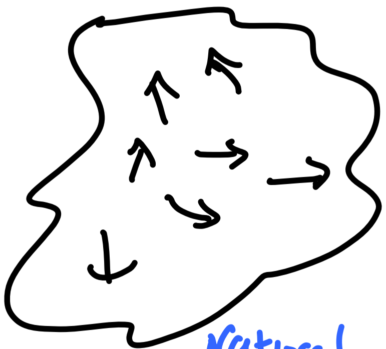
Physics 142 - November 11, 2010

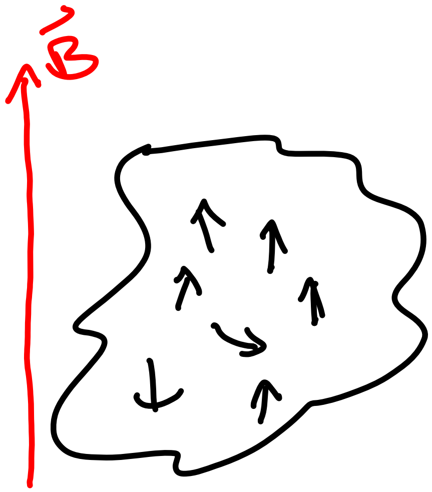
- Exam 2 Tues. Nov. 16 0800 → B+L 109
- Q+A session Mon. Nov. 15 4:30 pm → B+L 109
- Meet w/ presentation groups next week ?!

Magnetism in Materials

Paramagnetic



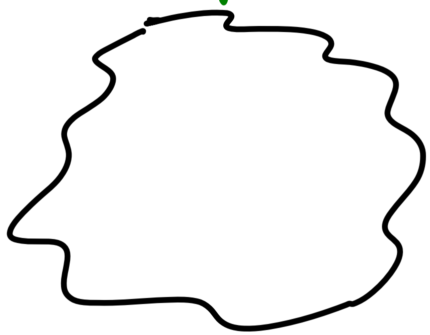
Natural Dipoles



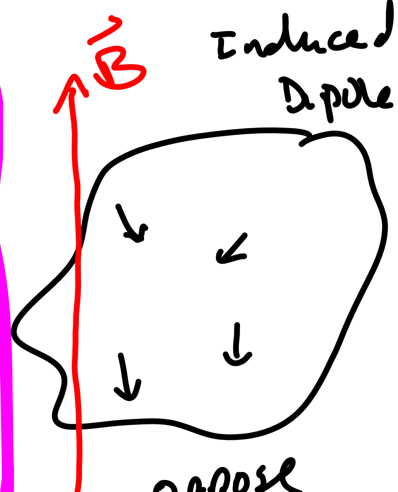
Torque from $\vec{B}$ Aligns more Dipoles along $\vec{B}$

$\Rightarrow$ $\vec{B}$ increased

Diamagnetic



No Dipole

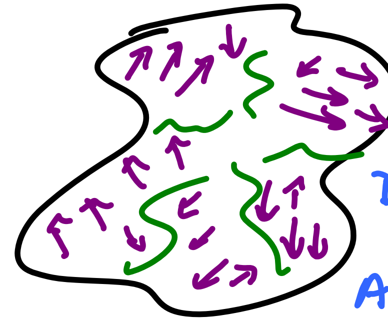


Induced Dipoles

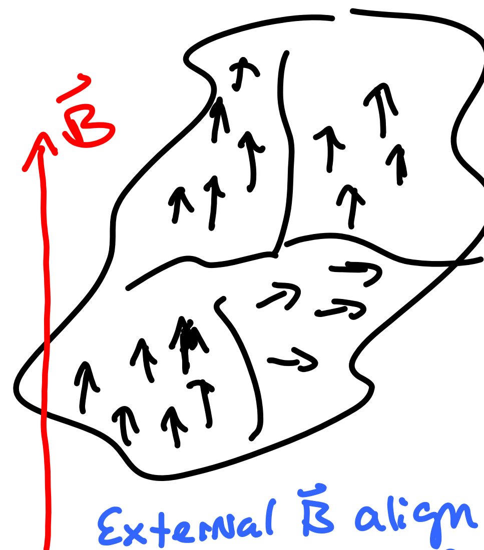
Oppose External $\vec{B}$

$\Rightarrow$ $\vec{B}$ weakened

Ferromagnetic



Strong Dipoles in Domains of local Alignment



External $\vec{B}$ aligns more Domains along $\vec{B}$

$\Rightarrow$ $\vec{B}$ increased

$$B = \mu_0 (1 + \chi_m) B_{free}$$

Magnetic Susceptibility

External B field

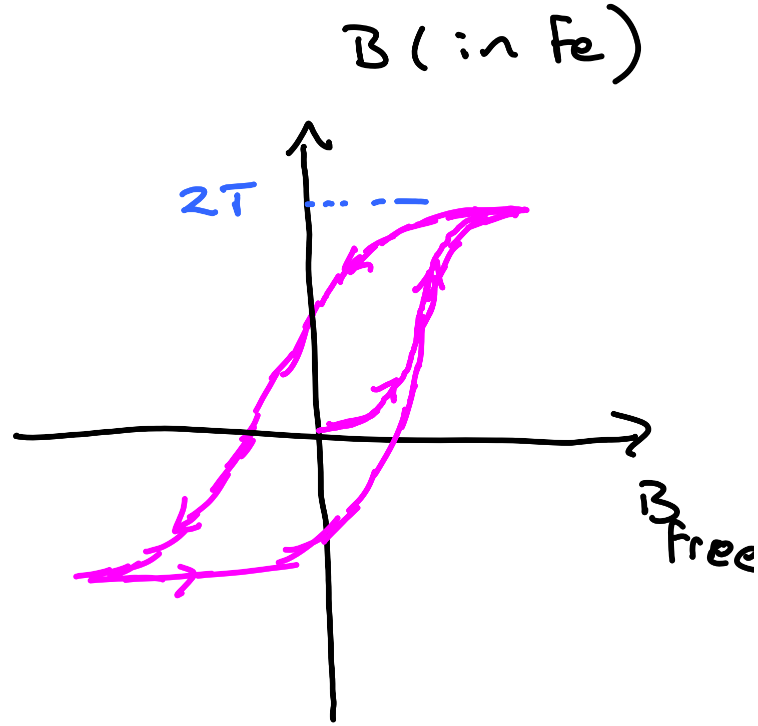
relative permeability

$\mu_0 K_m \sim$ permeability $\rightarrow \mu$

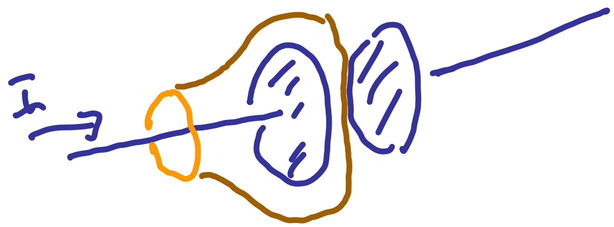
$K_m \gtrsim 1$ Paramagnetism

$\gg 1$ Ferromagnetism

< 1 Diamagnetism



Hysteresis loop



$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I + \underbrace{\mu_0 \epsilon_0 \frac{d}{dt}\oint_s \vec{E}\cdot d\vec{A}}_{\text{Maxwell's Displacement current}}$$

Integral form of Maxwell's equations

Gauss
$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

No magnetic monopole
$$\oint_S \vec{B}\cdot d\vec{A} = 0$$

Ampere
$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 \int_s \vec{J}\cdot d\vec{A} + \boxed{\mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}}$$

new term - "Maxwell's Displacement current"

Faraday
$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

Will now derive "differential" form ...

$$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

<u>Divergence of Vector Field</u> (cartesian coords)

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$ scalar

div $\vec{V}$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right)$$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} \cdots = 0$$

Gauss' Theorem
Green's Theorem
divergence Theorem

$$\left.\begin{array}{l}\text{Gauss' Theorem}\\ \text{Green's Theorem}\\ \text{divergence Theorem}\end{array}\right] \rightarrow \int_{vol} (\vec{\nabla}\cdot\vec{v})\,dv = \int_{surf} \vec{v}\cdot d\vec{a}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\int_{surf} \vec{E}\cdot d\vec{A} = \int_{V} (\vec{\nabla}\cdot\vec{E})\,dv = \frac{\int_{v} \rho\,dv}{\epsilon_0}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

$$\int_{S} \vec{B}\cdot dA = \int_{V} (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

Curl of Vector Field

$$\text{curl } \vec{V} = \vec{\nabla} \times \vec{V} \qquad \text{Vector}$$

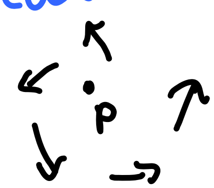
(Cartesian coords)

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

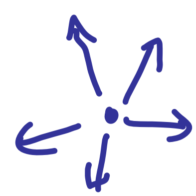
$$= \hat{i}\left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right] - \hat{j}\left[\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right] + \hat{k}\left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]$$

degree of circulation of field

large curl



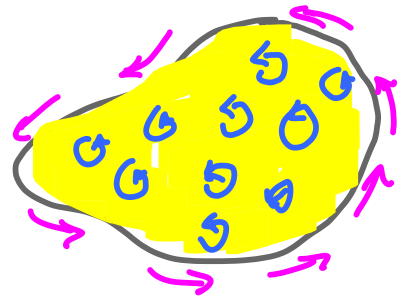
0 curl



0 curl

Stoke's theorem

$$\int_c \vec{V}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{V})\cdot d\vec{A}$$



$$\int_c \vec{E}\cdot d\vec{l} = -\frac{d\Phi_M}{dt} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

$$\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

Δ to ∂ because $\vec{B} = \vec{B}(x,y,z)$ + I've been lax

$$\underbrace{\oint_C \vec{B}\cdot d\vec{\ell}} = \mu_0 \underbrace{\int_S \vec{j}\cdot d\vec{A}}_{I_{encl}} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{A}$$

Stokes

$$\int_S (\vec{\nabla}\times\vec{B})\cdot d\vec{A} = \mu_0 \int_S \vec{j}\cdot d\vec{A} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\vec{E}}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

Maxwell's equations
in
differential form

Region with no current

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

Take curl of Both sides

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\epsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$$

Magic happens

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \vec{\nabla}(\cancelto{0}{\vec{\nabla}\cdot\vec{B}}) - \underbrace{\vec{\nabla}\cdot(\vec{\nabla}\vec{B})}_{\text{Laplacian of B}}$$

$$\nabla^2\vec{B}$$

Laplacian of Scalar Field T

$$\nabla^2 T = \vec{\nabla} \cdot (\vec{\nabla} T) =$$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$$

$$\boxed{\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}}$$

Laplacian of Vector field $\vec{V}$

$$\nabla^2 \vec{V} = (\nabla^2 V_x)\hat{i} + (\nabla^2 V_y)\hat{j} + (\nabla^2 V_z)\hat{k}$$