Physics 142 – November 16, 2010

- Presentations ...

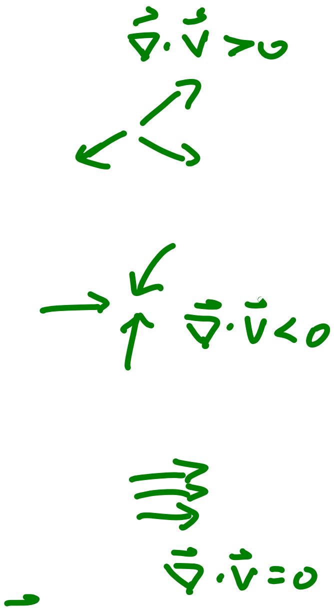
Last Time

Divergence of vector field $\vec{V}$

$$\text{div}\,\vec{V} \equiv \vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

recall

$$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

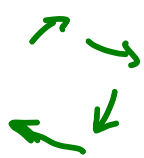


measures divergence or convergence of $\vec{V}$
extent to which there is a
sink or source of $\vec{V}$

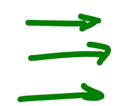
Curl of Vector field $\vec{V}$

$$\text{curl}\,\vec{V} = \vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

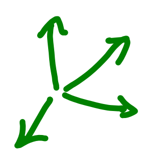
measures degree of circulation of field



$\vec{\nabla}\times\vec{V}$ not zero



$\vec{\nabla}\times\vec{V} = 0$



$\vec{\nabla}\times\vec{V} = 0$

Stokes Theorem

$$\oint \vec{V} \cdot d\vec{l} = \oint_S (\vec{\nabla} \times \vec{V}) \cdot d\vec{A}$$

Green's Theorem
Gauss' Theorem
divergence Theorem

$$\int_{vol} (\vec{\nabla} \cdot \vec{V})\, dv = \int_{surf} \vec{V} \cdot d\vec{A}$$

$$\oint_s \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \longrightarrow \int_v (\vec{\nabla}\cdot\vec{E})\,dv = \int_v \rho\, dv$$

$$\hookrightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho}$$

$$\oint_s \vec{B}\cdot d\vec{A} = 0 \longrightarrow \int_v (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

$$\hookrightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{d}{dt}\int_s \vec{E}\cdot d\vec{A}$$

$$\hookrightarrow \int_s (\vec{\nabla}\times\vec{B})\cdot dA = \mu_0 \int_s j\cdot dA + \mu_0\epsilon_0 \frac{d}{dt}\int_s \vec{E}\cdot d\vec{A}$$

$$\hookrightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{j} + \mu_0\epsilon_0 \frac{\partial E}{\partial t}}$$

$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

$$\hookrightarrow \int_s (\vec{\nabla}\times\vec{E})\cdot dA = -\frac{\partial}{\partial t}\int B\cdot dA \longrightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Differential form of Maxwell's eqns

Laplacian of Scalar Field T

$$\nabla^2 T = \vec{\nabla} \cdot (\vec{\nabla} T) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot$$

$$\left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

Laplacian of vector field $\vec{V}$

$$\nabla^2 \vec{V} = (\nabla^2 V_x)\hat{i} + (\nabla^2 V_y)\hat{j} + (\nabla^2 V_z)\hat{k}$$

region of $\underbrace{\text{NO current}}$

$$\vec{\nabla}\times\vec{B} = \cancel{\mu_0\vec{j}} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

0

curl of Both sides

$$\underbrace{\vec{\nabla}\times\vec{\nabla}\times\vec{B}} = \mu_0\epsilon_0\frac{\partial}{\partial t}\vec{\nabla}\times\vec{E}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} \equiv \vec{\nabla}\cancel{(\vec{\nabla}\cdot\vec{B})} - \underbrace{\vec{\nabla}\cdot(\vec{\nabla}\vec{B})}_{\nabla^2\vec{B}}$$

0

$$\nabla^2\vec{B} = \mu_0\epsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t} = -\mu_0\epsilon_0\frac{\partial}{\partial t}\frac{\partial\vec{B}}{\partial t}$$

$$\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 B_x = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t}$$

$$\nabla^2 B_y = -\mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t}$$

$$\nabla^2 B_z = -\mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}$$

Similarly $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

$$\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \qquad \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

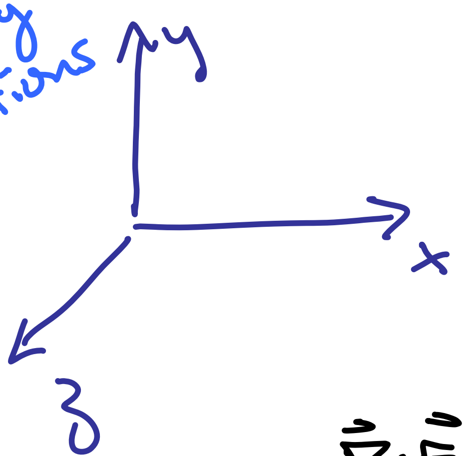
1 d wave propagating along x

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Wave equations in E, B w/ velocity of propagation

$$\sqrt{\frac{1}{\mu_0 \epsilon_0}} = c$$

Boundary conditions



$$\vec{E} = \vec{E}(x,t)$$

No charge $\quad \rho = 0$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\frac{\partial E_x}{\partial x} + \underbrace{\cancel{\frac{\partial E_y}{\partial y}}}_{0} + \underbrace{\cancel{\frac{\partial E_z}{\partial z}}}_{0} = 0$$

So, E_x is constant for all x $\rightarrow$ $E_x = 0$

E_y or E_z might be nonzero

- E is Transverse to direction of propagation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Boundary condition

$$\vec{E} = E(x,t)\,\hat{j}$$

"Polarization"

$$\cancel{\frac{\partial E_z}{\partial y}}^{\,0} - \cancel{\frac{\partial E_y}{\partial z}}^{\,0} = -\frac{\partial B_x}{\partial t}$$

$$-\left(\cancel{\frac{\partial E_z}{\partial x}}^{\,0} - \cancel{\frac{\partial E_x}{\partial z}}^{\,0}\right) = -\frac{\partial B_y}{\partial t}$$

$$\left(\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}}^{\,0}\right) = -\frac{\partial B_z}{\partial t}$$

Time Dependent $\vec{B}$

and

$\perp$ to $\vec{E}$

B_x constant in time

B_y constant in time

$\vec{E}, \vec{B}$ are TRANSVERSE

and mutually $\perp$

wave number

frequency

$$E_y(x,t) = E_{oy}\cos(\underbrace{k}_{\frac{2\pi}{\lambda}}x - \underbrace{\omega}_{\frac{2\pi}{T}}t + \varphi)$$

λ ← wavelength

T ← period

phase angle

initial condition

set to 0

$$B_z = -\int \frac{\partial E_y}{\partial x}\, dt$$

$$B_z = \int k E_{oy} \sin(kx - \omega t)\, dt$$

$$B_z = \frac{k}{\omega} E_{oy} \cos(kx - \omega t)$$

$$\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi/T} = \frac{T}{\lambda} = \frac{1}{c}$$

$$B_z = \frac{1}{c} E_y$$

E, B are coupled, time dependent

Mutually $\perp$

$|\vec{E}| = c\,|\vec{B}|$

in phase (time + space)