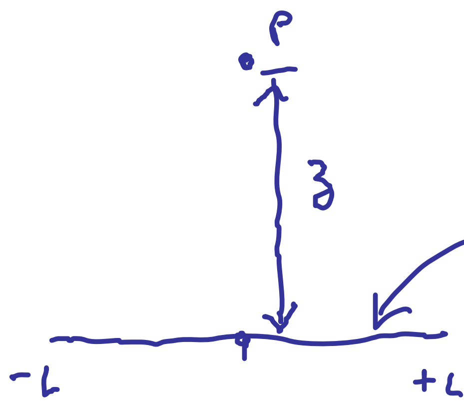
Physics 142 - September 11, 2014

- Prob Set 1 due by ~7 am-ish Tomorrow Morning
- Will release solns tomorrow
  - good idea to go thru them carefully
- Will post P.S. 2 Today/Tomorrow

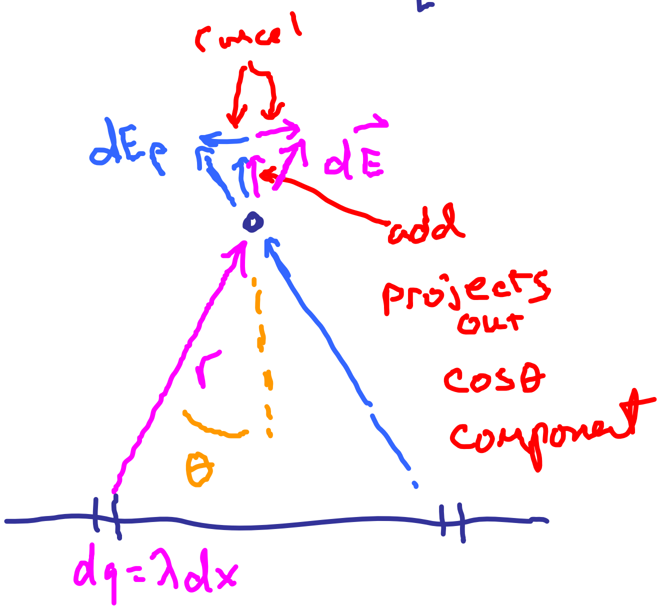
Last Time



What is $\vec{E}(p)$

$+\lambda$, constant

Total $Q = 2L\lambda$



$$\vec{E}_p = \int \frac{k\,dq\,\hat{r}}{r^2} = 2\int_0^L \frac{k\lambda \cos\theta\, dx}{r^2}\,\hat{z}$$

$$\vec{E}_p = 2\int_0^L \frac{k\lambda}{(x^2+z^2)}\frac{z}{(x^2+z^2)^{1/2}}$$

$$\vec{E}_p = \frac{2kL\lambda}{z(L^2+z^2)^{1/2}}\,\hat{z}$$

$$F = \frac{kq^2}{r^2}$$

$$k \sim \frac{N\,m^2}{C^2}$$

How do you check this??

Dimensional Analysis

Left side

$$\vec{E} = \vec{F}/q$$

$$N/C$$

Right side

$$\frac{\frac{N\,m^2}{C^2}\; m\; \frac{C}{m}}{m^2} \sim \frac{N}{C}$$

$$\vec{E}_p \quad \frac{2kL\lambda}{z(L^2+z^2)^{1/2}}\,\hat{z}$$

Limiting cases

Let $z \to$ large

Expect $\vec{E} \approx \dfrac{k\,\overbrace{2L\lambda}^{Q}}{z^2}$

z large

$$\frac{2kL\lambda}{z(L^2+z^2)^{1/2}} = \frac{\overbrace{2kL\lambda}^{kQ}}{z^2}$$

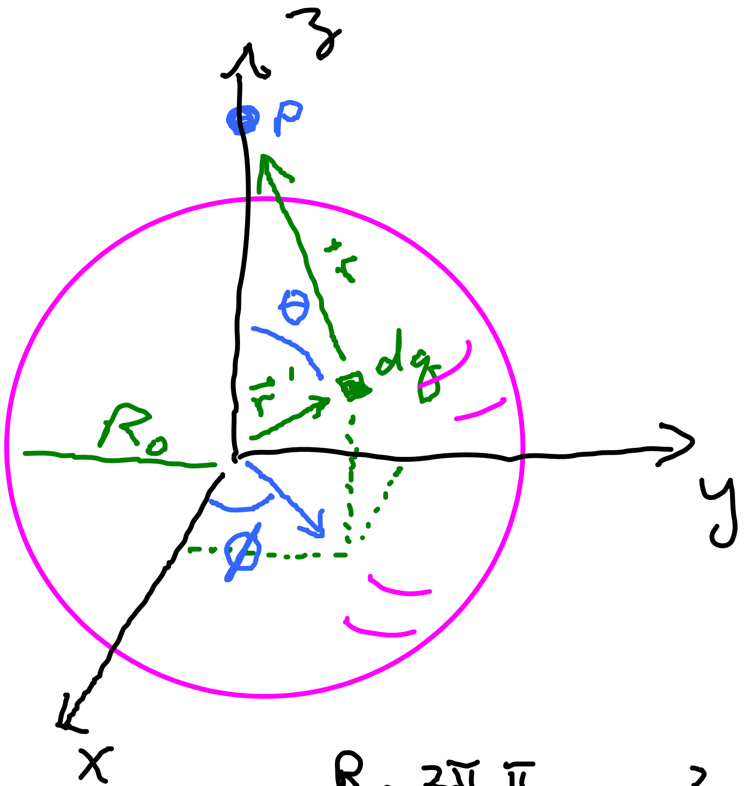
Field of point charge at large distance

Let $L \to$ large

use L'Hopital

$$\lim_{L\to\infty} \vec{E}_p = \frac{2k\lambda}{z}\,\hat{z}$$

Field around ∞ uniform line charge (as we will soon show)



Charge Q is distributed evenly in Sphere of radius R_0

what is $\vec{E}_p$

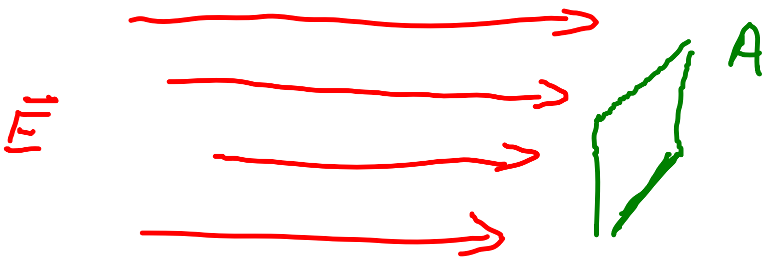
$$\vec{E}_p = \int_{\substack{\text{chg}\\ \text{Dist}}} \frac{k\rho\, dv}{r^2}\,\hat{r}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R_0^3}$$

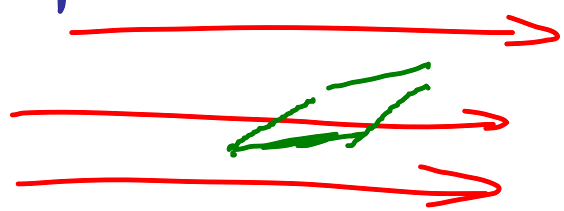
$$\vec{E}_p = \int_0^{R_0}\int_0^{2\pi}\int_0^{\pi} \frac{k\rho\, r'^2 \sin\theta\, d\theta\, d\phi\, dr}{r^2}$$

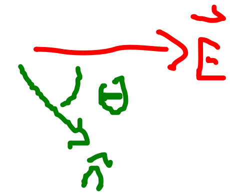
Holy Crap!

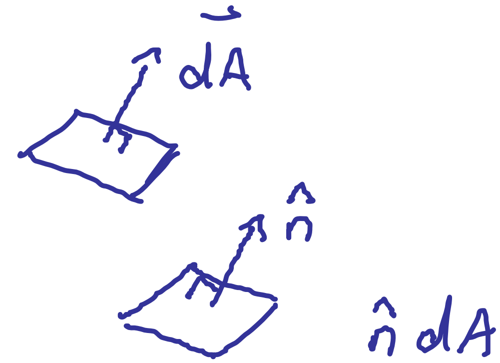
Electric Flux



$$\phi = A|\vec{E}|$$





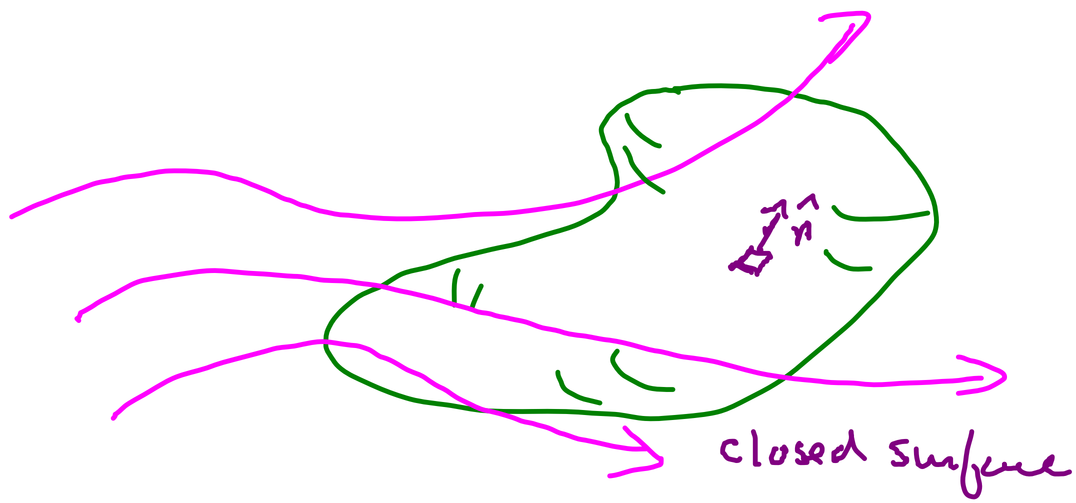


$$\boxed{\phi = \int_{\text{surface}} \vec{E}\cdot\hat{n}\, dA}$$

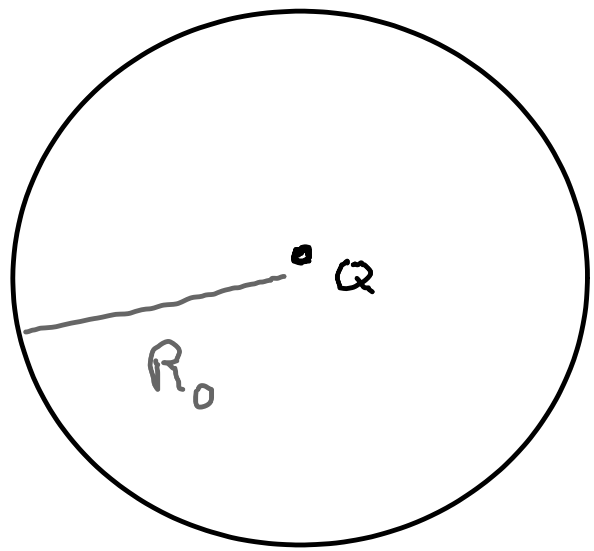
$$\phi = \int_{\text{surf}} \vec{E}\cdot\hat{n}\, dA$$

$$= \oint \vec{E}\cdot\hat{n}\, dA$$

$$= \oint \vec{E}\cdot\vec{dA}$$



Example

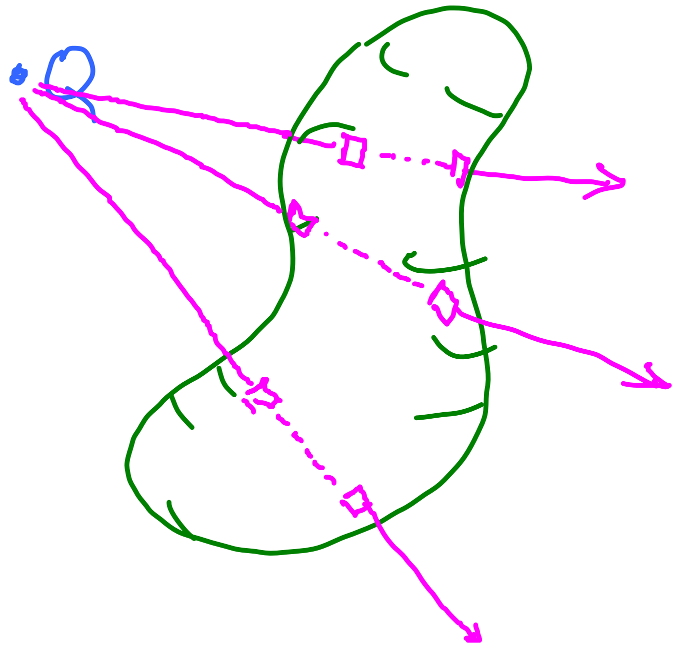


Pt at origin of Sphere, radius R_0

What is Flux thru surface

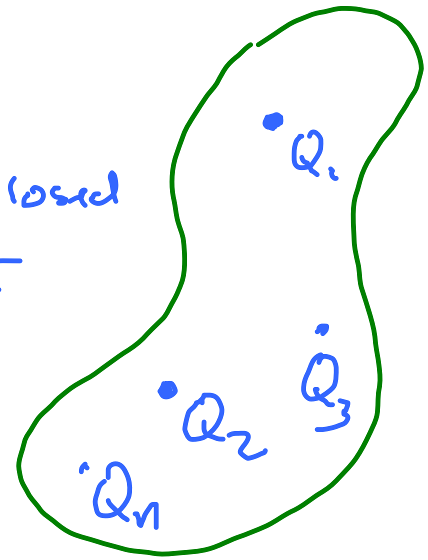
$$\Phi = \oint \underbrace{\vec{E}\cdot d\vec{A}}_{\hat{n}\,dA} = \oint E\,dA = E\oint dA = \frac{kQ}{R_0^2}4\pi R_0^2 = kQ4\pi$$

$$k = \frac{1}{4\pi\epsilon_0} \qquad \Phi = \frac{Q}{\epsilon_0}$$



Net Flux
Thru Surface = 0

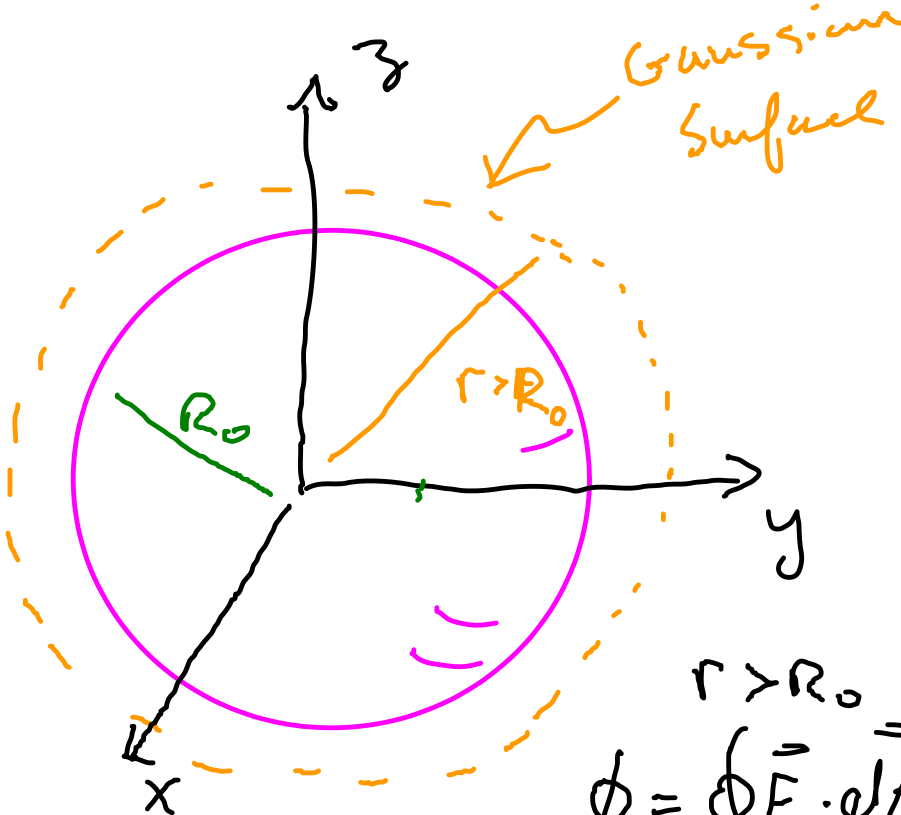
$$\text{Net flux} = \frac{Q_{net}^{\text{Enclosed}}}{\epsilon_0}$$



$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}}$$

Gauss' Law

True in general



spherical dist

$$\text{const } \rho = \frac{Q}{\frac{4}{3}\pi R_o^3}$$

what is $\vec{E}$ everywhere?

$r < R_o$ $\qquad$ $r > R_o$

$r > R_o$

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

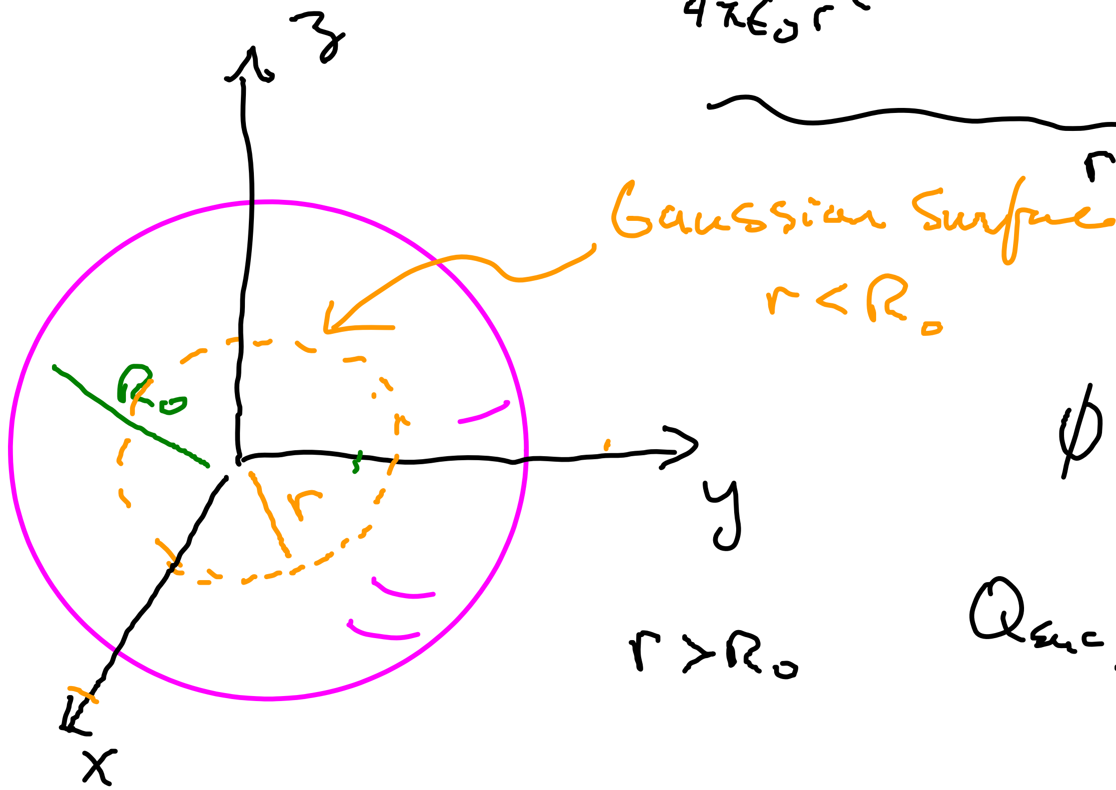
$$\oint E\, dA = E\, 4\pi r^2$$

$$E\,4\pi r^2 = Q/\epsilon_0$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \qquad \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$r > R_0$

$r < R_0$



$r > R_0$

$$\phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

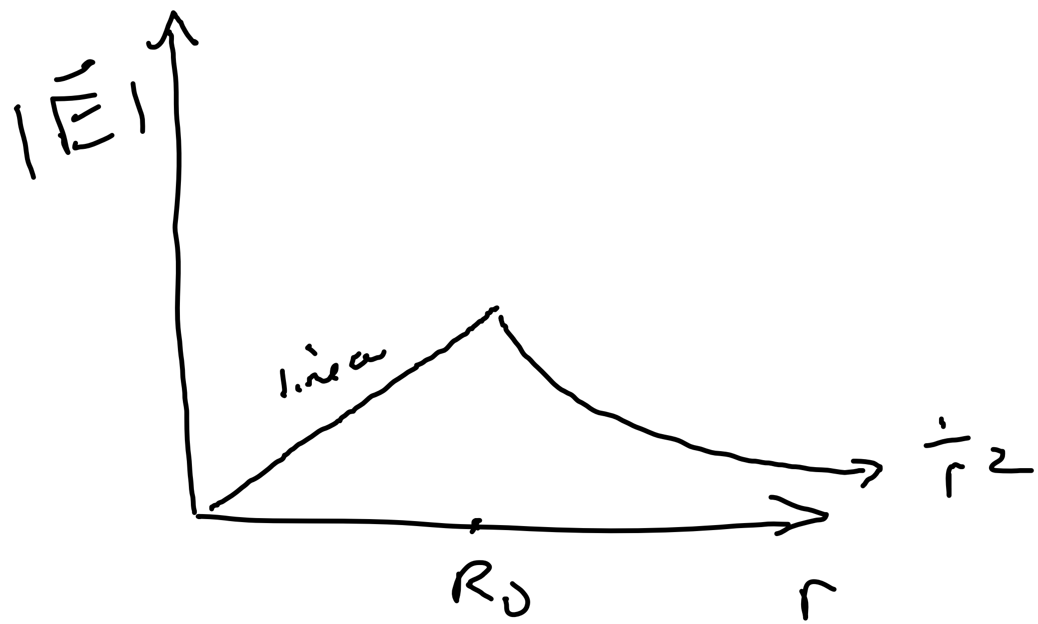
$$\phi = \frac{Q_{encl}}{\epsilon_0} = \rho\,\frac{4}{3}\pi r^3\,\frac{1}{\epsilon_0}$$

$$Q_{enc} = \frac{Q}{\frac{4}{3}\pi R_0^3}\,\frac{\frac{4}{3}\pi r^3}{\epsilon_0} = \frac{Q\,r^3}{\epsilon_0 R_0^3}$$

$$\oint \vec{E} \cdot d\vec{A} = \oint E\, dA = E\, 4\pi r^2$$

$$E\, 4\pi r^2 = \frac{Q r^3}{\epsilon_0 R_0^3}$$

$$\vec{E} = \frac{Q r}{4\pi \epsilon_0 R_0^3} \hat{r}$$



|E|
linear
1/r2
R0
r