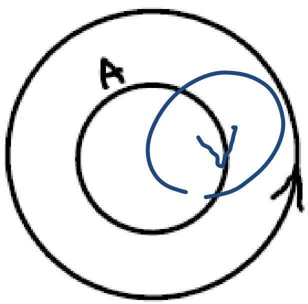
P142 University of Rochester NAME Soln Key – SM
S. Manly Fall 2014

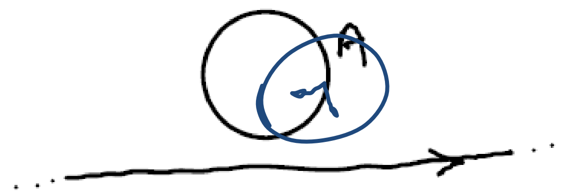
Final Exam (December 20, 2014)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (6 pts, no need to show work but ~~please use the correct hand!)~~:

In both sketches, A denotes a conducting wire loop. In each case, mark clearly the direction of the current induced in loop A due to the indicated current.





Problem 2 (3 pts, no need to show work):

A lens is used to image an object onto a screen. When the upper half of the lens is covered,

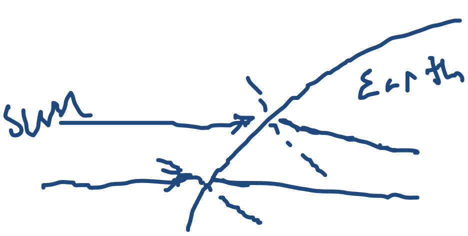
a) the upper half of the image disappears. The lower half of the image disappears.
b) The image becomes blurred.
c) The image become fainter.
d) The entire image disappears.

Problem 3 (6 pts):

You go out for a walk with Grandma as sunset approaches on Christmas eve. Amazingly (if you are near Rochester) the sky is clear! When you look at the sun, you notice it is

a) perfectly circular
b) squashed slightly at the edges closest and farthest from the horizon
c) elongated slightly at the edges closest and farthest from the horizon.

Select the answer that you believe to be true and justify it briefly in writing below. Feel free to use a sketch in your answer if it is helpful.



Rays at top of sun bent more than ones at bottom of sun as they hit atmosphere due to curvature of earth's atmosphere. So the image will appear slightly squashed at the edges closest and farthest from the horizon

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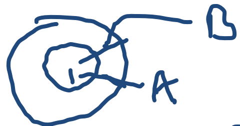
Problem 4 (8 pts, show your work):

Consider a coaxial cable made of two conductors. The inner conductor has a radius A and a positive charge per unit length of λ. The outer conductor has a radius B and a negative charge per unit length of λ. If the outer conductor is defined to have V=0, what is the potential of the inner conductor?



$\vec{E} = 0$ for $r < A$

$\vec{E} = 0$ for $r > B$

for $A < r < B$

use Gauss' law

$E\, 2\pi r \ell = \dfrac{\lambda \ell}{\epsilon_0}$ $\quad \vec{E} = (+\hat{r}) \dfrac{\lambda}{\epsilon_0 2\pi r}$

ΔV from $r = B$ to $r = A$ $= -\int_B^A \vec{E} \cdot d\vec{r} = -\dfrac{\lambda}{\epsilon_0 2\pi} \int_B^A \dfrac{1}{r}\, dr = -\dfrac{\lambda}{\epsilon_0 2\pi} \ln \dfrac{A}{B}$

Problem 5 (8 pts):

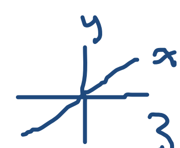
need more room here

An electromagnetic wave that has a frequency of 100 MHz has a magnetic field described by $\vec{B}(z,t) = (10\text{-}8\ \text{T})\cos(kz-\omega t)\hat{i}$.

a) What is the wavelength of this wave?

$c = \nu \lambda$

$\dfrac{c}{\nu} = \lambda \quad \dfrac{3\times 10^8\ \text{m/s}}{100\times 10^6\ \frac{1}{s}} = 3$ meters



b) What is the direction of propagation of this wave?

The wave is propagating in the $+\hat{z}$ direction

c) Write down the analogous description of the electric field for this wave?

$\vec{E} \times \vec{B} \sim +z$ so $\vec{E}$ along $-\hat{j}$, $|\vec{B}| = \frac{1}{c}\vec{E}$ $\quad \vec{E}(z,t) = (c)\, 10^{-8} \cos(kz - \omega t)(-\hat{j})$

d) Describe the polarization of this wave, i.e., type and direction (if any).

The wave is linearly polarized along the y-axis.

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Problem 6 (6 pts, show your work):

Consider two infinite solenoids, 1 and 2, that have axes of symmetry parallel to the z-axis as shown in the sketch. Solenoid 1 has radius R1 and n1 turns per unit length and solenoid 2 has radius R2 and n2 turns per unit length. Determine the constant of mutual inductance between the two solenoids.

$\mathcal{E}_1 = M \, di_2/dt$

$\mathcal{E}_2 = M \, di_1/dt$

The magnetic field of each solenoid is confined to the solenoid. So the mutual inductance is zero.

Problem 7 (10 pts, show your work):

As I'm sure most of you learned from the esteemed Professor Eberly, a blackbody is an object that is a perfect absorber (of electromagnetic radiation) and a perfect radiator. The power radiated by a blackbody is governed by the Stefan-Boltzman equation

$P=\sigma AT^4$

where $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4, and A equals the surface area of the blackbody and T is the temperature of the blackbody in degrees Kelvin. Calculate the temperature of the earth in the approximation that it is a perfect blackbody. Assume the radius of the earth is 6.4×10^6 m, the distance of the earth from the sun is 1.5×10^{11} m and the power radiated by the sun is 3.8×10^{26} W.

For Absorption, Assume Earth is Black Disk w/ Area πR^2

For emission, Assume radiation in 4π

Assume equilibrium bet power absorbed and emitted

$$\text{Power Absorbed} = 3.8 \times 10^{26} \, \frac{\pi (6.4 \times 10^6)^2}{4\pi (1.5 \times 10^{11})^2} \text{ Watts}$$

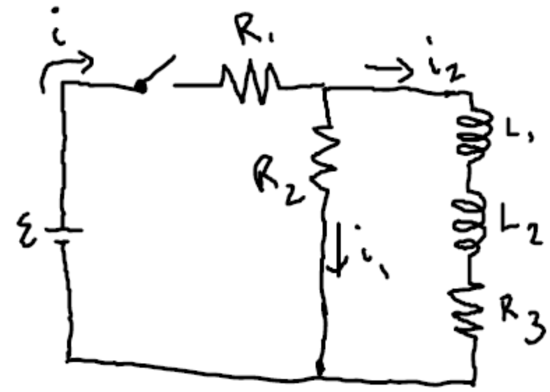
$$\parallel$$

$$\sigma \, 4\pi (6.4 \times 10^6)^2 \, T^4 \quad \text{Power radiated}$$

$T = 277$ K

Problem 8 (10 pts, show your work):

Consider the circuit shown in the sketch. Let ε=20 V, R_1=6 Ω, R_2=12 Ω, R_3=8 Ω, L_1=4 H, L_2=3 H.



a) What are the currents, i, i_1, and i_2 just after the switch is closed?

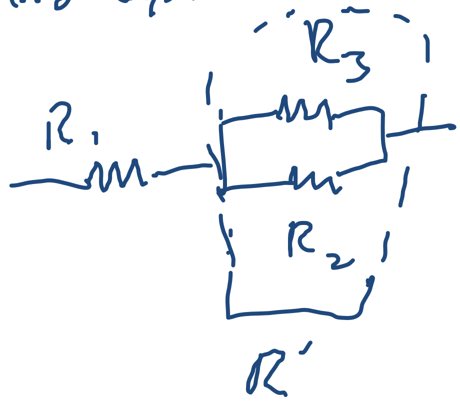
at $t=0$, I thru inductive branch is zero, $i_2=0$

$$i = i_1 = \varepsilon/(R_1+R_2) = \frac{20}{18} \text{ Amps} = 1.1 \text{ A}$$

b) What are the currents, i, i_1, and i_2 a long time after the switch is closed?

after a long time, as if inductors are not present

Find equivalent resistance to



$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{8} = \frac{20}{(12)(8)} = \frac{5}{(6)(4)} = \frac{5}{24}$$

$$R' = 24/5$$

$$\text{Total } R = R' + R_1 = \frac{24}{5} + 6 = \frac{54}{5}$$

$$\text{So } i = \frac{\varepsilon}{R_{TOT}} = \frac{20}{54/5} = \frac{100}{54}$$

$$i = \frac{50}{27} \text{ A} = 1.85 \text{ A}$$

$$\text{Voltage drop across } R_1 = \frac{50}{27} 6 = 11.1 \text{ V}$$

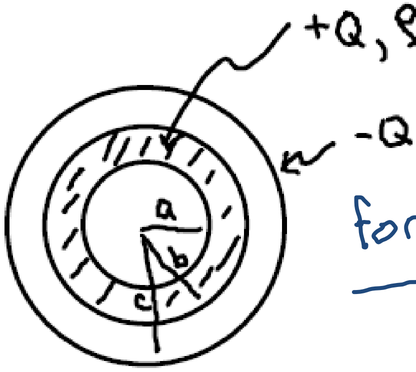
$$\text{Voltage across } R_2 \text{ and } R_3 = 20 - \frac{50}{27} 6 = 8.9 \text{ V}$$

$$i_2 = \frac{8.9}{8} = 1.1 \text{ A}$$

$$i_1 = \frac{8.9}{12} = .74$$

Problem 9 (8 pts, show your work):

Consider the spherically symmetric charge distribution in the sketch. In this configuration, there is a charge +Q distributed in a shell with a volume charge density $\rho=Q_o(1/r^2)$ that has an inner radius at r=a and an outer radius at r=b. In addition, a charge of –Q is distributed uniformly on a thin shell located at r=c. Determine the electric field in all regions of space.



use Gauss' Law to solve this

for r<a

$$\oint E\cdot dA = \frac{Q_{encl}}{\epsilon} = 0$$

$$\vec{E} = 0 \quad (r<a)$$

for a<r<b

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}|\,4\pi r^2 = \frac{1}{\epsilon_0}\int_a^r \rho\, 4\pi r^2 dr = \frac{4\pi}{\epsilon_0}\int_a^r Q_0\, dr$$

$$|\vec{E}|\,4\pi r^2 = \frac{4\pi Q_0}{\epsilon_0}(r-a)$$

$$\vec{E}_{a<r<b} = (+\hat{r})\,\frac{Q_0}{\epsilon_0}\,\frac{(r-a)}{r^2}$$

for b<r<c

$$|\vec{E}|\,4\pi r^2 = \frac{Q}{\epsilon_0} \rightsquigarrow \vec{E}_{b<r<c} = (+\hat{r})\,\frac{Q}{4\pi\epsilon_0 r^2}$$

for r>c

$$|\vec{E}|\,4\pi r^2 = 0 \rightsquigarrow \vec{E}_{r>c} = 0$$

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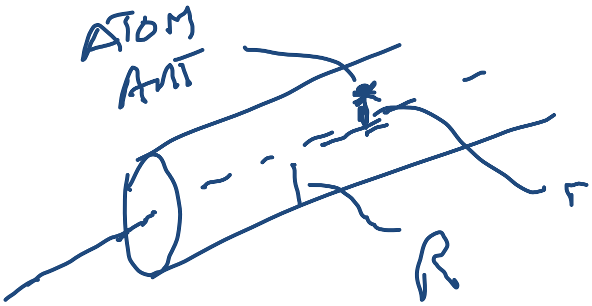
Problem 10 (9 pts, show your work):

Atom Ant, insect hero extraordinaire, fights an evil wizard. Suddenly our hero vanishes only to find himself in deep space in the midst of a very long, cylindrically symmetric distribution of negative electric charge. With his superpowers, Atom Ant determines that the radius of the charge distribution is R and that he is inside the distribution at r<R. He observes that the charge distribution has a uniform volume charge density ρ. Immediate Atom Ant knows he is in trouble. You see, part of Atom's superpowers come from the fact that he carries within him a net positive charge of q. If Atom Ant has mass m, describe his motion subsequent to his arrival in the strange deep space charge distribution.



Get Electric field at Atom Ant's position from Gauss

$$\int E\cdot dA = Q_{enc}/\epsilon_0$$

$$|\vec{E}|\, 2\pi r l = \frac{\rho}{\epsilon_0}\pi r^2 l \qquad \vec{E} = (-\hat{r})\frac{\rho r}{2\epsilon_0}$$

So,
Force on Atom Ant is $q\vec{E}$ . . . always toward center of distribution

$$m\frac{d^2 r}{dt^2} = -q\frac{\rho}{2\epsilon_0} r \qquad \frac{d^2 r}{dt^2} + \frac{q\rho}{2m\epsilon_0} r = 0$$

Atom Ant oscillates along a radial line through center of distribution with amplitude r and frequency $\omega^2 = \frac{q\rho}{2m\epsilon_0} \rightarrow \omega = \sqrt{\frac{q\rho}{2m\epsilon_0}}$

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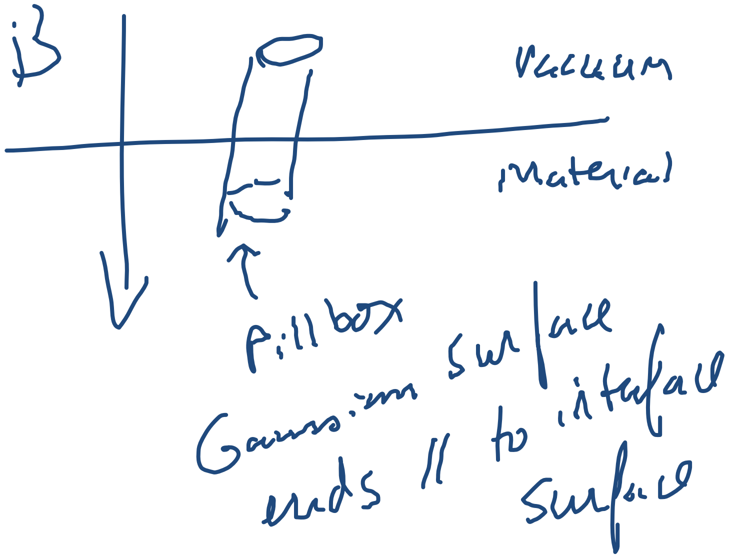
Problem 11 (8 pts, show your work):

Consider a flat interface between a vacuum and a material. Let a magnetic field be in the region of space being considered where the field is perpendicular to the surface. Prove that the magnetic field is continuous at the vacuum-material interface.



$$\int \vec{B}\cdot d\vec{A} = 0$$ Maxwell's eqn

$$B_{in}A - B_{out}A = 0$$

So $B_{in} = B_{out}$

Problem 12 (8 pts, show your work)

Consider two inertia reference frames S and S' with relative motion along the x-axis. Two events in reference frame S occur 10 μs apart at the same point in space. The distance between the two events is 2400 m in reference frame S'. What is the time interval between the events in S'? What is the velocity of S' relative to S?

More space

$$x_1' - x_2' = 2400\ m = \gamma(x_1 - vt_1) - \gamma(x_2 - vt_2)$$

$$= \gamma\underbrace{(x_1 - x_2)}_{0} + \gamma v\underbrace{(t_2 - t_1)}_{10^{-5}}$$

$$\boxed{2400 = \gamma v 10^{-5}}$$

$$t_1' - t_2' = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) - \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$$

$$t_1' - t_2' = \gamma\underbrace{(t_1 - t_2)}_{10^{-6}} + \frac{v}{c^2}\gamma\underbrace{(x_2 - x_1)}_{0}$$

$$\boxed{\Delta t' = \gamma 10^{-5}}$$

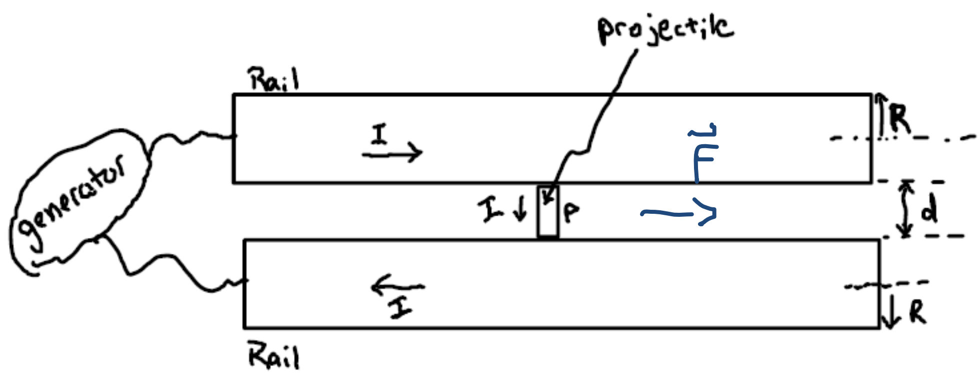
2 eqns
2 unknowns

$v = 1.87 \times 10^8$ m/s $= 0.62c$

$\gamma = 1.27$

Problem 13 (10 pts, show your work):

Consider the idealized rail gun shown in the schematic. This device consists of two conducting rails that can be considered as long solid cylinders of length L and radius R. The rails are a distance d apart. A conducting projectile, P, connects electrically and mechanically in a frictionless manner to the two rails. Assume that a generator creates a constant current I that travels up one rail, across the projectile and down the other rail. Derive an expression for the force on the projectile in terms of the basic variables in the problem.



2 Semiinfinite wires

$\longrightarrow$
$\longleftarrow$ create same $\vec{B}$ as 1 infinite wire by symmetry

So $\vec{B}$ in region of projectile is $\frac{\mu_0 I}{2\pi r}$ into paper

(can derive easily from Ampere's Law as needed.)

$$\vec{F}_{\text{on projectile}} = \int_R^{d+R} I B\, dr = \int_R^{d+R} \frac{\mu_0 I^2}{2\pi} \frac{1}{r}\, dr \quad \text{to right}$$

$$\vec{F} = \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{d+R}{R}\right) \quad \text{to right}$$