Physics 142 - September 18, 2014

- Please enjoy the sound file for this lecture. Should find link on class website
- P.S. 2 due this week
- Solns to P.S. 1 posted
  ... come to closure on those probs
- Feel free to email questions if needed

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Last Time –

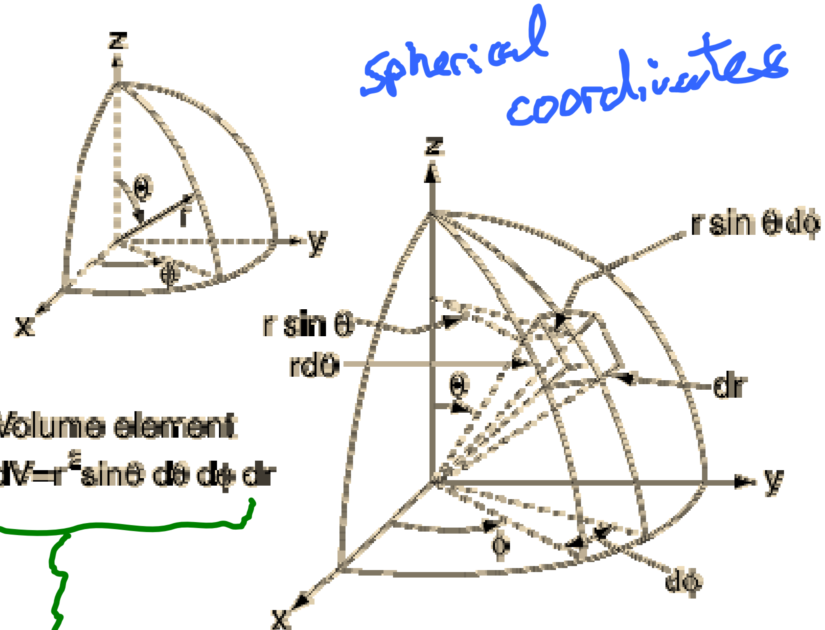
Gauss' Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

Always True . . . Most useful under certain conditions of symmetry

Curvilinear coordinates

- dV complicated compared to cartesian
- Can have generalized coordinates . . .
- We'll use spherical and cylindrical coords. often

spherical coordinates



Volume element
$$\underbrace{dV = r^2 \sin\theta \, d\theta \, d\phi \, dr}_{\text{understand this}}$$

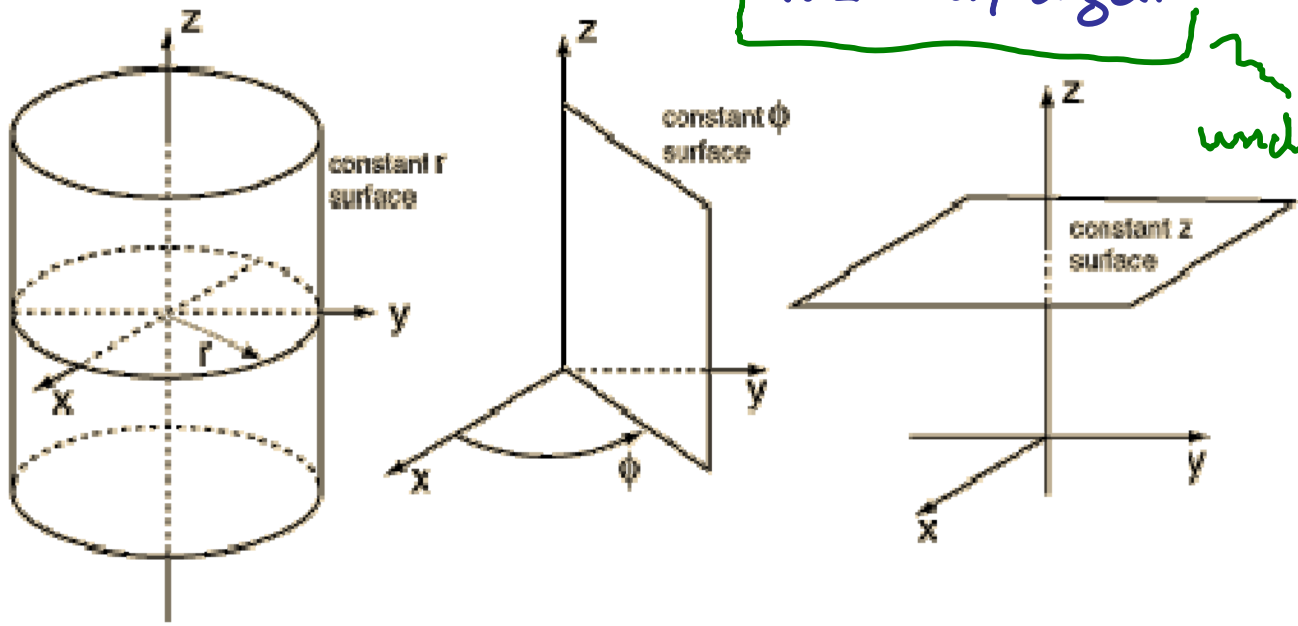
Griffiths E+M has nice discussion

cylindrical coordinates

③

$$\underbrace{dv = r\,d\phi\,dz\,dr}$$

understand



why does dv have to look ≈ cube-ish ??

Needs to be small in any dimension where the variable you integrate over has dependence.

Because I am such a Shell guy

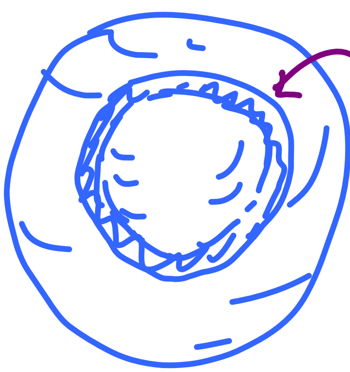
I'll limit us to problems in 1 variable

Usually that means radial dependence

Effectively integrates out Angular dependence.

dv does NOT need to be "small" in Angular dimensions in that case.

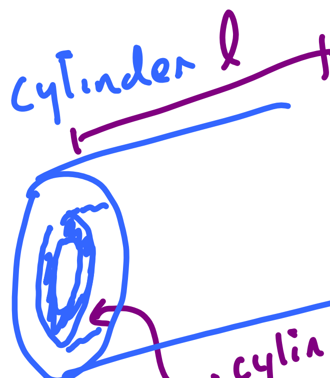
sphere



Shell

$dv = 4\pi r^2 dr$

cylinder ℓ



cylindrical shell

$dv = 2\pi r \ell dr$

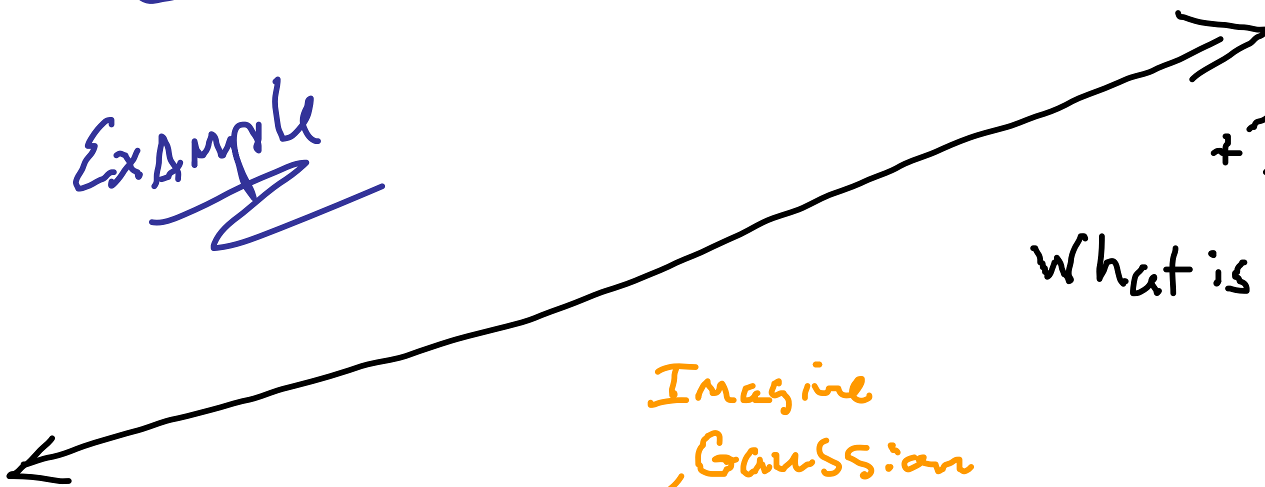
AT end of class last time

Example

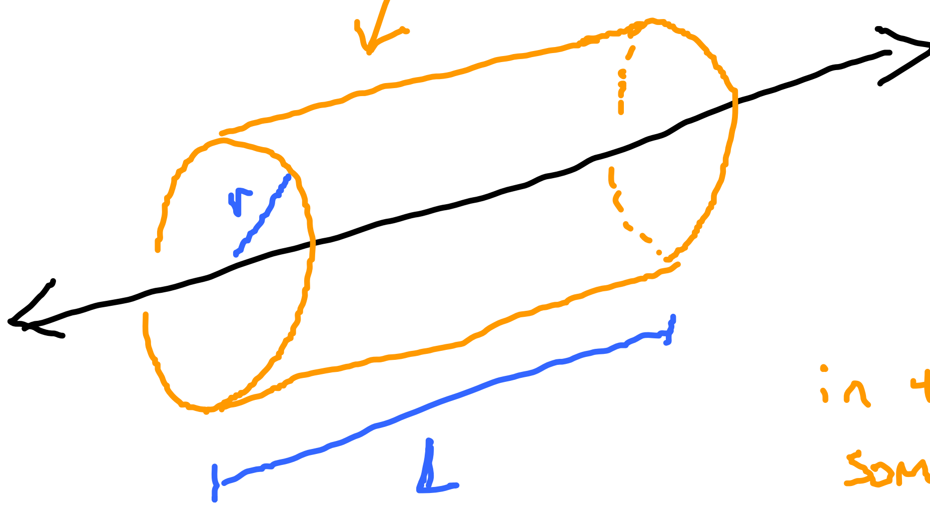
∞ straight line charge distribution

$+\lambda$, CONSTANT

What is $\vec{E}$ everywhere

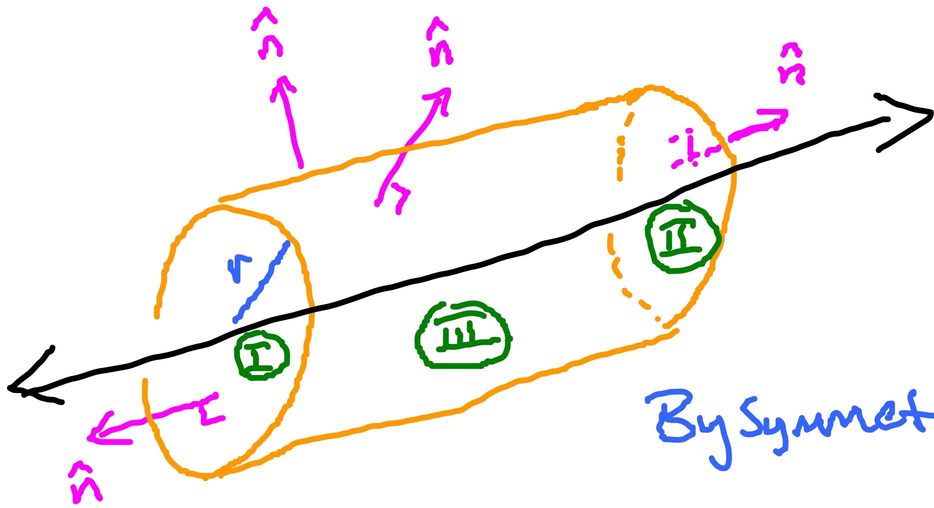


Imagine Gaussian surface ... choose arbitrary r in region where want to use Gauss' Law to evaluate $\vec{E}$



in this case, that is some r for cylinder centered on line charge (No other regions to consider)

L is chosen as an arbitrary length for now ... need it for calcs ... (will cancel)



Gauss' Law

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

By Symmetry, $\vec{E}$ is radial.

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{\mathrm{I}} \vec{E}\cdot\hat{n}\,da}_{=0\ \vec{E}\perp\hat{n}} + \underbrace{\int_{\mathrm{II}} \vec{E}\cdot\hat{n}\,da}_{=0\ \vec{E}\perp\hat{n}} + \int_{\mathrm{III}} \underbrace{\vec{E}\cdot\hat{n}}_{=|\vec{E}|=E}\,da = \int_{\mathrm{III}} E\,dA = E\,2\pi r L$$

constant on surface (symmetry)

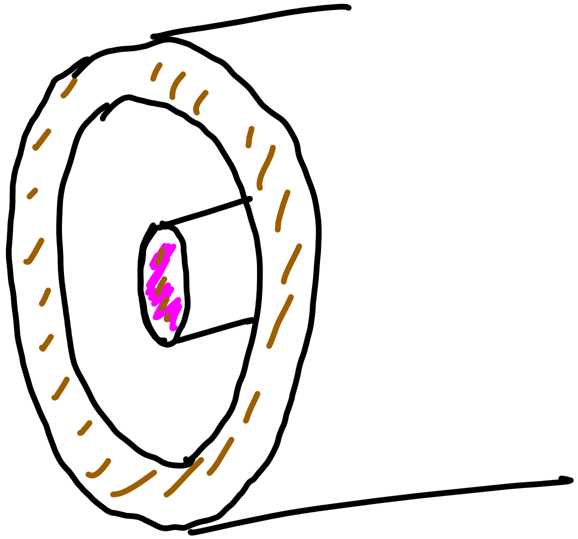
$Q_{encl} = L\lambda$

so $\oint \vec{E}\cdot d\vec{A} = Q_{encl}/\epsilon_0$

$E\,2\pi r L = L\lambda/\epsilon_0$

$$\boxed{\text{for } +\lambda \text{, radially outward } E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

Example



noncondcuting core

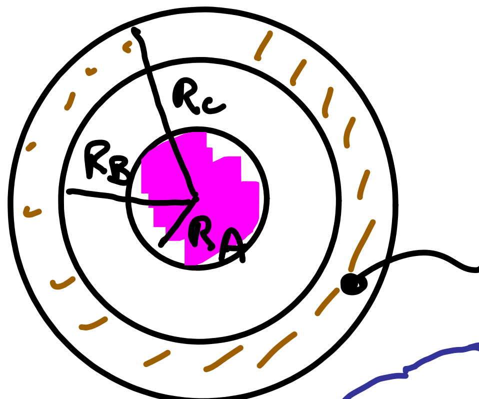
radius R_A

has $+\lambda$

distributed

(A) $\rho(r) = ar \quad r < R_A$

View from end



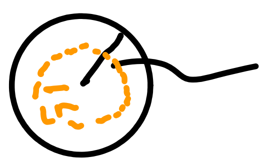
Conductor

Find $\vec{E}$ in all space

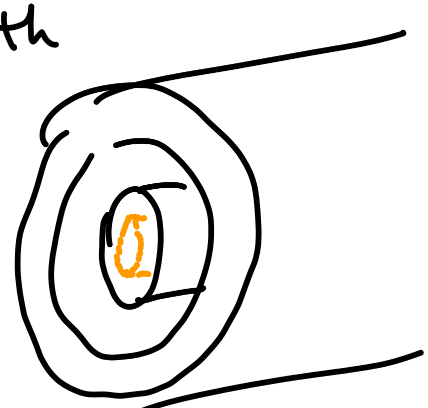
Sheath

Solution

$r < R_A$



R_A



imagine Gaussian surface w/ $r < R_A$

(cylindrically symmetric)

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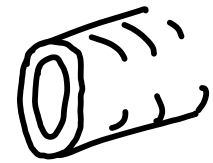
$\vec{E}$ radially outward by symmetry

$dv = 2\pi r L dr$

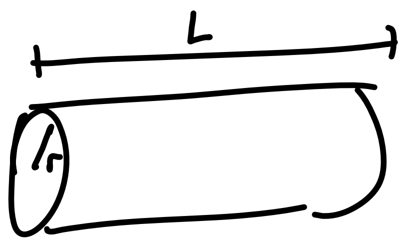


$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

endcaps do Not contribute $\vec{E} \perp d\vec{A}$

$$|\vec{E}| \int dA = |\vec{E}|\, 2\pi r L$$

pipe shell



$A = 2\pi r L$

Q_{encl} ?

$$\int \rho\, dv$$

$$= \int_0^r a r \, 2\pi r L \, dr \qquad \text{B}$$

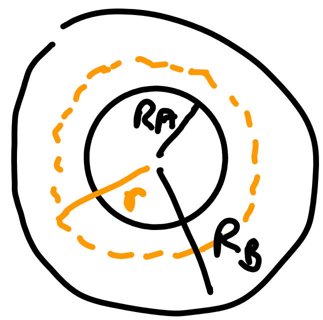
$$= a 2\pi L \int_0^r r^2 dr = \frac{a 2\pi L\, r^3}{3}$$

$$Q_{encl} = \frac{a 2\pi L\, r^3}{3}$$

$$|\vec{E}|\,2\pi r L = \frac{a\,2\pi L\,r^3}{3\epsilon_0}$$

A $\quad \vec{E} = \frac{a r^2}{3\epsilon_0}$ radially out $\quad r < R_A$

$R_A < r < R_B$



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}|\,2\pi r L = \frac{1}{\epsilon_0}\,\frac{a\,2\pi L\,R_A^3}{3}$$

$R_A < r < R_B$

SAME at $r = R_A$?

$r < R_A$

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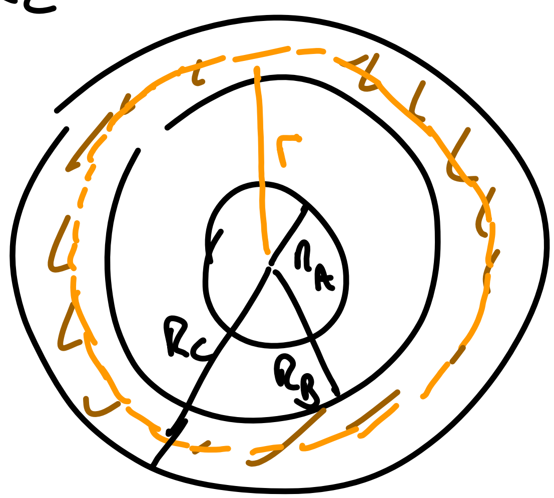
A $\vec{E} = \frac{a}{\epsilon_0 3} \frac{R_A^3}{r}$

yes!

radially outward

B $\vec{E} = \frac{a r^2}{3\epsilon_0}$

$R_B < r < R_C$



$\vec{E} = 0$ because region inside conductor

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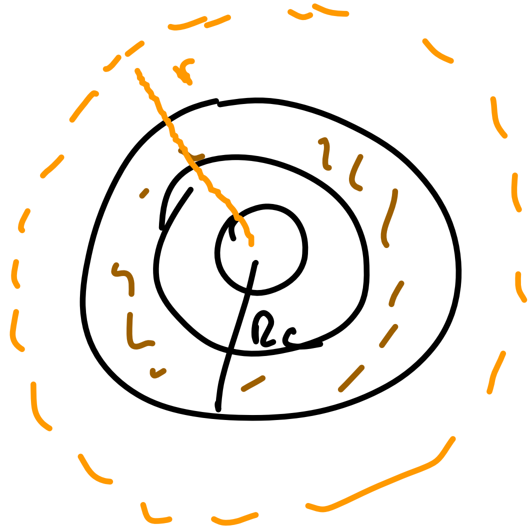
$r > R_c$



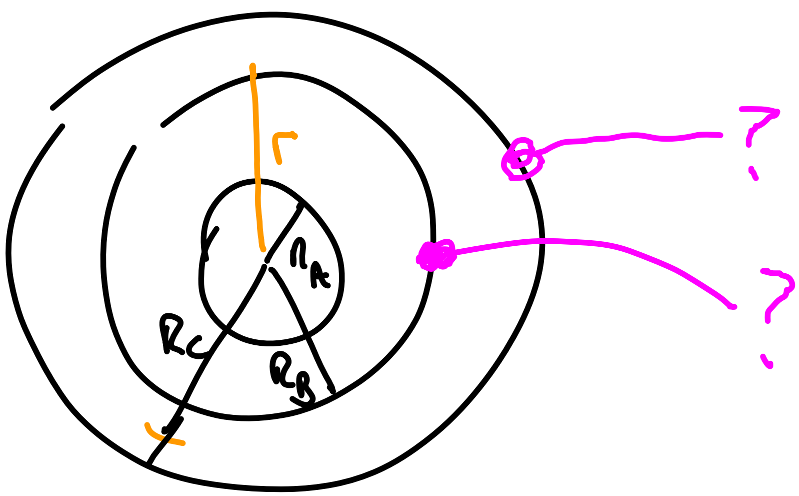
A

Ans. is
same as in 2^{ND} region ($R_A < r < R_B$)
because symmetry And Q_{encl}
are the same

$r > R_c$

$$\vec{E} = \frac{\alpha}{\epsilon_0 3} \frac{R_A^3}{r}$$ radially outward

What charge exists on the inside (outside) surface of the conducting sheath w/ inner radius R_B and outer radius R_C?



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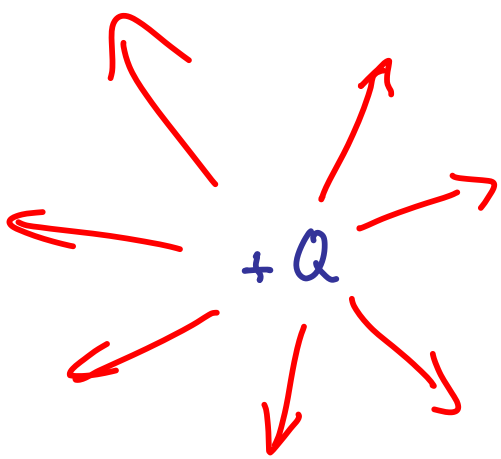
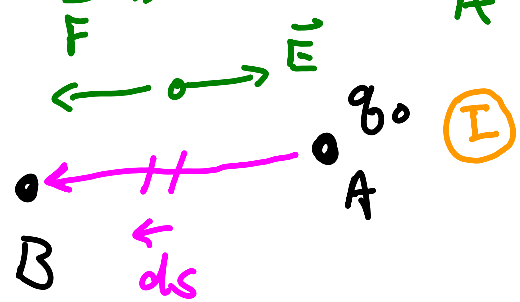
Recall how useful Energy considerations are for Mechanics

Electric field + Energy



Force I exert to move q_0 from A to B



How much work do I do to do this?

$$W = \int_A^B \vec{F} \cdot \vec{ds} = -\int_A^B F\,dr = -\int_A^B q_0 E\,dr = -\int_{R_A}^{R_B} q_0 E\,dr$$

$$= \int_A^B (\vec{F} \cdot -\vec{dr}) \quad ''$$

$$= -q_0 \int_{R_A}^{R_B} \frac{kQ}{r^2}\,dr = -q_0 kQ \left[-\frac{1}{r}\right]_{R_A}^{R_B}$$

$$= q_0 kQ \left[\frac{1}{R_B} - \frac{1}{R_A}\right]$$

Net $\oplus$ quantity

$$\Delta V \equiv \frac{W}{q_0} \equiv \text{potential difference} \qquad \frac{\text{work}}{\text{charge}}$$

$$\Big|\Big|\Big|$$

$$-\frac{\Delta U}{q_0}$$

$U \equiv$ potential energy of system

$$\Delta V \equiv V_B - V_A \equiv V_{AB}$$

$$\text{unit} = \frac{\text{Joules}}{\text{Coulomb}} \equiv \text{Volt}$$

What charge exists on the inside (outside) surface of the conducting sheath w/ inner radius R_B and outer radius R_C ?