P142 University of Rochester NAME Soln key - SM
S. Manly Fall 2010

Exam 1 (October 7, 2010)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (9 pts, show your work):

Magnitude of the

*

Charge q experiences a repulsive force of 0.50 mN when placed at a distance of 25 cm from charge Q_1, and it experiences an attractive force of 0.25 mN when placed a distance of 75 cm from charge Q_2. The ratio Q_1/Q_2 is

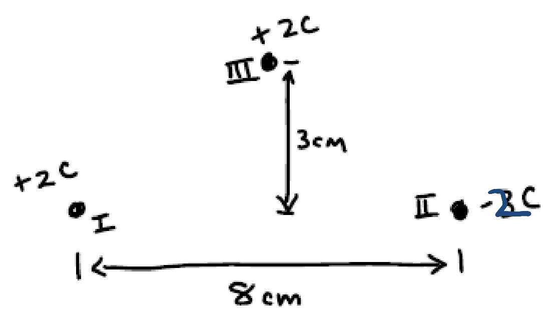
a) 2/9
b) 2/3
c) 9/2
d) 6
e) 3/2
f) 18

$$-0.0005 = \frac{kQ_1 q}{(.25)^2} \qquad 0.00025 = \frac{kQ_2 q}{(.75)^2}$$

$$-2 = \frac{Q_1/.25^2}{Q_2/.75^2} \qquad \frac{Q_1}{Q_2} =$$

Problem 2 (12 pts, show your work):

Consider the arrangement of charges shown in the sketch. Determine the electric field at the position of charge III.



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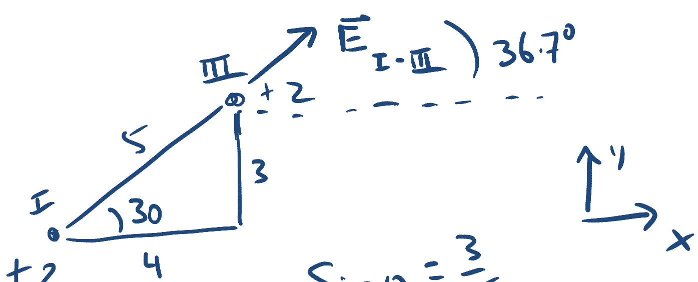
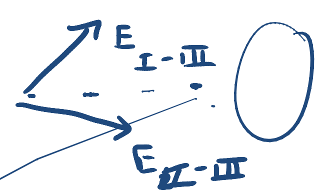


$$\sin\theta = \frac{3}{5}$$

$$\theta = 36.7^\circ$$

E direction to right



Vertical components cancel

$$E_{y_{I-III}} = -E_{y_{II-III}}$$

$$E_x = (2)\frac{kQ}{r^2}\cos\theta$$

$$E_{x_{I-III}} = E_{x_{II-III}}$$

$$E_x = (2)\frac{9\times10^9(2)}{(.05)^2}\cos\theta = 1.15\times10^{13}\ N/C$$

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1)	/9
2)	/12
3)	/9
4)	/17
5)	/15
6)	/18
7)	/20
tot	/100

Problem 3 (9 pts, show your work):

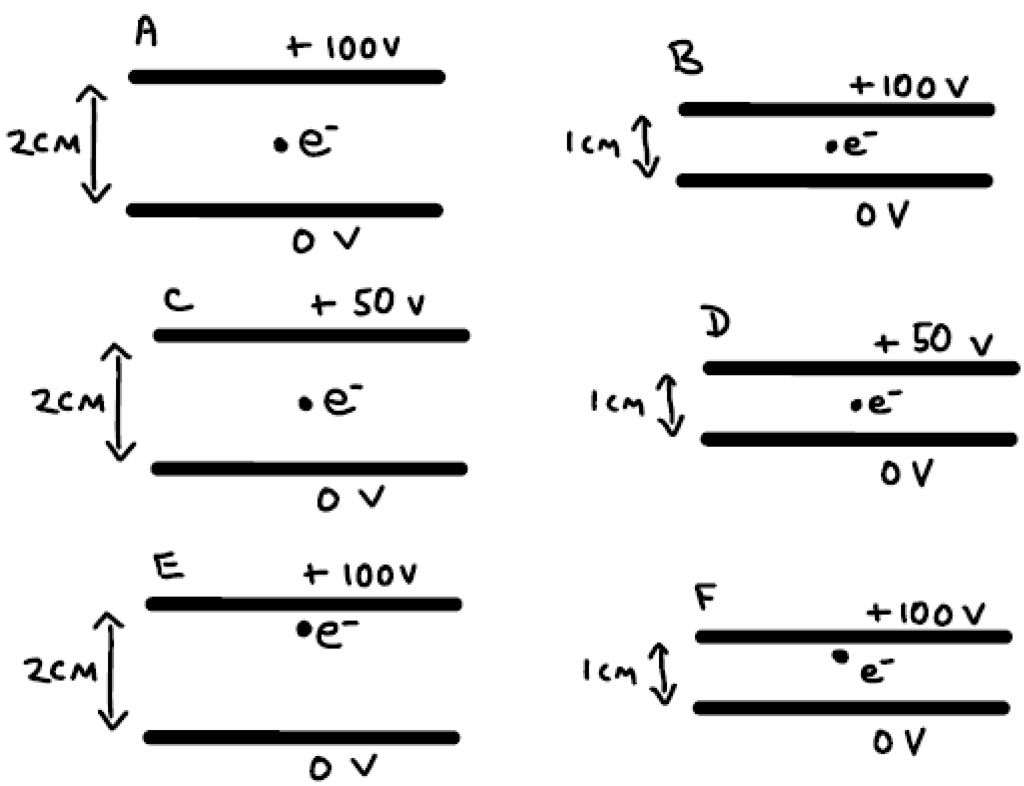
The net electrical flux through a spherical Gaussian surface of radius 1.0 cm is 24 Nm^2/C. If the radius of the surface is doubled to 2.0 cm, the electric flux will become

a) 96 Nm^2/C
b) 24 Nm^2/C
c) 12 Nm^2/C
d) 6 Nm^2/C
e) 3 Nm^2/C

Radius may be larger. But q_{encl} has not changed. Total flux through surface will be unchanged.

Problem 4 (17 pts, ~~justify your answer)~~:

Consider the six situations below labeled A-F. In each situation, two charged plates with the voltages shown are separated by a specified distance. Rank these situations from greatest to least according to the magnitude of the force felt by the electron. If the force is equal in two different situations, specify that with a "=" symbol. (Your answer should be in a form that looks something like C > E = A > B > D > F.)



$$\boxed{B=F > D=A=E > C}$$

$V = Ed$

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Problem 5 (15 pts, justify your answer):

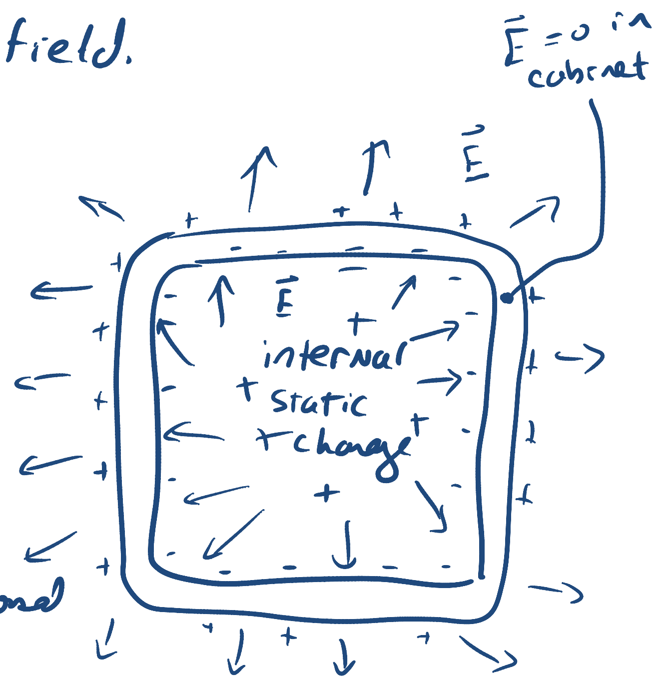
In a few years you take a job as a consulting physicist for Bob's Particle Accelerator and Washing Machine Design Company. After you've been on the job for a few weeks, you attend a meeting where some of the engineers are fretting about static electric charges building up on the inside of the new model of washing machines. The big boss Bob is in the meeting and he says, "Where did you bozos go to school? Huh? Don't you know anything? We don't need to worry about that big static charge in the middle of the washing machine. We're surrounding that charge with a big conducting metal case, right? You guys have heard about electrostatic shielding haven't you?" Then Bob looks at you and says, "Explain it to these bozos."

Answers may vary

Briefly defend or criticize Bob's argument, assuming you are independently wealthy and need not fear for your job.

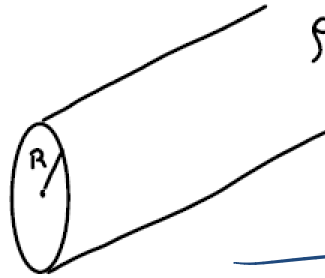
Bob, I know you're the big boss man and all, but you really don't know much physics. Electrostatic Shielding is where the charge in a conductor will rearrange itself in response to an electric field so that the electric field inside the conductor is zero. This can be used to shield the inside of an enclosed region from an external electric field.

In the case under discussion, the internal $\vec{E}$ will cause a separation of charge in the conducting metal case such that the internal field does not penetrate into the case. But, that will leave induced charge on the outside of the case that will give rise to an external $\vec{E}$. A gaussian surface drawn outside the case will have a net $Q_{enclosed}$ and a non-zero external field.



Problem 6 (18 pts, justify your answer):

Consider a very long straight cable of radius R carrying a total charge Q per unit length L distributed as $\rho(r)=Br^2$, where B is a constant.



$\rho(r) = Br^2$

Charge Q per unit length L

λ

*

a) Determine B in terms of the variables R, Q and L.

$Q = \int_{\substack{\text{Vol} \\ \text{of length } L}} \rho \, dv$

$Q = \int_0^R Br^2 \, 2\pi r L \, dr$

$Q = B 2\pi L \int_0^R r^3 dr$

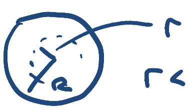
$Q = B 2\pi L \frac{R^4}{4}$

$\boxed{B = \frac{Q\,2}{\pi L R^4}}$

check units

$B \to \frac{C}{m^5}$ $\quad C/m^5$ ✓

b) Determine the electric field for r<R.



$r<R$

$\oint \vec{E} \cdot d\vec{a} = Q_{encl}/\epsilon_0$

Choose cylindrical gaussian surface with $r<R$

$\vec{E}$ radial so encaps have $\vec{E} \perp d\vec{a}$ and do not contribute.

$|\vec{E}| 2\pi r L = \int_0^r \rho \, dv \frac{1}{\epsilon_0}$

$|\vec{E}| 2\pi r L = \frac{1}{\epsilon_0} \int_0^r Br^2 2\pi r L \, dr = \frac{B 2\pi L}{\epsilon_0} \frac{r^4}{4}$

$\boxed{\vec{E} = \frac{B r^3}{\epsilon_0 4} \hat{r} \quad \text{or} \quad \frac{\lambda r^3}{\epsilon_0 2\pi R^4} \hat{r}}$

*

c) Determine the electric field for r>R.

Choose gaussian surface with $r>R$

$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$

encaps do not contribute due to $\vec{E} \perp d\vec{a}$ for those surfaces

$|\vec{E}| 2\pi r L = Q/\epsilon_0$

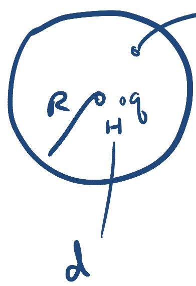
$\boxed{\vec{E} = \frac{Q}{L\, 2\pi r \epsilon_0} \hat{r} = \frac{\lambda}{2\pi r \epsilon_0} \hat{r}}$

Same as field of infinite line charge with λ.

Problem 7 (20 pts, justify your answer):

Before the success of the Bohr model in explaining atomic spectra, the plum-pudding model of the atom was advocated by some physicists. In this model, the atom consists of a diffuse cloud of positive charge in which the electrons are embedded like bits of fruit in pudding. Light was emitted or absorbed by the electrons as they oscillated in the positive medium. The frequency of the light corresponded to the frequency of the oscillation of the electron. (We've not yet seen it in our course, but light is emitted by accelerating electric charge; so, this picture of the atom has merit).

Consider a positive charge +4e, distributed uniformly in a sphere ✓of radius R. Suppose a single electron with mass m and charge e is placed in the center of this spherical distribution of charge. Assume that the electron is displaced by a very small distance, d, from the center and released. Show that the electron undergoes simple harmonic motion and calculate the frequency of oscillation for that electron. *



$$\rho = \frac{4e}{\frac{4}{3}\pi R^3} = \frac{3e}{\pi R^3}$$

SHM

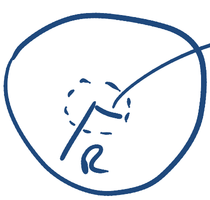
$$F = m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\omega^2 = k/m$$

What is $\vec{E}$ at position r=d from center?

Draw gaussian surface $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$



$$|\vec{E}|4\pi r^2 = \frac{\rho}{\epsilon_0}\frac{4}{3}\pi r^3$$

$$E_{at\,d} = \frac{\rho r}{3\epsilon_0} = \frac{er}{\pi R^3 \epsilon_0}$$

recall

$$F = qE$$

So, Force on e^- radially in and

$$F = m\frac{d^2r}{dt^2} = -\frac{e^2 r}{\pi R^3 \epsilon_0}$$

$$\frac{d^2r}{dt^2} + \left(\frac{e^2}{m\pi R^3\epsilon_0}\right) r = 0$$

SHM form with

$$\omega^2 = \frac{e^2}{m\pi R^3 \epsilon_0}$$

Do dimensional analysis of ω^2 answer

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

$$\epsilon_0 \rightarrow \frac{C^2}{N\,m^2}$$

$$\frac{C^2 N m^2}{kg\, m^3 C^2} = \frac{C^2 kg\, m\, m^2}{s^2 kg\, m^3 C^2} = \frac{1}{s^2}$$

$$N = kg\, m/s^2$$

correct for frequency ✓

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Exam 1 Formulas

$$\vec{F} = q\vec{E}$$

$$\vec{F} = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$\phi_E = \oint \vec{E}\cdot d\vec{A}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\vec{E} = \int_{vol} \frac{k\,dQ}{r^2}\hat{r}$$

$$V = \text{work}/\text{charge}$$

$$V_{\substack{\text{point}\\ \text{charge}}} = \frac{kQ}{r}$$

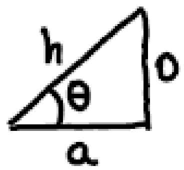
$$V = \int_{vol} \frac{k\,dQ}{r}$$

$$E_s = -dV/ds$$

$|e| = 1.6\times10^{-19}$ coulombs

$$k = 8.99\times10^{9}\ \frac{N m^2}{C^2}$$

$$\epsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$$



$$\sin\theta = \frac{o}{h} \quad \cos\theta = \frac{a}{h}$$

$$\tan\theta = o/a$$

Sphere: $A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$

cylinder: $A = 2\pi r L + 2\pi r^2$

$V = \pi r^2 L$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

$$\int \frac{du}{u} = \ln|u|$$

$$\int e^u du = e^u$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = \sqrt{x^2+a^2}$$