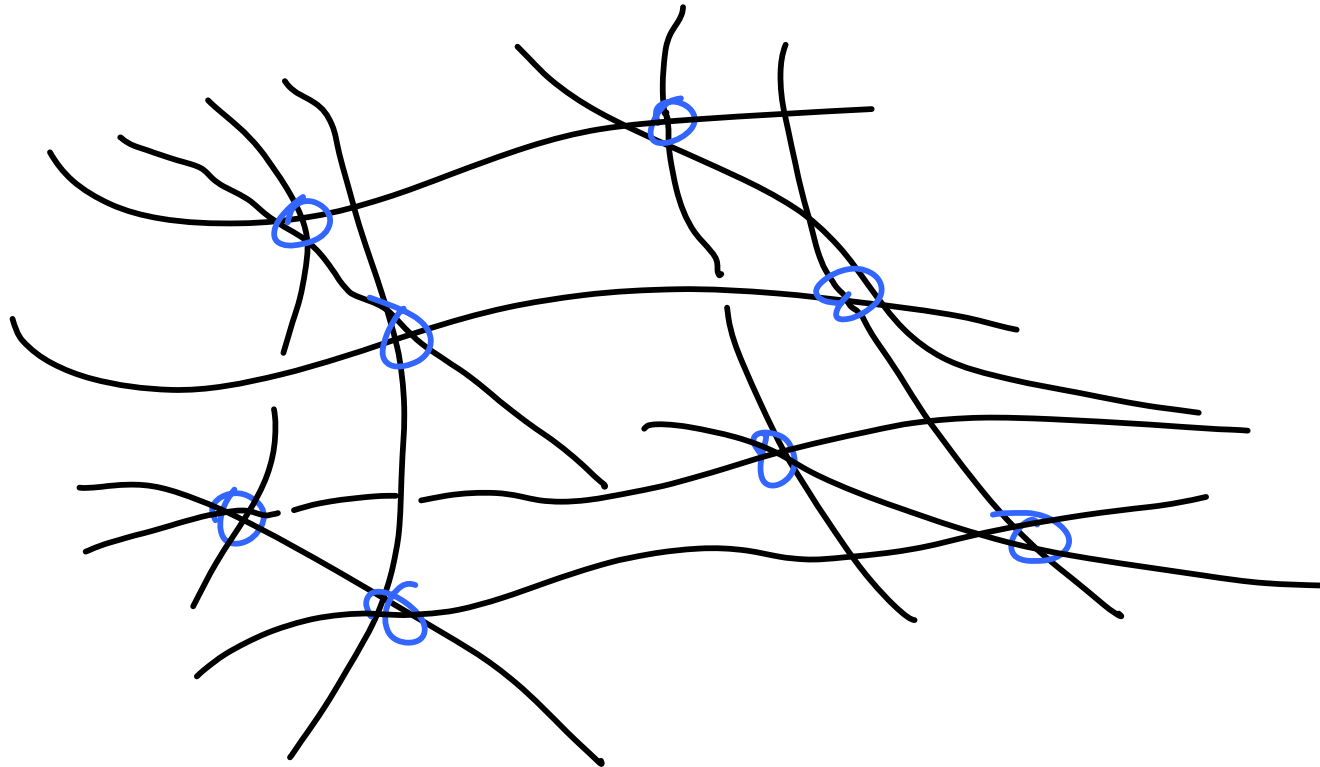
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

= 0

charge
resides on
surface of
conductor

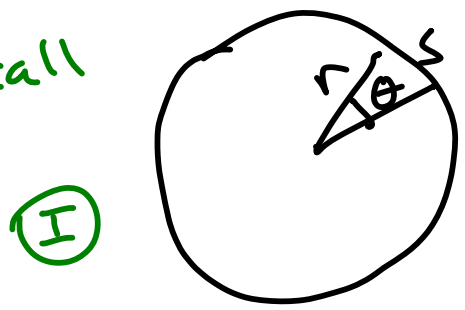
5

Curvilinear Coordinate System



6

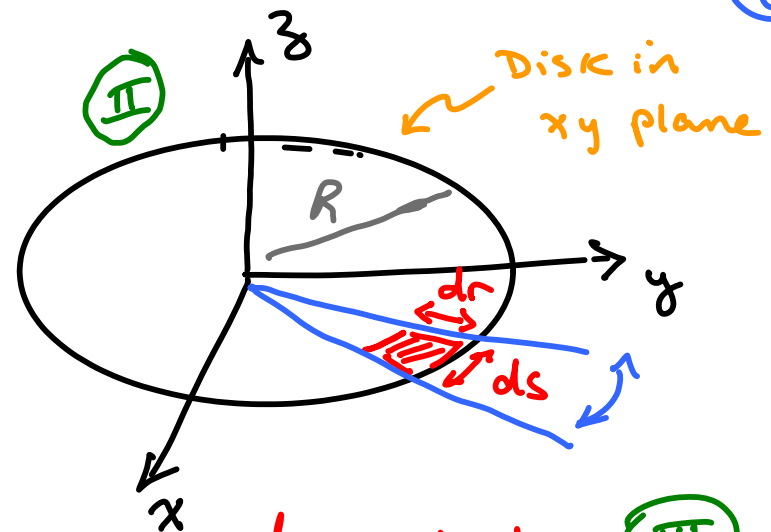
Recall



(I)

$$S = r\theta$$

Find area of
circular disk



(II)

Disk in xy plane

$$da = dr ds \quad \text{(III)}$$

$$= r d\theta dr \quad \text{(IV)}$$

$$ds = r d\theta$$

R integral

theta integral

(V)

$$Area = \int da = \int_0^R \int_0^{2\pi} r d\theta dr$$

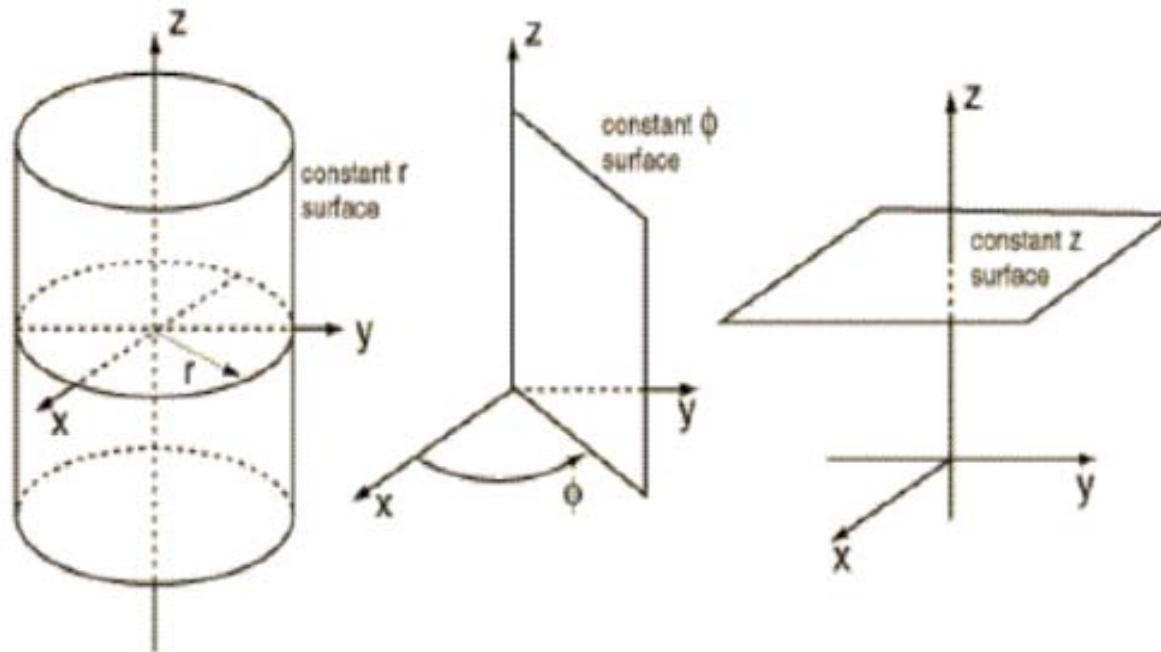
(VI)

$$= \int_0^R r dr \int_0^{2\pi} d\theta = \int_0^R r dr (2\pi)$$

(VII)

$$= \frac{R^2}{2} 2\pi = \pi R^2$$

cylindrical coordinates



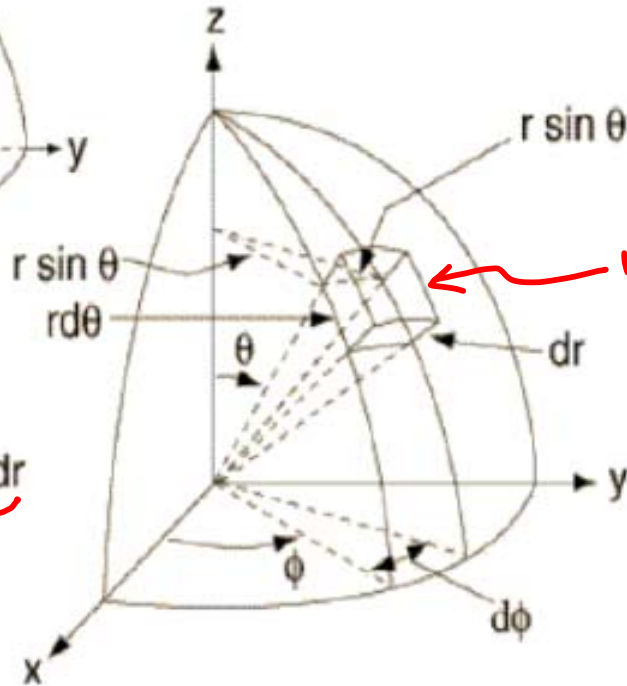
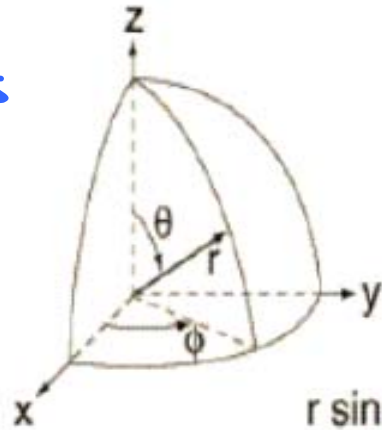
Some figures in this section from:

<http://hyperphysics.phy-astr.gsu.edu/hbase/sphc.html>

Also Griffiths, Intro to Electromagnetism

Spherical polar coordinates

Variables Defined

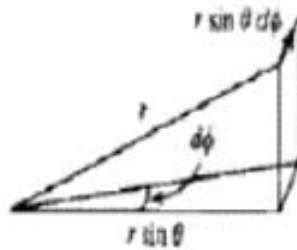


Volume element dv

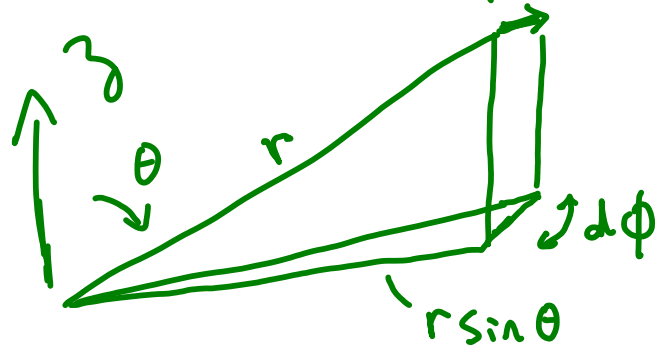
Differential Volume element $dV = r^2 \sin \theta \, d\theta \, d\phi \, dr$

This is important
Spend some time
studying this
and make
sure you
understand it.

$$dl_\phi = r \sin \theta \, d\phi$$

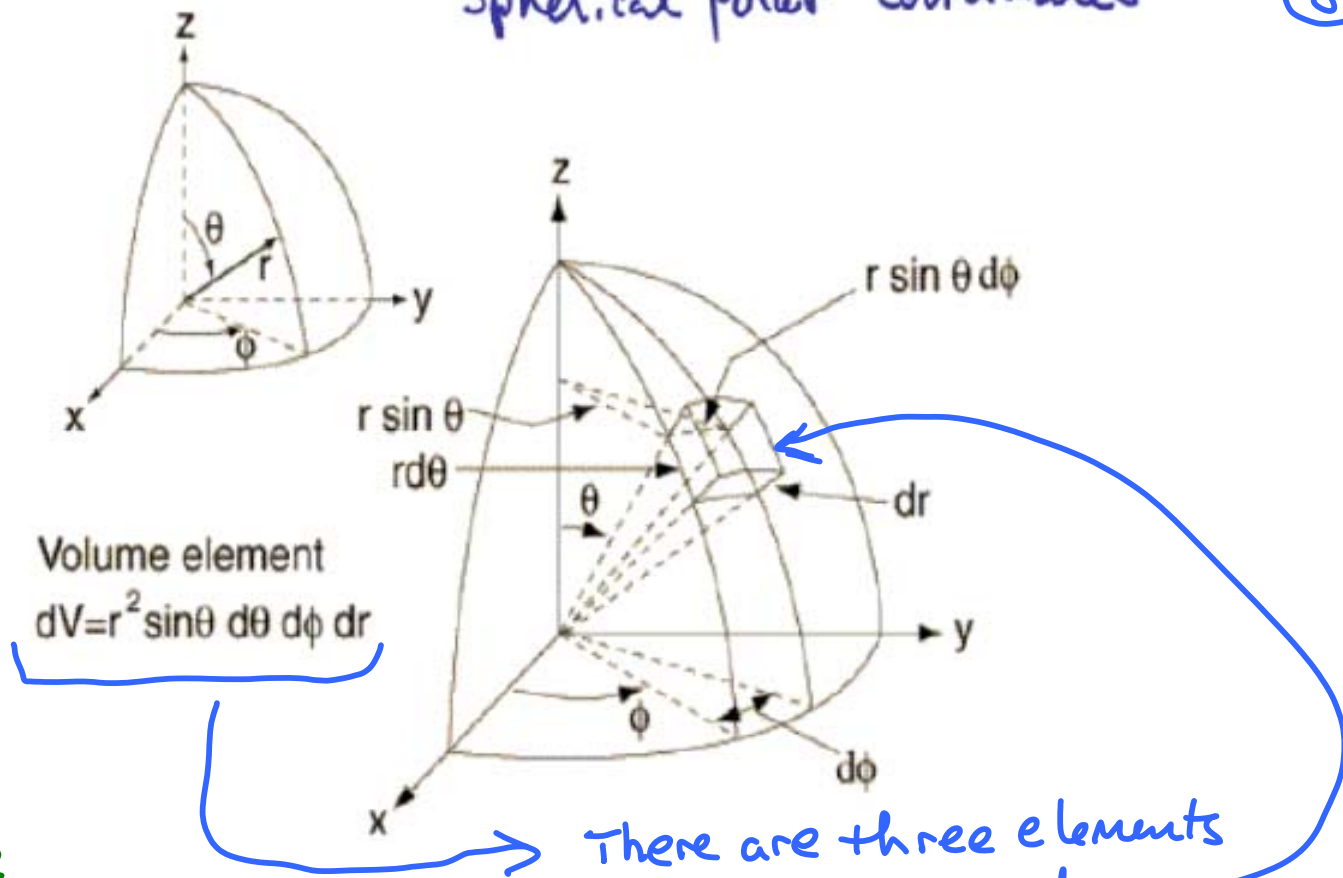


$$dl_\phi = r \sin \theta \, d\phi$$



Spherical polar coordinates

8a

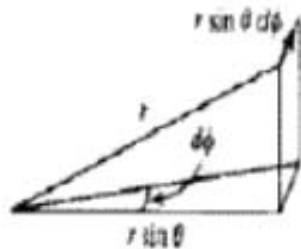


if This were
 a cartesian
 coordinate
 problem

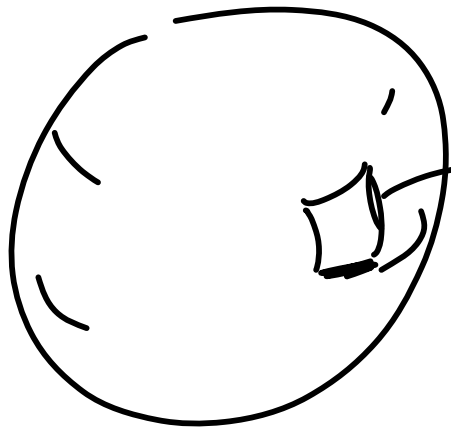
$$dv = dx \, dy \, dz$$

There are three elements
 that make up dv

$$dl_\phi = r \sin\theta \, d\phi$$



$\frac{1}{r}$ in r dr
 $\frac{1}{\sin\theta}$ in θ $r \, d\theta$
 $\frac{1}{\sin\theta}$ in ϕ $r \sin\theta \, d\phi$



Find Area of Sphere

9

θ integral

ϕ integral

(I)

$$\text{Area} = \int da = \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\theta d\phi$$

CONSTANT

(II)

$$= r^2 2\pi \int_0^\pi \sin\theta d\theta = 4\pi r^2$$

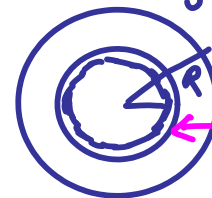
Can Choose shape of dv to simplify problem

or integrate over θ by construction



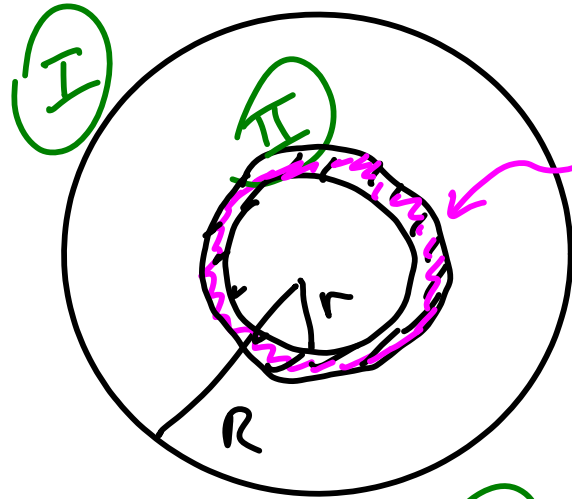
$$da = dr r d\theta$$

$$A = \int_0^R \int_0^{2\pi} r d\theta dr$$



$$da = 2\pi r dr$$

area of circle
- Solved as a 1-dimensional integral

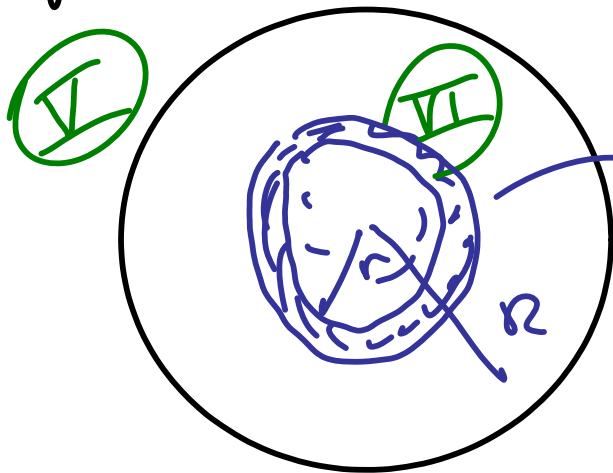


III $da = 2\pi r dr$

IV $\int_0^R 2\pi r dr$

$= \pi r^2$

Vol of Sphere



Shell

$dv = 4\pi r^2 dr$

$Vol\ sph = \int_0^R dv = \int_0^R 4\pi r^2 dr = \frac{4}{3}\pi R^3$



$$\rho = k r^3 \quad r < R$$

$$= 0 \quad r \geq R$$

II

units of k must make
things work

III

$$\rho \rightarrow \frac{C}{m^3}$$

so

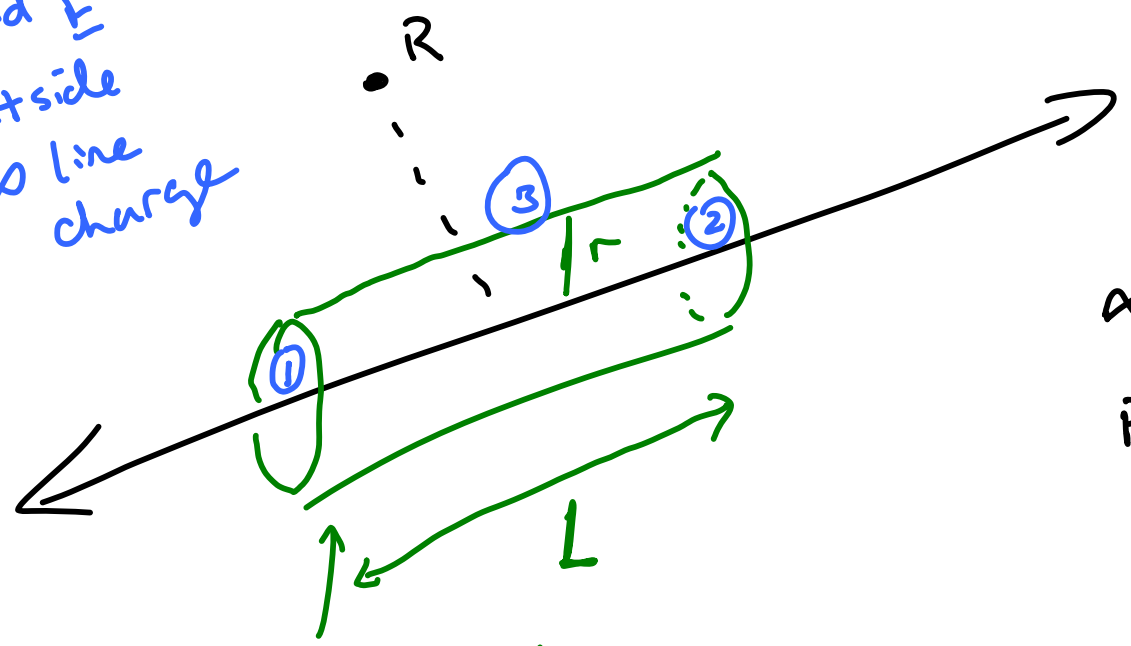
$$k \rightarrow \frac{C}{m^6}$$

Find total charge

$$\text{I} \quad Q_{\text{TOT}} = \int \rho \, dv = \int_0^R \rho \underbrace{4\pi r^2 \, dr}_{dv}$$

$$\text{II} \quad Q_{\text{TOT}} = \int_0^R k r^3 4\pi r^2 \, dr = k 4\pi \int_0^R r^5 \, dr \\ = k 4\pi \frac{R^6}{6}$$

Find \vec{E}
outside
 ∞ line
charge



$+ \lambda$ chg/length

∞ line charge
Find electric
field at R

cylindrically symmetric gaussian surface

\vec{E} radially out (symmetry)

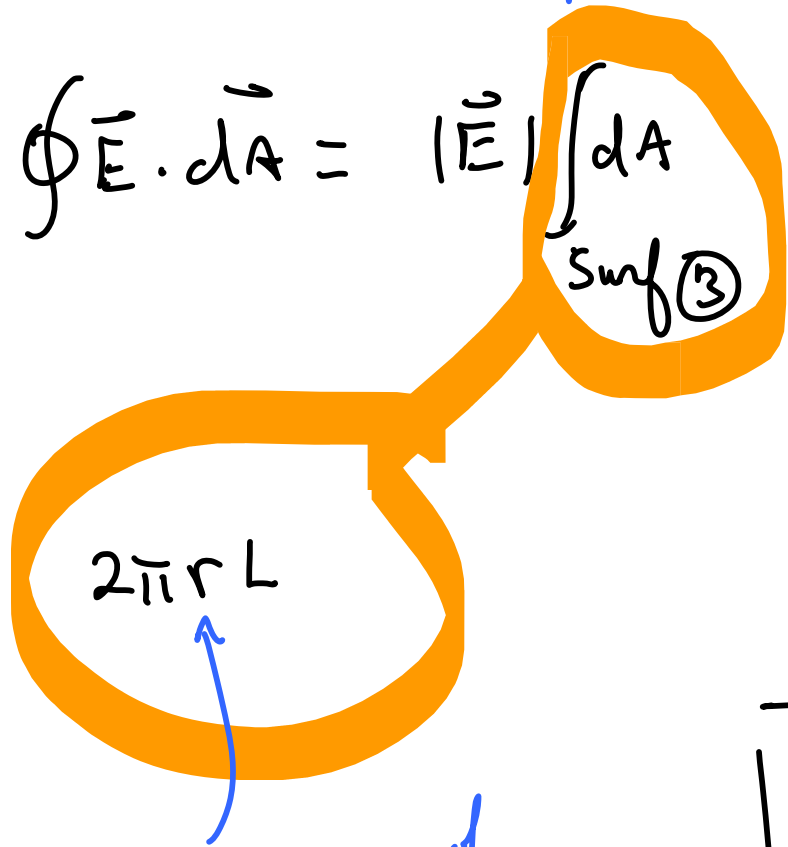
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$\vec{E} \cdot d\vec{A} = 0$ for ① + ② because $d\vec{a} \perp \vec{E}$
for "endcaps"

$$\vec{E} \cdot d\vec{A} = |\vec{E}|_r dA \text{ for surf } \textcircled{3}$$

(13)

$$\oint \vec{E} \cdot d\vec{A} = |\vec{E}| \int_{\text{surf } \textcircled{3}} dA = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$



Surface area of
Pipe part of
cylinder + caps

$$|\vec{E}| 2\pi r L = \frac{\lambda L}{\epsilon_0}$$

$$|\vec{E}| = \frac{\lambda}{2\pi r \epsilon_0}$$

\vec{E} is radial out
by symmetry
(Assuming λ is positive)