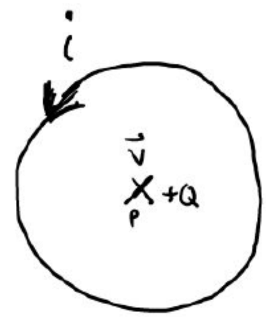
Exam 2 (November 6, 2014)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (15 pts, no need to show work but please use the correct hand!):

In each of the following specify the direction of the force on the positive charge Q. Specifically, write down the appropriate choice from the list: there is no force on Q; into the paper; out of the paper; to the left (in the plane of the paper); to the right (in the plane of the paper); toward the top (in the plane of the paper); toward the bottom (in the plane of the paper).

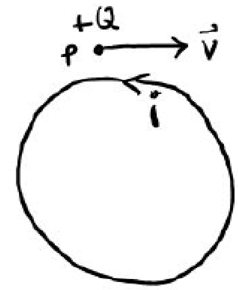
a) Point p is in center of positive current loop shown in plane of paper. Q moves into the paper.



$\vec{B}$ due to i at p is out
$\vec{v} \times \vec{B} = 0$

No force

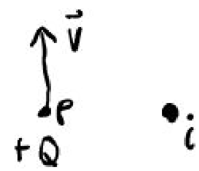
b) Point P lies outside of the positive current loop shown in the plane of the paper. Q moves to the right.



$\vec{B}$ at p in into paper
$\vec{v} \times \vec{B}$ is up

Force is up ↑$\vec{F}$

c) Current comes out of the paper toward you. Q moves upward in plane of the paper.



$\vec{B}$ at p due to i is down
$\vec{v} \times \vec{B} = 0$

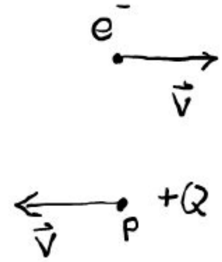
No force

d) Current comes out of the paper toward you. Q moves to the right.

$\vec{B}$ at p due to i is down
$\vec{v} \times \vec{B}$ is into paper

Force into paper ×$\vec{F}$

e) An electron moves to the right as shown while Q moves to the left.



$\vec{B}$ at p due to e^- is out of paper
$\vec{v} \times \vec{B}$ is up

↑$\vec{F}$

Force is up

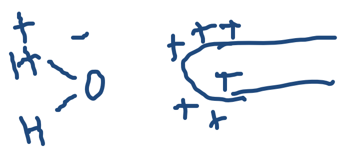
P142 University of Rochester NAME Solution key - SLM
S. Manly Fall 2014

Problem 2 (12 pts, show your work):

If you place an electrically charged rod near a thin stream of water, the stream is deflected. Please briefly explain why this deflection occurs and in what direction it occurs (relative to the position of the rod with respect to the stream).

The $\vec{E}$ of the rod will cause water molecules to be oriented with the opposite charge of the dipole toward the rod.

In this induced orientation the opposite charge is closer to the rod charge giving a net coulomb force toward the rod. So the thin water stream is deflected toward the rod.



Problem 3 (13 pts, show your work):

Bubba Joe belongs to the Planetary Patrol speed enforcement brigade. His job is to monitor the speed of interstellar spacecraft as they pass through the solar system. If he sees a ship that's going too fast, he levies a fine on the pilot and the ship owner. Also, having grown up in North Carolina, he levies a fine on any ship he sees that has New York plates. All in all, he's happy with his job. Bubba Joe sees a spacecraft passing the moon and measures the length of the craft to be 85 meters. From the markings, Bubba Joe identifies the ship as a Kiaford Starcruiser. When Bubba checks his handy dandy spaceship information manual, he sees that the Kiaford Starcruiser is manufactured with a length of 100 meters. From this information, Bubba Joe can determine the speed of the ship and decide whether or not to levy a speeding fine. How fast is the Kiaford Starcruiser going when it is seen by Bubba?

100 meters is the proper length

85 meters is what Bubba measures - Lorentz contracted

$$\Delta x_{Bubba} = \Delta x_{proper}/\gamma$$

$$85 = 100/\gamma$$

$$\gamma = 100/85 \simeq 1.18$$

$$1.18 = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\frac{1}{1.18} = 0.85 = \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$.85^2 = 1-\left(\frac{v}{c}\right)^2$$

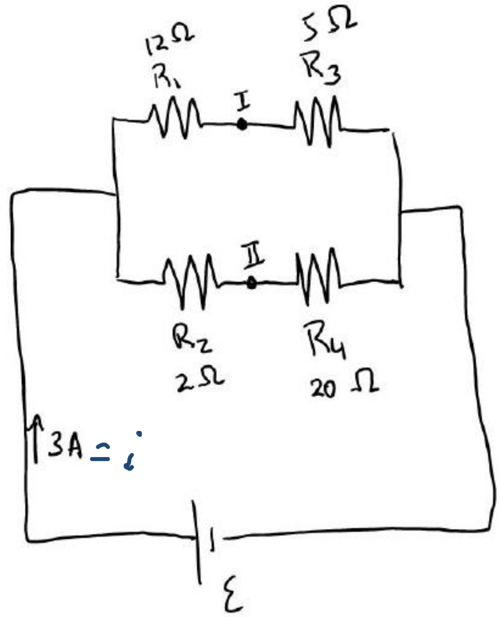
$$\left(\frac{v}{c}\right)^2 = 1-.72$$

$$\boxed{v = 0.53\ c}$$

P142 University of Rochester NAME Solution key - Slm
S. Manly Fall 2014

Problem 4 (15 pts, show your work):

Consider the circuit shown in the figure. What is the potential difference between the points I and II shown on the figure? Is point I or point II at the higher potential? *(Please note that in this problem "I" and "II" represent Roman numerals used to indicate positions in the circuit and do not represent currents. The only current you are given in this problem is the 3A in the left segment shown in the figure.)*



Need to determine currents in Top + Bottom branches of resistors and value of ε.

Top Branch $R_{TOP\,TOT} = R_1 + R_3 = 17\,\Omega$

Bott Branch $R_{BOTT\,TOT} = R_2 + R_4 = 22\,\Omega$

$$\frac{1}{R_{Total}} = \frac{1}{R_{TOP\,TOT}} + \frac{1}{R_{BOTT\,TOT}}$$

$$\frac{1}{R_{TOT}} = \frac{1}{17} + \frac{1}{22} = \frac{22+17}{(22)(17)}$$

$$R_{TOT} = \frac{(22)(17)}{39} = 9.6\,\Omega$$

so $\varepsilon = i R_{TOT}$

$\varepsilon = (3)(9.6) = 28.8\text{ V}$

current in top branch is

$28.8 = i_{TOP}\, 17$

$i_{TOP} = 1.7\text{ A}$

current in bottom branch

$3 - 1.7 = 1.3\text{ A}$

$V_I = 28.8 - (1.7)(12) = 8.4$

$V_{II} = 28.8 - (1.3)(2) = 26.2$

$\Delta V = V_{II} - V_I = 26.2 - 8.4$

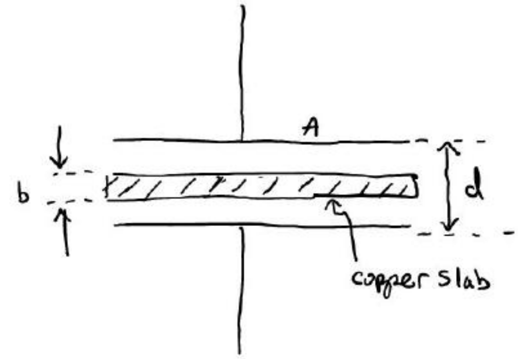
$= 17.8\text{ V}$

Point II is at the higher potential

1)	/15
2)	/12
3)	/13
4)	/15
5)	/20
6)	/25
tot	/100

Problem 5 (20 pts, show your work):

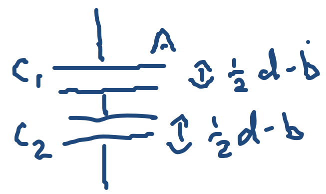
Suppose you thrust a slab of copper of thickness b into a parallel plate capacitor as shown in the figure. It is centered exactly halfway between the plates. The parallel plates have area A and are separated by a distance d (d>b).



$C = \frac{\epsilon_0 A}{d}$ for ∥ plates

a) What is the capacitance of the system after the slab is introduced?

looks like



$$C_{orig} = \frac{\epsilon_0 A}{d}$$

$$C_1 = \frac{\epsilon_0 A}{\frac{1}{2}(d-b)} = \frac{2\epsilon_0 A}{d-b} \qquad \frac{1}{C_{New}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{2(d-b)}{2\epsilon_0 A}$$

$$C_2 = \frac{\epsilon_0 A}{\frac{1}{2}(d-b)} = \frac{2\epsilon_0 A}{d-b} \qquad C_{New} = \frac{\epsilon_0 A}{d-b}$$

b) If a charge Q is maintained on the plates, what is the ratio of the stored energy before to that after the slab is inserted?

$Q = CV$

$E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$

$$E_{bef} = \frac{1}{2}\frac{Q^2}{C_{orig}} = \frac{1}{2}Q^2\frac{d}{\epsilon_0 A}$$

$$E_{new} = \frac{1}{2}\frac{Q^2}{C_{New}} = \frac{1}{2}\frac{Q^2(d-b)}{\epsilon_0 A}$$

$$\frac{E_{before}}{E_{New}} = \frac{d}{d-b}$$

c) How much work is done to insert the slab?

$$E_{New} - E_{before} = \frac{1}{2}Q^2\left[\frac{1}{C_{New}} - \frac{1}{C_{orig}}\right] = \frac{1}{2}Q^2\left[\frac{d-b}{\epsilon_0 A} - \frac{d}{\epsilon_0 A}\right]$$

$$= \frac{1}{2}Q^2\left[-\frac{b}{\epsilon_0 A}\right]$$

d) Is the slab "sucked in" or do you have to push it in between the plates or does it take no effort at all to insert the slab? Give some indication of why you answer this the way you do.

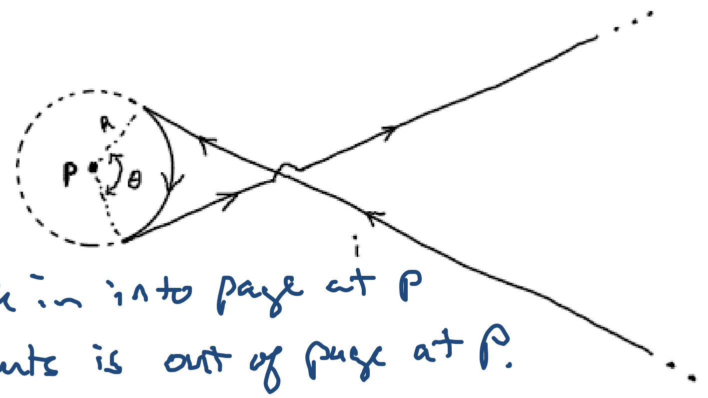
slab is sucked in

New config has lower energy

must do negative work to insert slab.

Problem 6 (25 pts, show your work):

Consider two semi-infinite wires attached (tangentially) to a semicircular wire. A current i flows, as shown in the figure, along one semi-infinite wire to the semicircle and then along the other semicircular wire. What is the size of the angle θ such that the magnetic field is zero at point P at the center of curvature of the semicircle?



First note that $\vec{B}$ due to semicircle is into page at P and $\vec{B}$ due to semi-infinite segments is out of page at P.

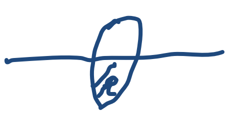
$\vec{B}$ at P = 0 when $|\vec{B}|_{\text{semicircle}} = |\vec{B}|_{\text{line segments}}$

Find $\vec{B}$ due to line segments

Can use Biot-Savart and integrate from 0 to ∞ and mult by 2

This will work

Easier way is to recognize from the symmetry of the problem that $\vec{B}$ due to the two semi-infinite line segments is equivalent (at point P) to the field of an infinite wire at r = R



$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$ $B 2\pi R = \mu_0 i$ $B_{P\,\text{segments}} = \frac{\mu_0 i}{2\pi R}$

for semicircle, use Biot-Savart

$$B_P = \int_0^\theta \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 i}{4\pi R^2}\int_0^\theta R\,d\theta = \frac{\mu_0 i}{4\pi R}\,\theta$$

$dl = R\,d\theta$

These are equal in magnitude when θ = 2 rad.

So, Answer is

θ = 2 radians