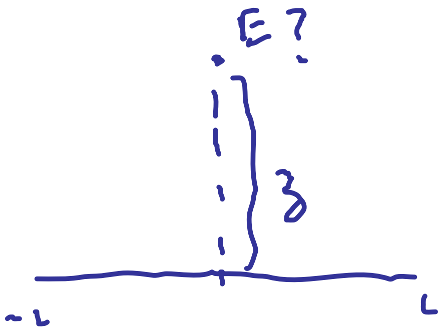
Physics 142 - September 16, 2014

- SM out of town this week
- No Lecture in Hoyt this Thursday
- Slides + Audio will be posted on class website



confusion abt integral
rushed at end of 9/9
Skipped a step at start of 9/11

$$\vec{E}_p = \int \frac{k\,dq\,\hat{r}}{r^2} = 2\int_0^L \frac{k\lambda \cos\theta\, dx}{r^2}\,\hat{z}$$

$$\vec{E}_p = 2\int_0^L \frac{k\lambda \quad z\,dx}{(x^2+z^2)(x^2+z^2)^{1/2}} = 2k\lambda z\int_0^L \frac{dx}{(x^2+z^2)^{3/2}}\,\hat{z}$$

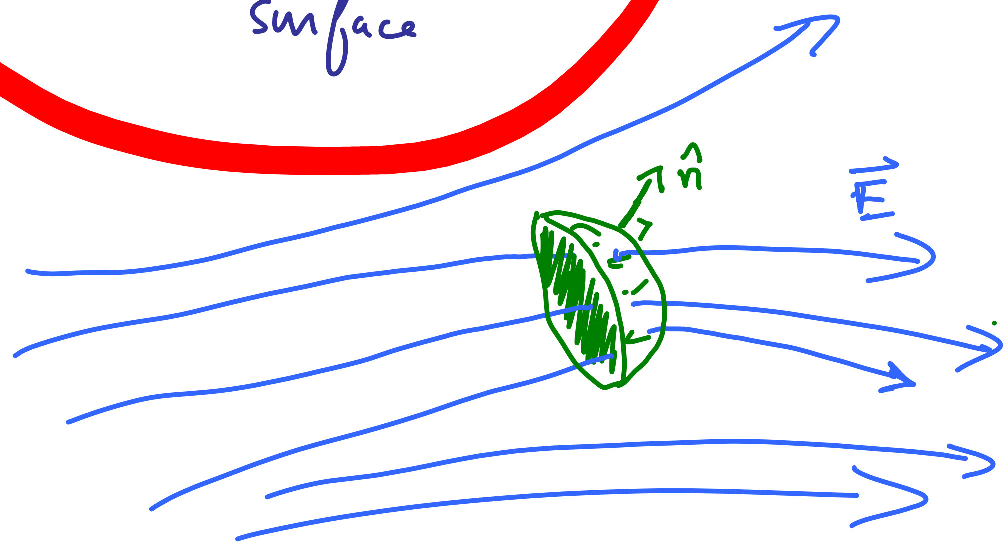
$$= \hat{z}\,2k\lambda z\left[\frac{x}{z^2(x^2+z^2)^{1/2}}\right]_0^L = \frac{2k\lambda z L}{z^2(L^2+z^2)^{1/2}}\,\hat{z} = \frac{2k\lambda L}{z(L^2+z^2)^{1/2}}\,\hat{z}$$

Last Time

Electric Flux

$\hat{n}\, dA$

$$\Phi_{\text{surf}} = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



Gauss' Law

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

True in general

integral over volume inside gaussian surface
(NOT necessarily all Charge)

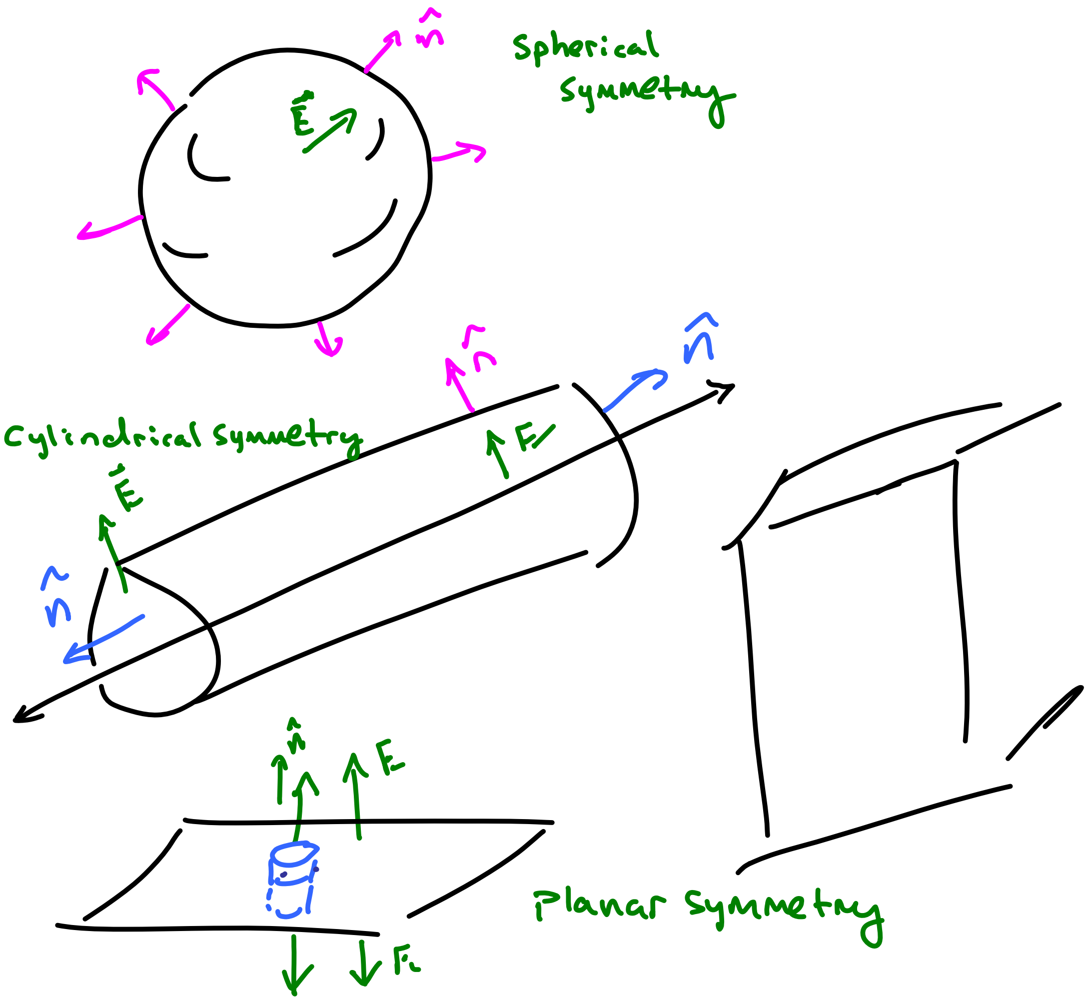
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\int_V \rho \, dv$$

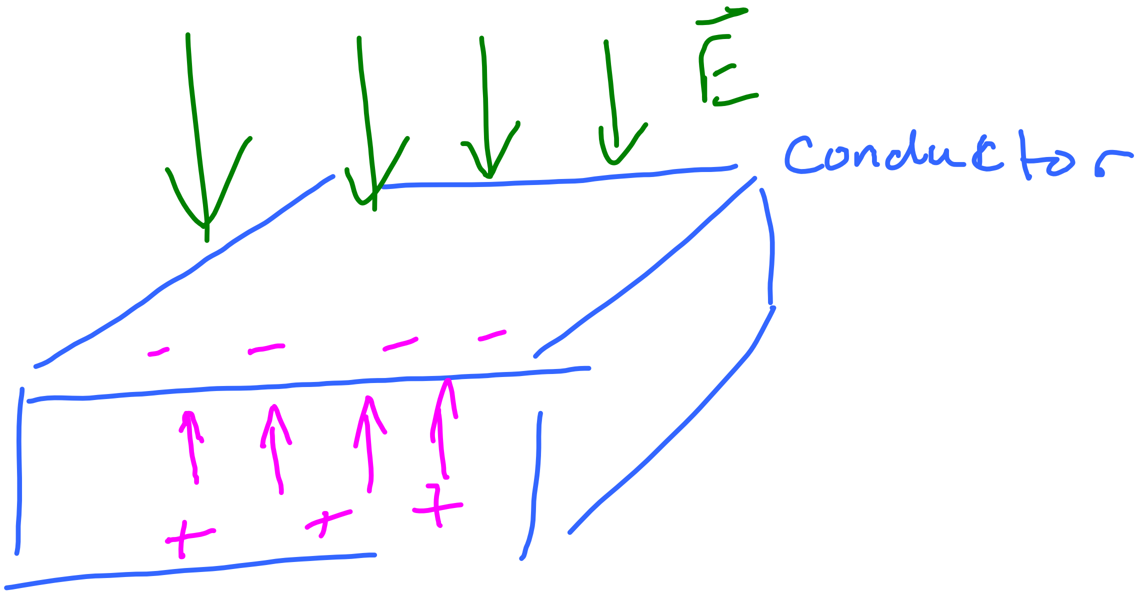
integral over Gaussian surface

Easy if $\vec{E} \perp d\vec{A}$ or $\vec{E} \parallel d\vec{A}$

Easy if $|\vec{E}|$ is constant on surface – can pull out of integral

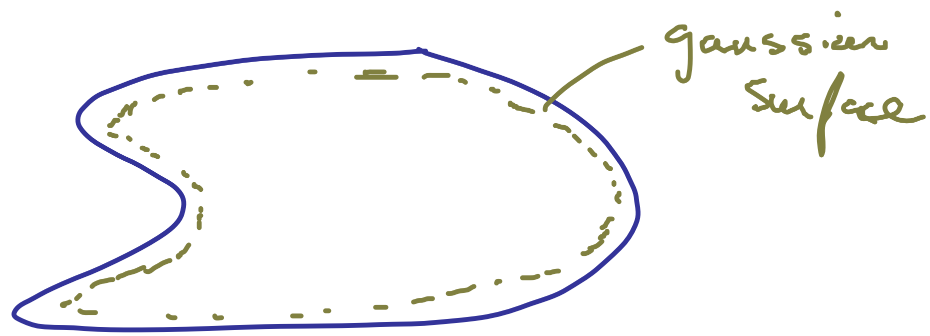


Spherical Symmetry
Cylindrical symmetry
Planar Symmetry



$\vec{E} = 0$

inside conductor

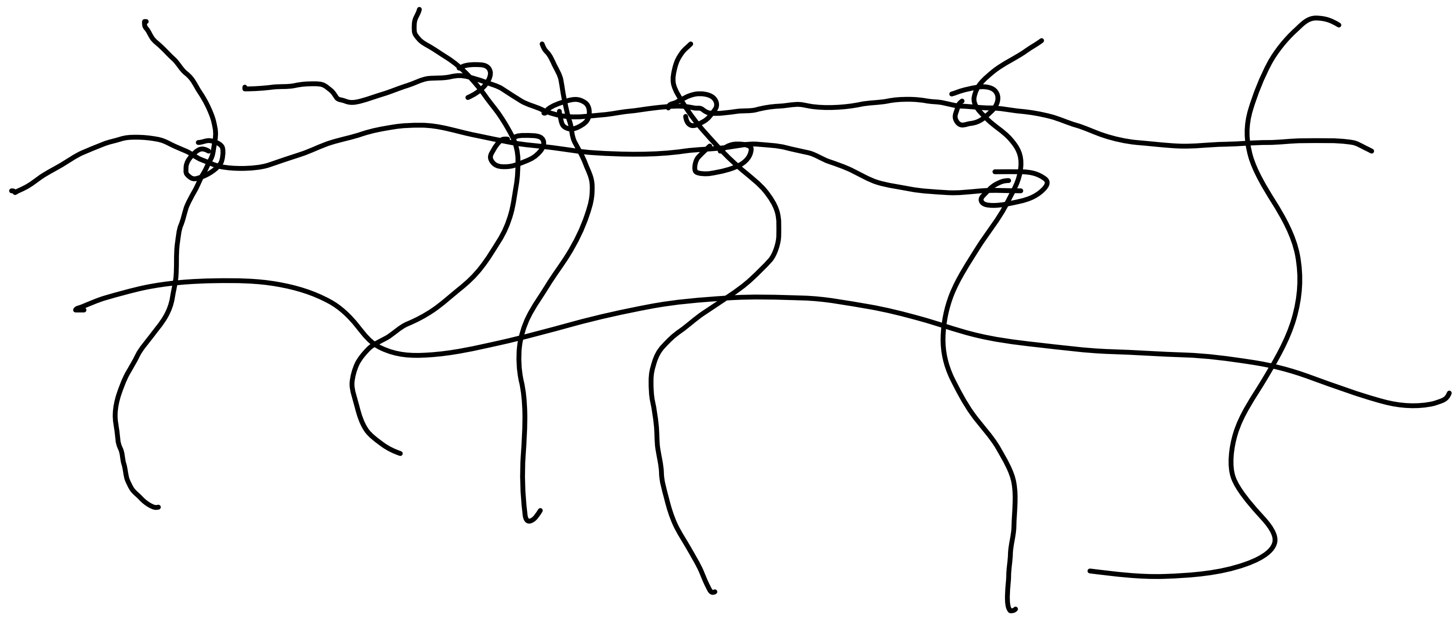


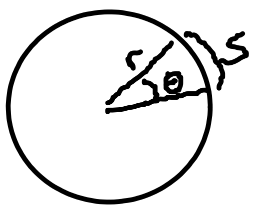
$$\oint \cancel{\vec{E}} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

0

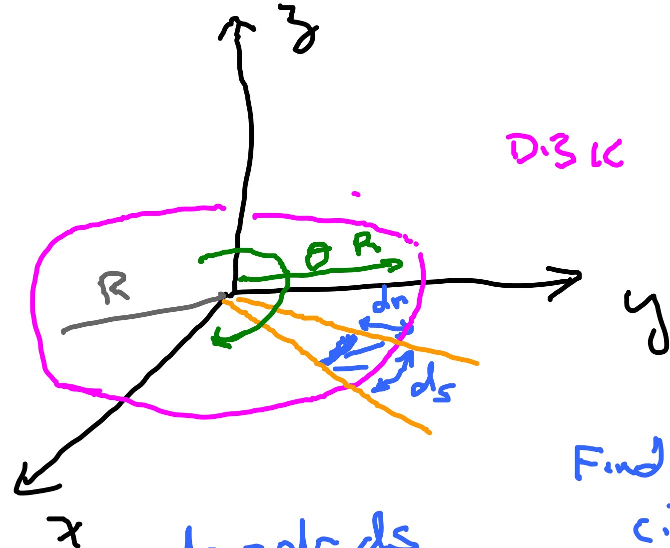
$\vec{E} = 0$

curvilinear coordinates





$s = r\theta$



Disk in x-y plane

Find Area of circular disk

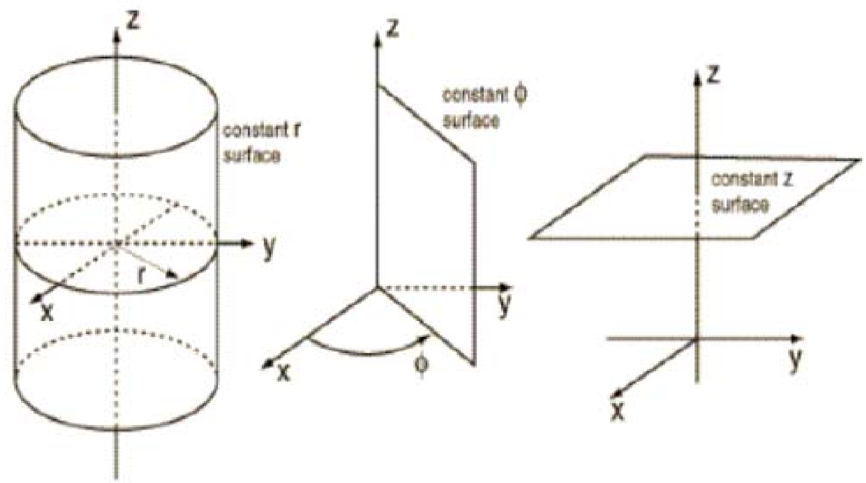
$da = dr \underbrace{ds}_{r\,d\theta}$

$da = r\,d\theta\,dr$

$$\text{Area} = \int da = \int_0^R \int_0^{2\pi} r\,d\theta\,dr$$

$$= 2\pi \int_0^R r\,dr = \pi R^2$$

cylindrical coordinates



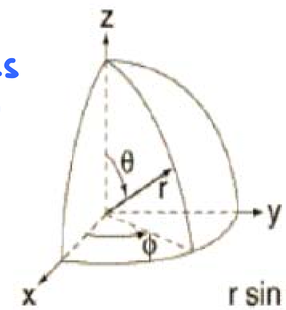
Some figures in this section from:

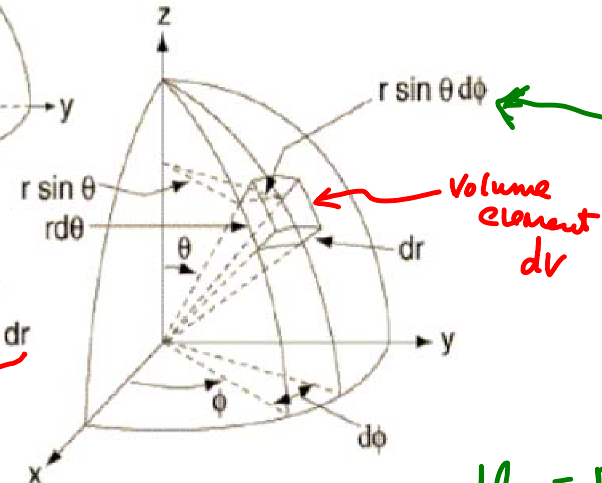
http://hyperphysics.phy-astr.gsu.edu/hbase/sphc.html

Also Griffiths, Intro to Electromagnetism

Spherical polar coordinates

Variables Defined



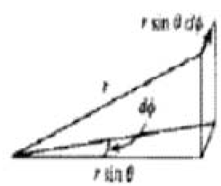


Differential Volume element

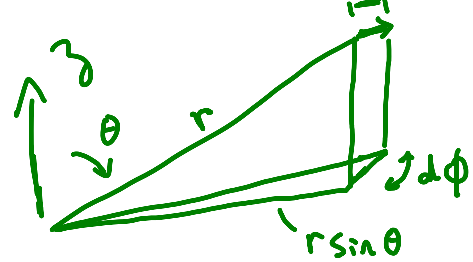
$dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$

This is important
Spend some time
studying this
and make
sure you
understand it.

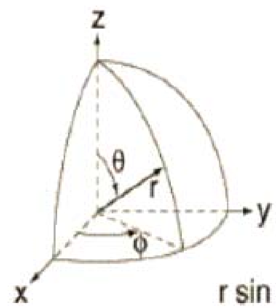
$dl_\phi = r \sin\theta \, d\phi.$

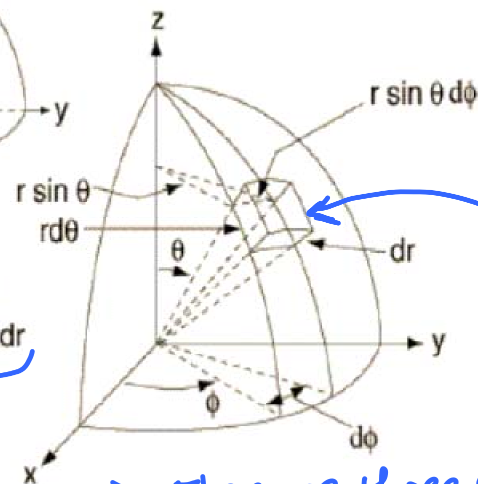


$dl_\phi = r\sin\theta \, d\phi$



Spherical polar coordinates





Volume element

$\underbrace{dV = r^2 \sin\theta \, d\theta \, d\phi \, dr}$

There are three elements that make up dv

in r $\quad dr$

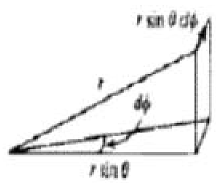
in θ $\quad r d\theta$

in φ $\quad r \sin\theta \, d\phi$

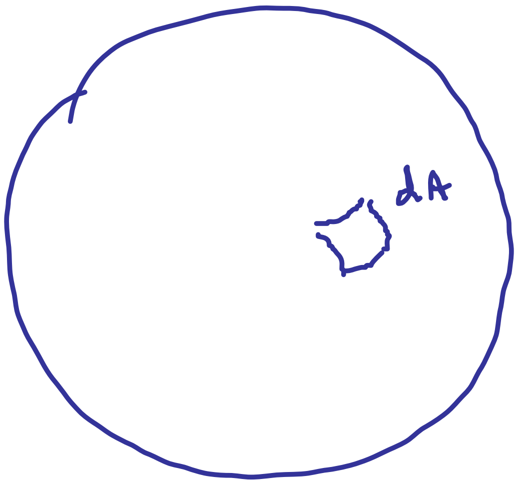
if this were a cartesian coordinate problem

$dv = dx\, dy\, dz$

$dl_\phi = r \sin\theta \, d\phi.$



what is Area of Sphere



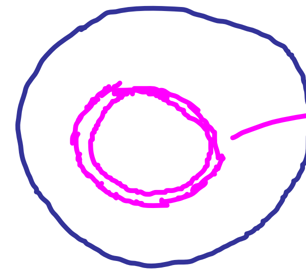
$$\text{Area} = \int dA = \int_{\underset{\theta}{0}}^{\pi} \int_{\underset{\phi}{0}}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$dA = r\,d\theta \; r\sin\theta \, d\phi$$
$$= r^2 \sin\theta \, d\theta \, d\phi$$

$$2\pi r^2 \underbrace{\int_0^{\pi} \sin\theta \, d\theta}_{2} = 4\pi r^2$$

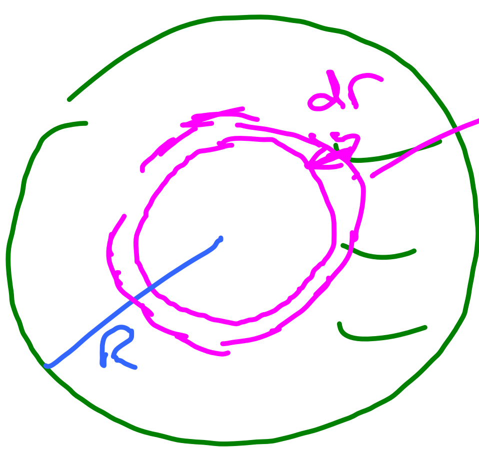
$da = dr\, r\, d\theta$

$A = \int_0^R \int_0^{2\pi} r\, d\theta\, dr$



$dA = 2\pi r\, dr$

$A = \int_0^R 2\pi r\, dr$



$4\pi r^2$

Sphere

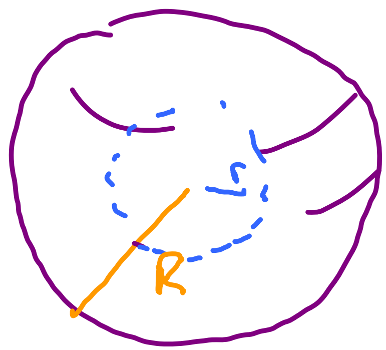
vol. of sphere

$V = \int dv =$

$\iiint r^2 \sin\theta\, d\theta\, d\phi\, dr$

$dv = 4\pi r^2\, dr$

$V = \int dv = \int_0^R 4\pi r^2\, dr = \frac{4}{3}\pi r^3$



$\rho = k r^3 \quad r<R$

$= 0 \quad r>R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{TOT}}{\epsilon_0}$$

what is Q_{TOT} for gaussian surf w/ $r<R$

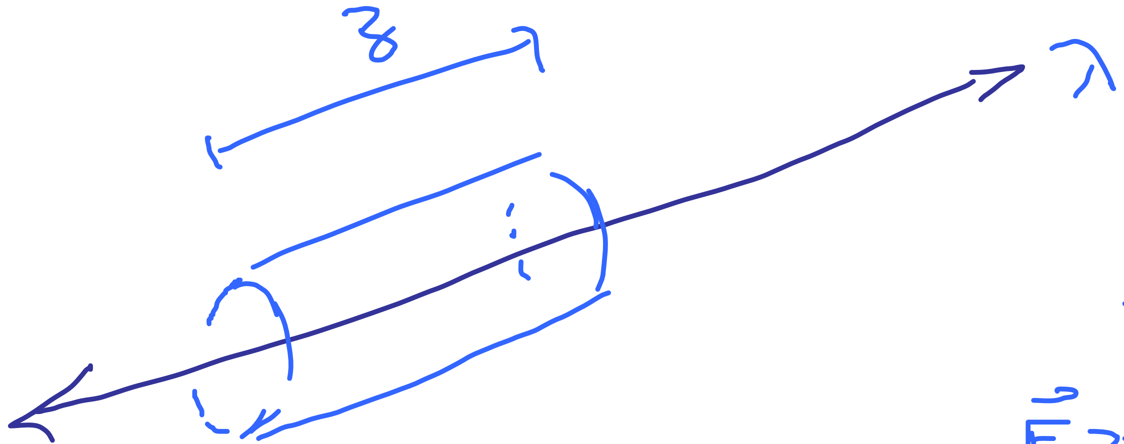
k – units of C/m^6

$$\underset{r<R}{Q_{TOT}} = \int \rho\, dv = \int_0^r k r^3\, 4\pi r^2\, dr$$

$$= \int_0^r k 4\pi r^5 dr = \frac{k 4\pi r^6}{6}$$

$r>R$

For $\underset{r>R}{Q_{TOT}} = \int_0^R k 4\pi r^5 dr = \frac{k 4\pi R^6}{6}$



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\vec{E}\, 2\pi r z = \frac{z\lambda}{\epsilon_0}$$

$$|\vec{E}| = \frac{\lambda}{2\pi r \epsilon_0}$$