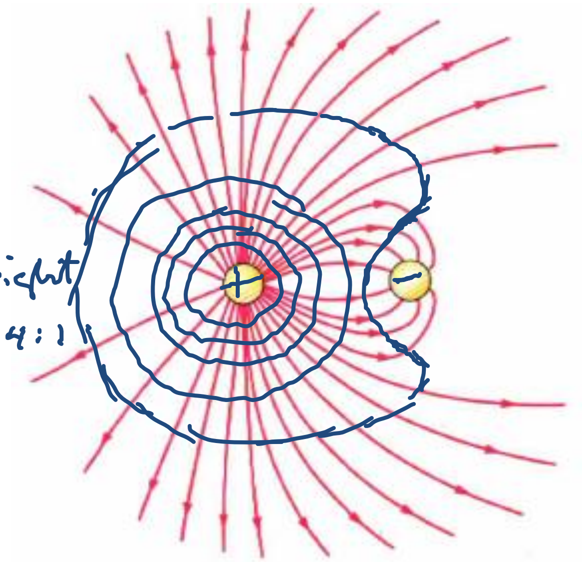
P142 University of Rochester NAME Solution key – SM
S. Manly Fall 2014

Exam 1 (October 7, 2014)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (12 pts, show your work):

Consider the configuration of two charges shown in the figure along with a representation of the electric field lines of the system.



a) Indicate on the figure the signs of each of the charges.

b) What are the relative magnitudes of the two charges?

32 lines for + charge on left
8 lines for – charge on Right

$Q_{LEFT} : Q_{right} = 32:8 = 4:1$

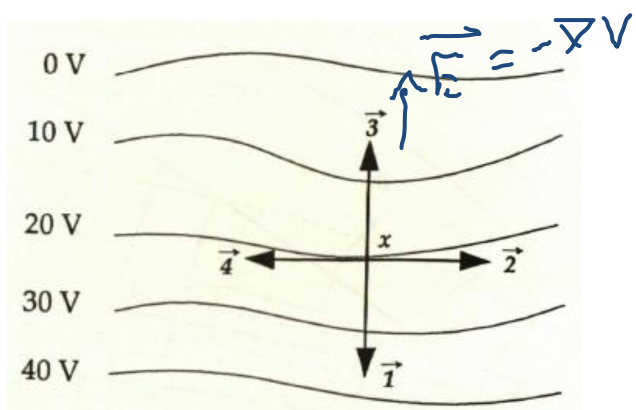
c) Sketch on the figure the equipotential lines for this system of charges.

See sketch

Problem 2 (8 pts, show your work):

The vector that best describes the direction of the electric field at the point x on the 20 V equipotential line is

a) vector 1
b) vector 2
c) vector 3
d) vector 4
e) none of these is correct

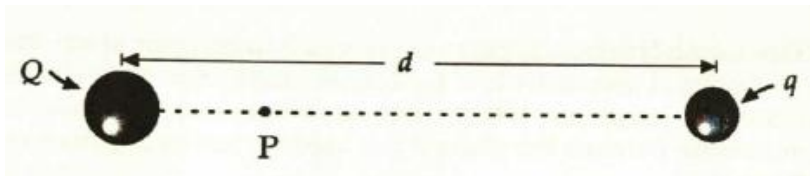


gradient is in this direction

Problem 3 (8 pts, show your work):

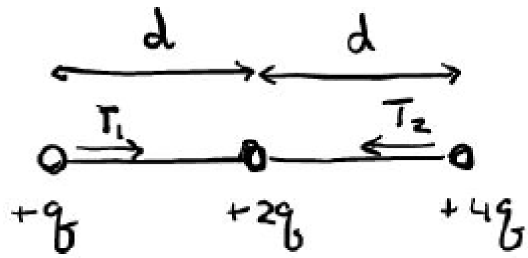
Charges Q and q (Q≠q), separated by a distance d, produce a potential $V_P=0$ at point P. This means that

a) no force is acting on a test charge placed at point P.
b) Q and q must have the same sign.
c) the electric field must be zero at point P.
d) ~~the net work in bringing Q to distance d from q is zero.~~
e) the net work needed to bring a charge from infinity to point P is zero.



Problem 4 (15 pts, show your work):

Suppose three charges are tied together (collinearly) by two massless strings. Determine the tension in the strings in terms of k, q and d.



$$\Sigma F_{left} = 0 = T_1 - \frac{k\,2q^2}{d^2} - \frac{k\,4q^2}{4d^2}$$

$$0 = T_1 - \frac{k\,3q^2}{d^2} \qquad \boxed{T_1 = \frac{k\,3q^2}{d^2}}$$

$$\Sigma F_{Right} \quad 0 = -T_2 + \frac{k\,4q^2}{4d^2} + \frac{k\,8q^2}{d^2}$$

$$\boxed{T_2 = 9\frac{kq^2}{d^2}}$$

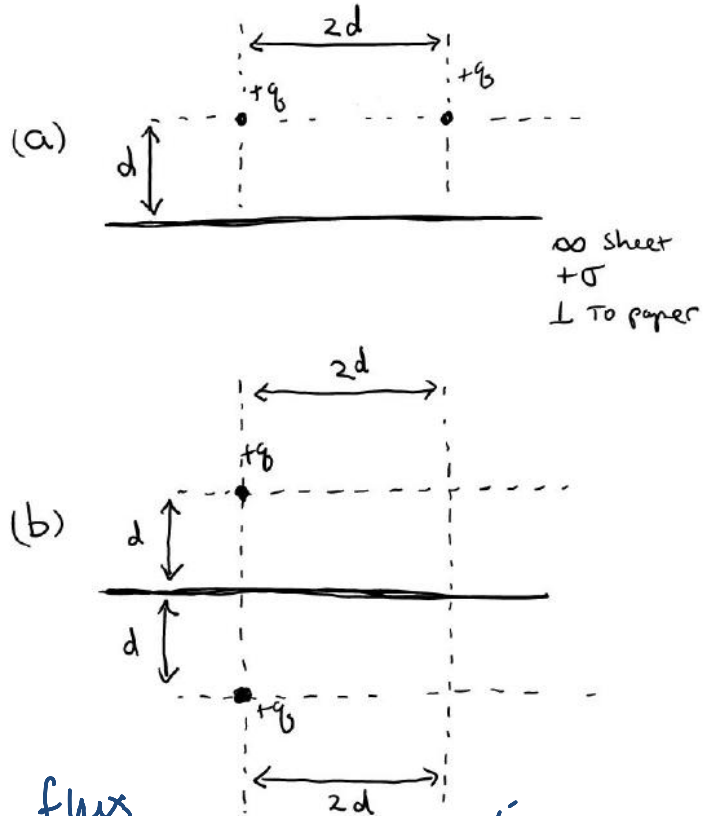
Check by looking at force on middle

$9 - 3 + 2 - 8$ ✓

$$\Sigma F_{middle} = T_2 - T_1 + \frac{k\,2q^2}{d^2} - \frac{k\,8q^2}{d^2} = \frac{9kq^2}{d^2} - \frac{k3q^2}{d^2} + \frac{k2q^2}{d^2} - \frac{k8q^2}{d^2}$$

Problem 5 (15 pts, show your work):

Figure (a) shows an infinite plane (perpendicular to the plane of the paper) with a constant positive charge per unit area, σ. In addition, as shown in the figure, two positive charges are located a distance d above the plane and separated by a distance of 2d. Figure (b) shows a similar configuration (same plane, same charges) except for one of the charges is now located a distance 2d from the other charge on the other side of the plane along a line that is perpendicular to the plane of charge.



(a) What is the total electric flux passing through the plane in configuration (a)?

by symmetry and def of flux

$$\phi = \frac{1}{2}\frac{q}{\epsilon_0} + \frac{1}{2}\frac{q}{\epsilon_0} = \frac{q}{\epsilon_0}$$

(b) What is the total electric flux passing through the plane in configuration (b)?

$$\phi = 0 = +\frac{q}{2\epsilon_0} - \frac{q}{2\epsilon_0}$$

flux from each charge goes opp directions thru surface

(c) What is the relative magnitude of the energy stored in the two configurations?

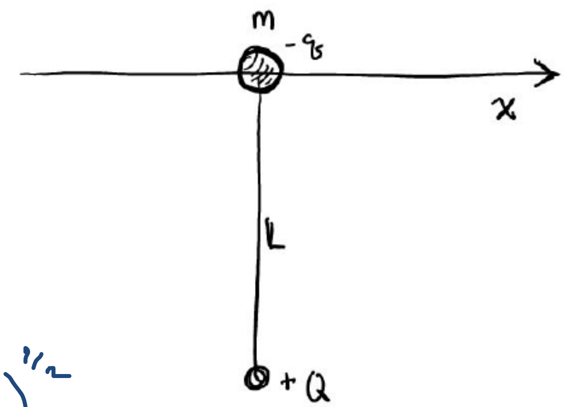
Energy stored is the same in the two configurations

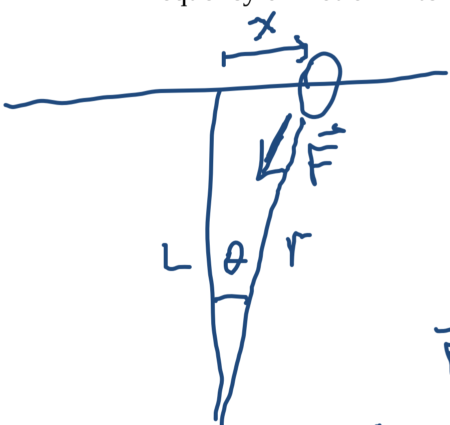
by symmetry . . .

charges are same distance from each other and same distance from the plane in both configurations

Problem 6 (21 pts, show your work):

A charge +Q is located on the y axis a distance L from the origin. A bead of mass m and charge –q is located at the origin and is free to slide frictionlessly along the x axis. Suppose the bead is displaced by a small amount x, where x<<L. When released, show that the bead executes simple harmonic motion and determine the frequency of motion in terms of L, Q, q and m.

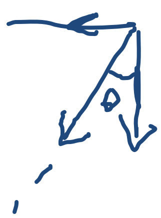




$$r = (x^2 + L^2)^{1/2}$$

$$r = L\left(\left(\frac{x}{L}\right)^2 + 1\right)^{1/2}$$

$\vec{F}\sin\theta$ is restoring force along x axis



$$m\frac{d^2x}{dt^2} = -\frac{kQq}{r^2}\sin\theta$$

For small x

$\sin\theta \approx \theta$

$x \cong L\theta$

$\theta \cong \frac{x}{L}$

$$m\frac{d^2x}{dt^2} = -\frac{kQq}{x^2+L^2}\,\frac{x}{L} = -\frac{kQq\,x}{L^3\left(1+\frac{x^2}{L^2}\right)}$$

$$(1+\alpha)^{-1} = 1 - \alpha + \alpha^2 \ldots$$

$$\left(1+\frac{x^2}{L^2}\right)^{-1} = 1 - \frac{x^2}{L^2} + \text{higher order terms}$$

$\frac{x^2}{L^2}$: let be zero since x/L small

higher order terms $\to 0$ because x/L small

1)	/12
2)	/8
3)	/8
4)	/15
5)	/15
6)	/21
7)	/21
tot	/100

$$m\frac{d^2x}{dt^2} = -\frac{kQq}{L^3}x \rightsquigarrow \boxed{\frac{d^2x}{dt^2} + \frac{kQq}{ML^3}x = 0}$$

SHM with $\omega^2 = \frac{kQq}{ML^3}$

Check units

$$\frac{kQq}{ML^3} = \frac{kQq}{L^2}\frac{1}{ML} = \frac{N}{kg\,m}$$

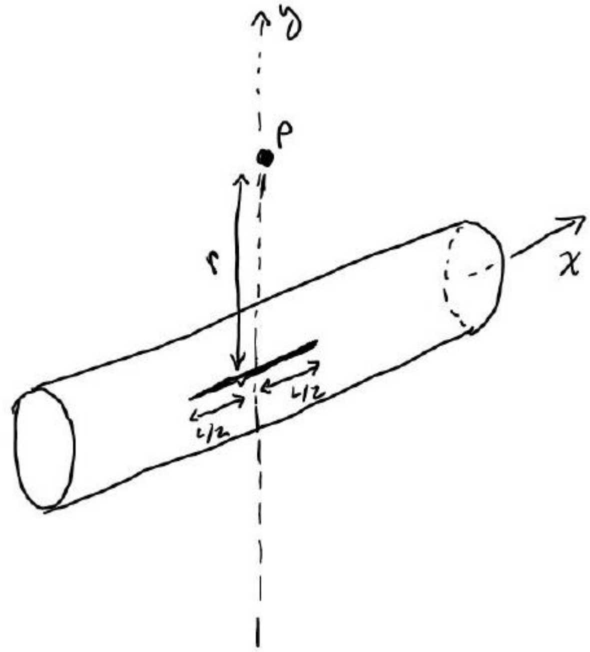
N is in units of kg m/s²

$$kg\frac{m}{s^2}\frac{1}{kg\,m} \rightsquigarrow \frac{1}{s^2} \checkmark$$

good unit of frequency

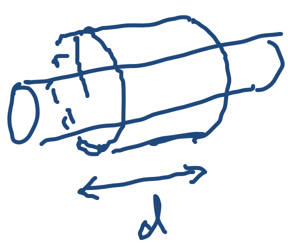
Problem 7 (21 pts, show your work):

Consider an infinitely long cylindrically symmetric charge distribution with an axis of symmetry that is the x-axis. This charge distribution has a constant positive charge per unit length λ and a radius R. Something very strange happened to this charge distribution on the way to the exam. A line of charge of length L along the x-axis, centered at the origin, disappeared. The amount of charge per unit length that disappears along this line is λ'. Given this distribution, sketched to the right, determine the net electric field at a point P at y=r above the midpoint of the missing charge line segment.



$\vec{E}_p$ = Superposition of $\vec{E}_{cylinder}$ and $\vec{E}$ of line segment w/ $-\lambda'$

use Gauss' Law for cylinder



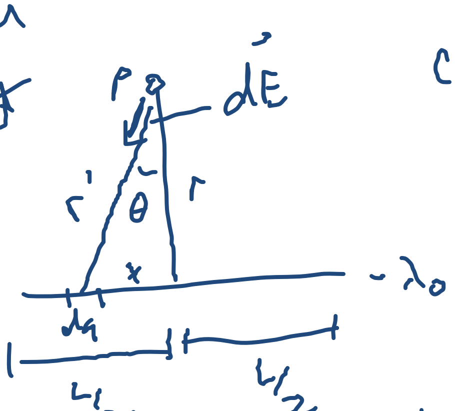
$$\oint E\cdot dA = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}|2\pi r d = \frac{\lambda d}{\epsilon_0}$$

for cylinder

$$\boxed{\vec{E} = \frac{\lambda}{2\pi r\epsilon_0}\hat{y} \text{ at } p}$$

Consider line segment



Contributing component of dE is along y

$$\vec{dE} \rightarrow dE\cos\theta = dE\frac{r}{r'} = dE\frac{r}{(x^2+r^2)^{1/2}}$$

$$dE = \frac{k\,dq}{(x^2+r^2)}$$

$$dE_y = -\frac{k\,r\,\lambda'\,dx}{(x^2+r^2)^{3/2}}$$

$$E_{y\,\text{line segment}} = -2\int_0^{L/2} k\lambda' \frac{r}{(x^2+r^2)^{3/2}}\,dx$$

integral table $\displaystyle\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$

$$E_{y\,\text{line segment}} = -2k\lambda' r \left.\frac{x}{r^2(x^2+r^2)^{1/2}}\right|_0^{L/2} = -2k\lambda' \left.\frac{x}{r(x^2+r^2)^{1/2}}\right|_0^{L/2}$$

$$E_{y\,\text{line segment}} = -\frac{2k\lambda' \frac{L}{2}}{r\left(\left(\frac{L}{2}\right)^2+r^2\right)^{1/2}} = -\frac{k\lambda' L}{r\left(r^2+\left(\frac{L}{2}\right)^2\right)^{1/2}}$$

Let $r \to$ big $\quad E_y \to -\dfrac{k\overbrace{\lambda' L}^{Q}}{r^2}$ ✓

So $$\boxed{\vec{E}_p = \hat{y}\left[\frac{\lambda}{2\pi r\epsilon_0} - \frac{k\lambda' L}{r\left(r^2+\left(\frac{L}{2}\right)^2\right)^{1/2}}\right]}$$

Exam 1 Formulas

$$\vec{F} = q\vec{E}$$

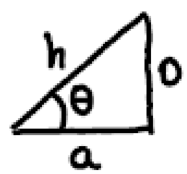
$$|e| = 1.6\times10^{-19} \text{ coulombs}$$

$$k = 8.99\times10^{9} \ \frac{Nm^2}{C^2}$$

$$\vec{F} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$$

$$\epsilon_0 = 8.85\times10^{-12} \ C^2/Nm^2$$

$$\phi_E = \oint \vec{E}\cdot d\vec{A}$$



$$\sin\theta = \frac{O}{h} \quad \cos\theta = \frac{a}{h}$$

$$\tan\theta = O/a$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\text{Sphere: } A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$$

$$\vec{E} = \int_{vol} \frac{k\,dQ}{r^2}\hat{r}$$

$$\text{cylinder: } A = 2\pi rL + 2\pi r^2$$

$$V = \pi r^2 L$$

$$V = \text{work}/\text{charge}$$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

$$V_{\text{point charge}} = \frac{kQ}{r}$$

$$\int \frac{du}{u} = \ln|u|$$

$$V = \int_{vol} \frac{k\,dQ}{r}$$

$$\int e^u du = e^u$$

$$E_s = -dV/ds$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = \sqrt{x^2+a^2}$$