How did Thurs. "lecture" work out?

- Exam 1 - Oct 7 0800-0920 Loc TBA
  - NOT Oct 2 during class time
  - Calculators
  - 1 3x5 index card (both sides) formulas
  - Post P1412 exams
  - Formula sheet + integral tables supplied
  - Material coverage forth coming
Last time, we discussed the work done to move a charge from state I to state II. The potential difference between these two states is given by:

$$\Delta \text{Energy of system} = \Delta U$$

The work done, $W$, to move charge $Q$ from state I to state II is given by:

$$W = -\frac{\Delta U}{Q} = \text{Potential difference}$$

Well defined, absolute potential requires that a "zero" be defined:

$$V = V_{II} - V_I = \Delta V$$

Units: Joules per Coulomb.
Man of the Hour

1 Volt = 1 Joule/Coulomb

Count Alessandro Giuseppe Antonio Anastasio Volta

Como, Lombardy, Italy

1745 - 1827

Invented the Voltaic pile

ForeRunner of the modern battery
Electrostatics (Electromagnetism) is a conservative force.

Potential difference is path independent.

Point charge \( \mathbf{r} \rightarrow \mathbf{r}_p \)

Note this is a scalar.

Potential of sum is scalar sum of potentials.

\[
V_p = \sum Q_i \frac{k}{r_i}
\]
\[ V_\rho = \int \frac{k \, d\rho}{r} \quad \text{Volume} \]

\[ dV_\rho = \frac{1}{2} \, d\rho \]

Potential due to \( d\rho \)
Why do you care??

\[ E_s = -\frac{dV}{ds} \]

can get \( \bar{E} \) from \( V \).

\[ V(x) \rightarrow E_x = -\frac{dV}{dx} \]

\[ V(x, y, z) \]

\[ \bar{E} = \hat{i} E_x + \hat{j} E_y + \hat{k} E_z \]

\[ \bar{E} = \hat{i} \left( -\frac{dV}{dx} \right) + \hat{j} \left( -\frac{dV}{dy} \right) + \hat{k} \left( -\frac{dV}{dz} \right) \]

\[ \frac{\partial V}{\partial s} = \frac{dV}{ds} \quad \text{where all other variables are treated as constant} \]
\[ \vec{E} = \hat{i} \left( -\frac{\partial V}{\partial x} \right) + \hat{j} \left( -\frac{\partial V}{\partial y} \right) + \hat{k} \left( -\frac{\partial V}{\partial z} \right) \]

\[ \vec{E} \equiv -\vec{\nabla} V = -\nabla \cdot V \]

\[ \vec{\nabla} \equiv \text{gradient} \equiv \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \]

\[ V = f(x, y, z) = e^x y^2 z^3 \]

\[ \vec{E} = -e^x y^2 z^3 \hat{i} - 2y e^x z^3 \hat{j} - 3z^2 e^x y^2 \hat{k} \]

**Vector operator**
2 conducting spheres connected by conducting wire

\[ \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \]

\[ \frac{Q_1}{Q_2} \text{ as } \frac{R_1}{R_2} \]

where does breakdown occur?

\[ E_1 = \frac{kQ_1}{R_1^2} \]

\[ E_2 = \frac{kQ_2}{R_2^2} \]

\[ \frac{kQ_1}{R_1} \quad \frac{kQ_2}{R_2} \quad \frac{1}{R_1} \quad \frac{1}{R_2} \]

Deposit Q

How does it get distributed?
\[ \nu_p = \frac{1}{r} \]

\[ \Delta V_{AB} = 0 \]

Lines of equal potential

Equi-potential lines

\[ \Delta V_{CD} = \frac{bQ}{r_D} - \frac{bQ}{r_C} \]

Equipotential lines always at right-angles to electric field
lines of equipotential

Always at Right Angles to
electric field lines

(Recall $\vec{E} = -\nabla V$ ... meaning the gradient
or direction of greatest slope in
Potential function)
Determine $V_p$ and $E_p$.

$$dg_\theta = \lambda \, ds$$

$$\lambda = \frac{Q}{2\pi a}$$

$$r = \sqrt{x^2 + a^2}$$

$$V_p = \int_{\text{ring}} \frac{k \lambda \, ds}{r}$$

$$V_p = \frac{kQ}{r} \int ds = \frac{kQ}{r} \cdot \frac{2\pi a}{r}$$
\[ V_p = \frac{kQ}{\sqrt{x^2 + a^2}} = \frac{kQ}{\sqrt{a^2 + x^2}} \]

as \( x \to \infty \), \( V_p \to \frac{kQ}{x} \) (looks like point charge from distance)

\[ \vec{E}_p = -\nabla V = -\frac{d}{dx} \frac{kQ}{\sqrt{x^2 + a^2}} \]

as \( x \to \infty \), \( \vec{E} \to \frac{kQ}{x^2} \hat{\mathbf{x}} \)

Field of a point charge