Physics 142 – Fall 2010 – Solutions to Problem Set 1

Problem 1 (Ohanian 22-2)

$$|F| = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = 8.99 \times 10^9 \times \frac{1.60 \times 10^{-19} \times 1.60 \times 10^{-19}}{(2.82 \times 10^{-10})^2}\ \text{N}$$

$$\underline{= 2.89 \times 10^{-9}\ \text{N}}$$

Problem 2 (Ohanian 22-3)

$$|F| = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{1.60 \times 10^{-19} \times 1.60 \times 10^{-19}}{(2 \times 10^{-15})^2}\ \text{N} \ \underline{= 58\ \text{N}}$$

$$a = \frac{F}{m} = \frac{58\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = 3.4 \times 10^{28}\ \text{m/s}^2$$

Problem 3 (Ohanian 22-13)

Proton has charge 1.60×10^{-19} C. 1 mole has 6.02×10^{23} particles. Therefore, Faraday's constant is $1.60 \times 10^{-19} \times 6.02 \times 10^{23}$ C $\underline{= 96300\ \text{C}}$

Problem 4 (Ohanian 22-14)

1 electron has 1.6×10^{-19} C. Therefore

$$1.5\ \text{C/s} = \left(\frac{1.5\text{C}}{1.6 \times 10^{-19}\ \text{C/electron}}\right)\Big/\text{s} \ \underline{= 9.4 \times 10^{18}\ \text{electron/s}}$$

Problem 5 (Ohanian 22-17)

Number of moles in copper penny = 2.7 g/63.5 g mol = 0.0425 mol.
Number of electrons is

$$0.0425 \text{ mol} \times 6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}} \times 29 \frac{\text{electrons}}{\text{atom}} = 7.42 \times 10^{23} \text{ electrons}$$

Total charge is 7.42×10^{23} electrons $\times (-1.60 \times 10^{-19})$C/electron
$= -1.19 \times 10^5$ C.

There will be an equal positive charge of $+1.19 \times 10^5$ C due to protons.
Therefore,

$$|\mathbf{F}| = \left|\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\right| = 9 \times 10^9 \times \frac{1.19 \times 10^5 \times 1.19 \times 10^5}{(2.0)^2} \underline{= 3.2 \times 10^{19} \text{ N}}$$

Addendum to problem 5

$$F_{em} = 3.18 \times 10^{19} \text{ N}$$

to lift a carrier, $F_{up} = mg$, at least

$$F_{up} = \left(100{,}000 \text{ tons} \times \frac{\text{kg}}{1.1 \times 10^{-3} \text{ ton}}\right)(9.8 \text{ m/s}^2)$$

$$= 8.9 \times 10^8 \text{ N}$$

$$\# = \frac{F_{em}}{F_{up}} = \boxed{3.6 \times 10^{10} \text{ carriers}}$$

Problem 6 (Ohanian 22-27)

Let Q be charge on Earth. Then charge on Moon will be
$\frac{1.74}{6.38} Q = 0.273\ Q.$

Electric force $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{0.273\ Q^2}{r^2}$

Gravitational force $F_g = GMm/r^2$. We want $F_g = F_e$; therefore

$$\frac{1}{4\pi\varepsilon_0}\frac{0.273\ Q^2}{r^2} = \frac{GMm}{r^2}, \text{ or } Q^2 = (GMm)(4\pi\varepsilon_0/0.273)$$

$$Q = \sqrt{\frac{6.67\times10^{-11}\times5.98\times10^{24}\times7.35\times10^{22}}{9\times10^{9}\times0.273}}\ \text{C} = 1.09\times10^{14}\ \text{C}$$

At 1.6×10^{-19} C per electron, this means $\frac{1.09\times10^{14}\ \text{C}}{1.6\times10^{-19}\ \text{C/electron}}$

$= 6.8\times10^{32}$ electrons on Earth

and $(0.273\ Q) = 0.273(6.8\times10^{32}) = 1.9\times10^{32}$ electrons on Moon

... 1.60×10^{-19} C.

Problem 7 (Ohanian 22-53)

The following are impossible:

$p + p \rightarrow n + n + \pi^+ \quad (e + e \rightarrow 0 + 0 + e)$

$p + p \rightarrow n + p + \pi^0 \quad (e + e \rightarrow 0 + e + 0)$

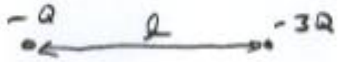
$p + p \rightarrow n + p + \pi^0 + \pi^- \quad (e + e \rightarrow 0 + e + 0 + (-e))$

In all cases here, charge is not conserved.

Problem 8 … no solution to give

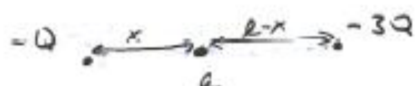
Problem 9

$\boxed{a}$



What is charge and placement of other charge?

Symmetry tells us that charge must be on line with other two. If we put a negative charge to left of both, that could stabilize $-Q$, but definitely not $-3Q$. If we use a positive charge there, would stabilize $-3Q$ but not $-Q$. Similarly with putting a charge to right of both. So let's put our charge in between, a distance x from $-Q$.



Call right positive.

Net force on $-Q$ must be zero.

$$\Sigma \vec{F}_{-Q} = \vec{F}_{\text{from } q} + \vec{F}_{\text{from } -3Q} = 0$$

$$\vec{F}_{\text{from } q} = -\frac{k(-Q)(q)}{x^2}$$

b/c $\hat{r}$ points to left here for both.

$$\vec{F}_{\text{from } -3Q} = -\frac{k(-Q)(-3Q)}{\ell^2}$$

$$\Sigma \vec{F}_{-Q} = \frac{kQq}{x^2} - \frac{3kQ^2}{\ell^2} = 0$$

$$\frac{\cancel{kQ}q}{x^2} = \frac{3\cancel{kQ}Q}{\ell^2} \rightarrow \ell^2 q = 3Qx^2$$

This equation has both our unknowns, but we need another!

$$\Sigma \vec{F}_{-3Q} = \vec{F}_{\text{from } q} + \vec{F}_{\text{from } -Q} = 0$$

$$\vec{F}_{\text{from } q} = \frac{k(-3Q)q}{(\ell - x)^2} \quad (\hat{r} \text{ points right here})$$

$$\vec{F}_{\text{from } -Q} = \frac{k(-3Q)(-Q)}{\ell^2} \quad (\text{here too})$$

$$\Sigma \vec{F}_{-3Q} = -\frac{3kQq}{(\ell - x)^2} + \frac{3kQ^2}{\ell^2} = 0$$

$$\frac{\cancel{3k}\cancel{Q}q}{(\ell - x)^2} = \frac{\cancel{3k}Q^{\cancel{2}}}{\ell^2} \rightarrow q\ell^2 = (\ell - x)^2 Q$$

Another equation! So we have:

$$\ell^2 q = 3Qx^2 \qquad q\ell^2 = (\ell - x)^2 Q$$

so,

$$3\cancel{Q}x^2 = (\ell - x)^2 \cancel{Q}$$
$$= \ell^2 - 2\ell x + x^2$$

$$2x^2 + 2\ell x - \ell^2 = 0$$

$$x = \frac{-2\ell \pm \sqrt{(2\ell)^2 + 4 \cdot 2 \cdot \ell^2}}{2 \cdot 2}$$

$$= \frac{-2\ell \pm \sqrt{12\ell^2}}{4}$$

$$x = -\frac{1}{2}\ell \pm \frac{1}{2}\sqrt{3}\,\ell$$

We should choose +, b/c otherwise we'll get a negative x, which would correspond to putting our charge on the left side of both.

$$x = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right) l$$

$$= \left(\frac{\sqrt{3}-1}{2}\right) l \approx 0.37\, l$$

take one of earlier eqns:

$$l^2 q = 3Qx^2$$

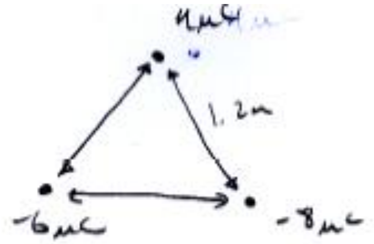
$$q = \frac{3Qx^2}{l^2} = \frac{3Q(0.37\cancel{l})^2}{\cancel{l}^2}$$

$q = 0.4\,Q$ positive, which makes sense!

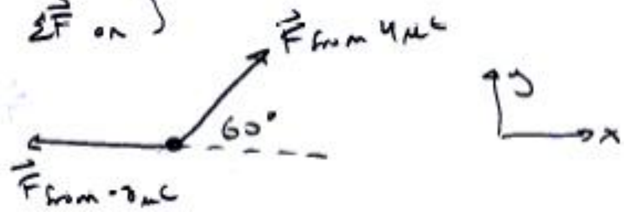
$$\boxed{\text{so placement is } 0.37\,l \text{ from } -Q, \quad \text{charge is } 0.4Q}$$

Problem 10 –

10



Calc $\Sigma\vec{F}$ on



$$\Sigma\vec{F} = \vec{F}_{\text{from } 4\mu C} + \vec{F}_{\text{from } -8\mu C}$$

$$|F_{\text{from } 4\mu C}| = \frac{k(6\times10^{-6}C)(4\times10^{-6}C)}{(1.2m)^2} = 0.15\,N$$

$$|F_{\text{from } -8\mu C}| = \frac{k(6\times10^{-6}C)(8\times10^{-6}C)}{(1.2m)^2} = 0.30\,N$$

$$\vec{F}_{\text{from } -8\mu C} = -0.30\,N\,\hat{x}$$

$$\vec{F}_{\text{from } 4\mu C} = 0.15\,N \quad 60° \text{ N of E}$$

$$= (0.15\,N)\cos 60\,\hat{x} + (0.15\,N)\sin 60\,\hat{y}$$

$$= (0.08\,N)\hat{x} + (0.13\,N)\hat{y}$$

$$\Sigma\vec{F} = -0.30\,N\,\hat{x} + 0.08\,N\,\hat{x} + 0.13\,N\,\hat{y}$$

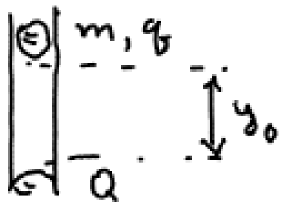
$$\boxed{\Sigma\vec{F} = (-0.23\,N)\hat{x} + (0.13\,N)\hat{y}}$$

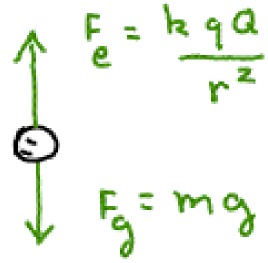
or $[(-0.23N)^2 + (0.13N)^2]^{1/2} = \boxed{0.26\,N}$

$\theta = \tan^{-1}\left(\frac{0.13}{0.23}\right) = \boxed{30° \text{ N of W}}$

Problem 11 –

Problem set 1 #11 soln





at equilibrium $F_e = F_g$ and $r = y_0$

$$\frac{kqQ}{r^2} = mg \Rightarrow \boxed{y_0 = \sqrt{\frac{kqQ}{mg}}}$$

Think about Simple Harmonic Motion

$$F = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

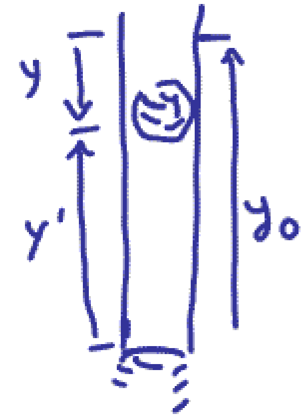
differential eqn of this form leads to SHM with frequency ω.

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow \text{SHM w/}$$

$$\omega^2 = \frac{k}{m}$$

Back to our problem

make small displacement y
from equilibrium position



$$-y = y_0 - y'$$
$$y = y' - y_0$$

I $\quad F_{NET} = \dfrac{kqQ}{(y')^2} - mg = \dfrac{kqQ}{(y_0+y)^2} - mg$

$$= \frac{kqQ}{y_0^2\left(1+\frac{y}{y_0}\right)^2} - mg$$

but Taylor's series $\quad (1+x)^{-2} = 1 - 2x + 3x^2 \ldots$

$$\frac{1}{\left(1+\frac{y}{y_0}\right)^2} = 1 - 2\frac{y}{y_0} + \text{higher order terms}$$

(neglect since $\frac{y}{y_0} \ll 1$)

$\Rightarrow$ SMALL oscillations

Substitute Binomial expansion into eqn I above

$$F_{NET} = \frac{kQq}{y_0^2}\left(1 - 2\frac{y}{y_0}\right) - mg = m\frac{d^2y}{dt^2}$$

$$y_0^2 = \frac{kQq}{mg}$$ Sub in

$$m\frac{d^2y}{dt^2} = \frac{\cancel{kQq}\,mg}{\cancel{kQq}}\left(\cancel{1} - 2\frac{y}{y_0}\right) - \cancel{mg}$$

$$\frac{d^2y}{dt^2} + \frac{2g}{y_0}y = 0$$

This is eqn for SHM with $\omega^2 = \frac{2g}{y_0}$