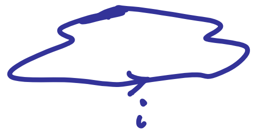
Physics 142 - November 6, 2014

- Meet w/ your project group
- Begin to formulate a plan
- Contact person to contact me and set up a time for group to meet w/ me

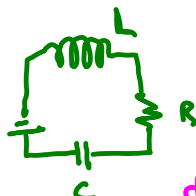
Last Time



$\phi_m = L\,i$

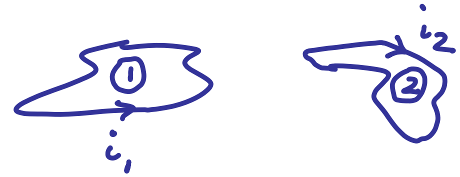
$L \equiv$ constant of self-inductance

$$\mathcal{E} = -\frac{d\phi_m}{dt} = -L\frac{di}{dt}$$



$\equiv L$ units Henrys

$$\boxed{\mathcal{E} = -L\frac{di}{dt}}$$



$\phi_{m_2} = M\,i_1$

$\phi_{m_1} = M\,i_2$

Same "M" cross-talk dictated by geometry

$M \equiv$ constant of mutual inductance

$$\mathcal{E}_{2 \text{ by } 1} = -M\frac{di_1}{dt}$$

$$\mathcal{E}_{1 \text{ by } 2} = -M\frac{di_2}{dt}$$

unless problem is specifically about mutual inductance ... usually treat inductance in a circuit as self-inductance

Energy density in the fields:

$$\boxed{u_E = \frac{\epsilon_0 E^2}{2} \qquad u_B = \frac{B^2}{2\mu_0}}$$

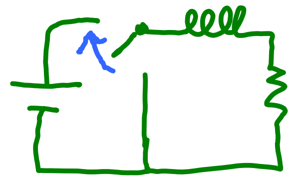
general — say nothing about circuits or sources or boundary conditions!

Energy in Inductor

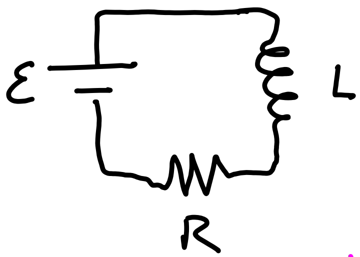
$$U = \frac{1}{2} L I^2$$

similar to $U_{capacitor} = \frac{1}{2} C V^2$

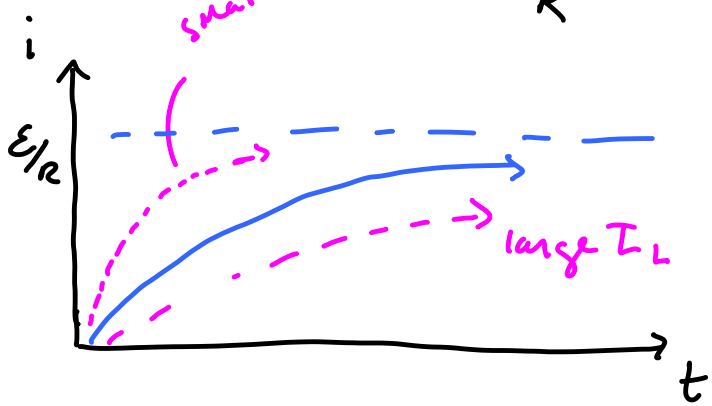
LR circuit



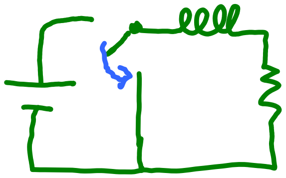
$$\mathcal{E} - L\frac{di}{dt} - iR = 0$$



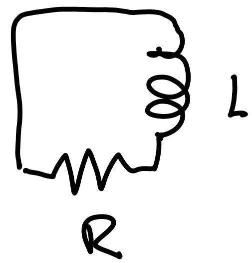
$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$



Inductive time constant $\equiv \frac{L}{R} = \tau_L$

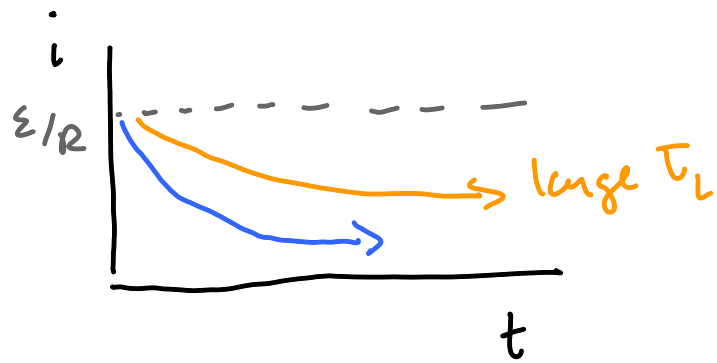


At some time t later, short L across R

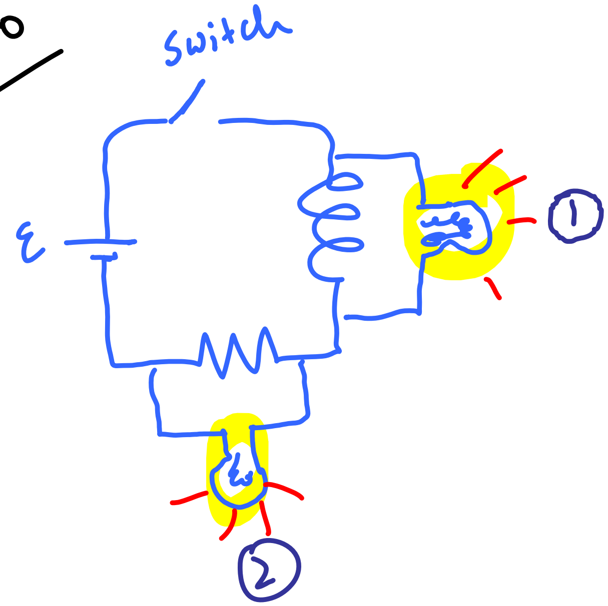


$$0 = iR + L\frac{di}{dt}$$

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L}$$

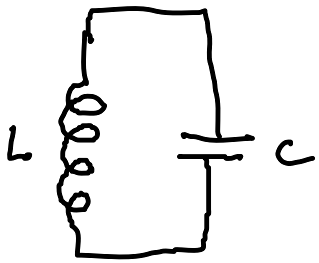


Demo



Close switch ... What happens ?

LC circuit



Fully charged capacitor

at $t=0$

$R \to 0$

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{Li^2}{2}$$

$$0 = \frac{dU}{dt} = \frac{dU_E}{dt} + \frac{dU_B}{dt} = \frac{2q}{2C}\frac{dq}{dt} + \frac{L}{2}2i\frac{di}{dt}$$

dq/dt

$$0 = L\frac{di}{dt} + q/C$$

$$\underbrace{0 = L\frac{d^2q}{dt^2} + \frac{q}{C}}$$

$$\underbrace{0 = \frac{d^2q}{dt^2} + \frac{1}{LC}q}_{\text{SHM!}}$$

SHM in $q(t)$ $\Rightarrow$ $\omega^2 = \frac{1}{LC}$

$$q(t) = Q\cos(\omega t + \varphi)$$

$$i(t) = \frac{dq(t)}{dt} = -\omega Q \sin(\omega t + \varphi)$$

Look at energy flow

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t+\varphi)$$

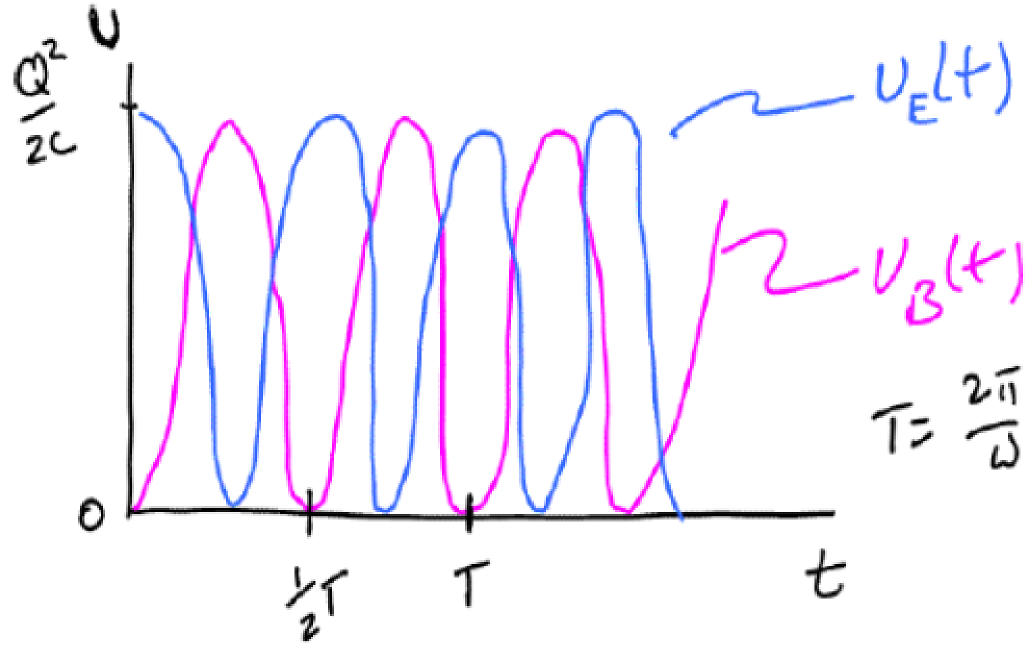
E stored in capacitor or E field

$$U_B = \frac{1}{2}L\dot{i}^2 = \underbrace{\frac{1}{2}L\,Q^2\omega^2}\sin^2(\omega t+\varphi)$$

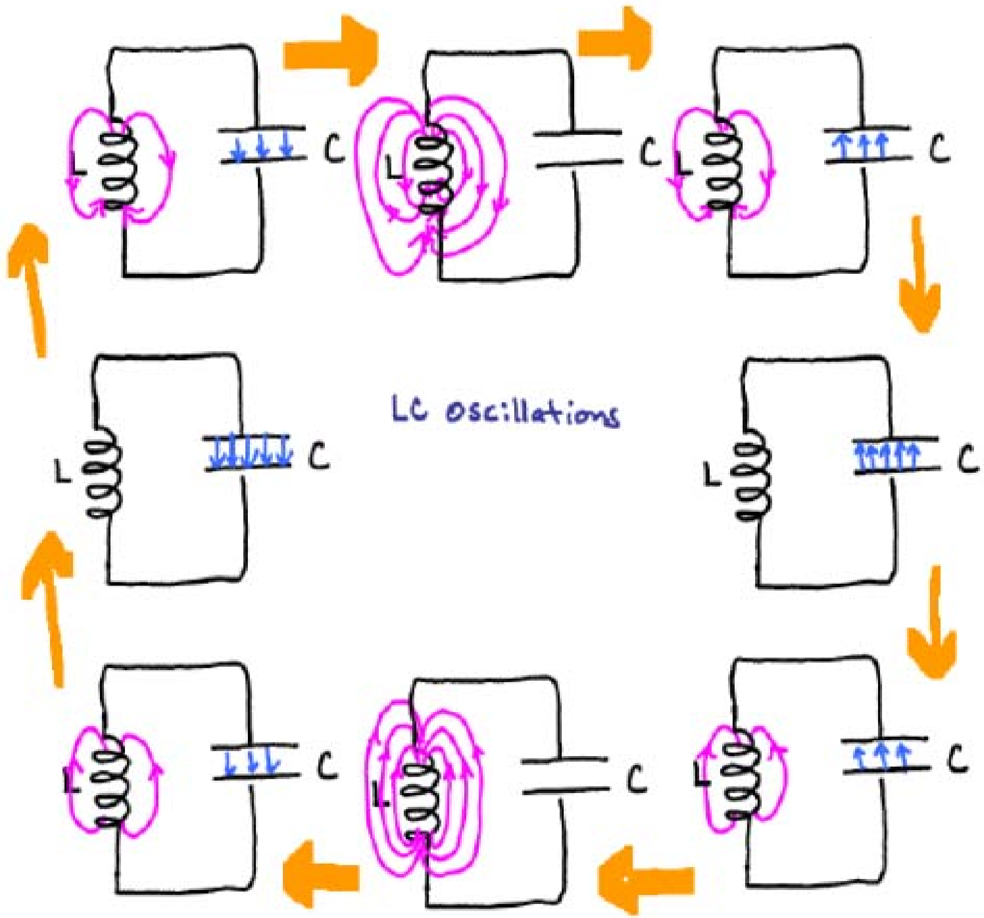
E stored in Inductor or B field

$$\omega^2 = \frac{1}{LC} \quad = \frac{1}{2}\cancel{L}Q^2\frac{1}{\cancel{L}C} = \frac{1}{2}\frac{Q^2}{C}$$

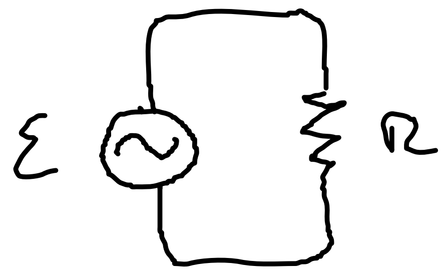
LC Oscillations



$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$


LC oscillations
L
C

AC circuits



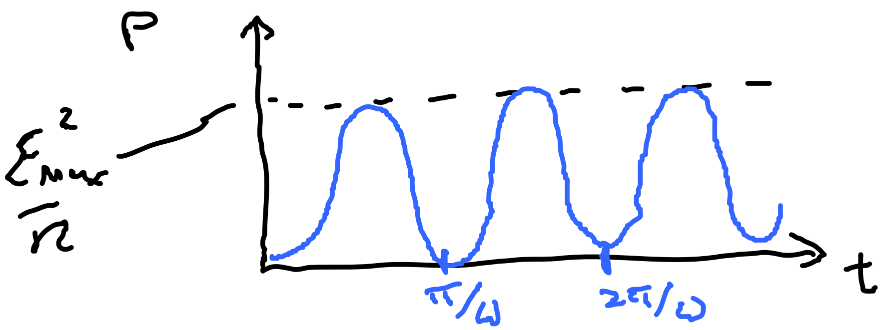
$$\mathcal{E} = \mathcal{E}_{max} \sin(\omega t)$$

$$\mathcal{E} - IR = 0$$

$$I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_{max}}{R} \sin(\omega t)$$

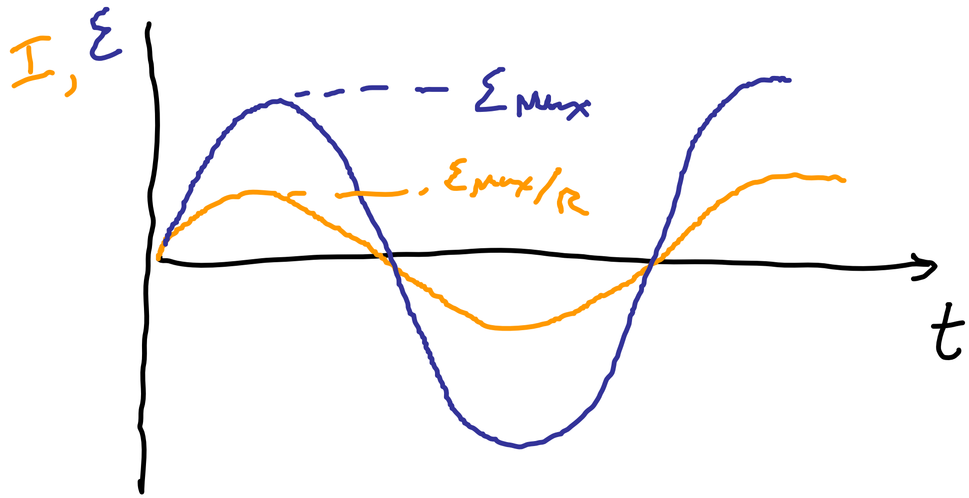
Instantaneous power

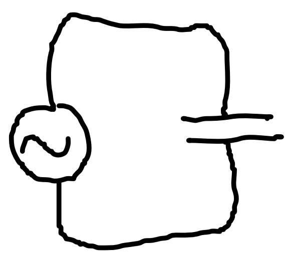
$$P = IV = I\mathcal{E} = \frac{\mathcal{E}_{max}^2}{R} \sin^2(\omega t$$



$$\overline{\sin^2\theta} \rightarrow \frac{1}{2}$$

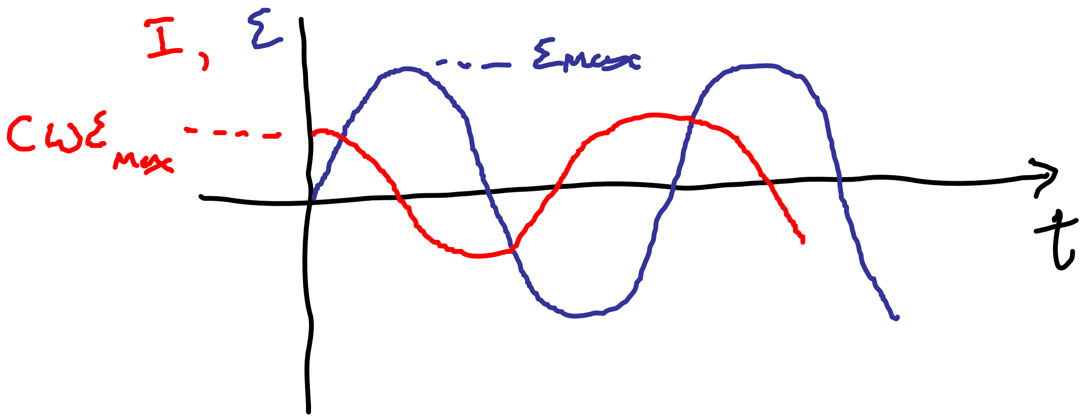
Ave power $= \bar{P} = \dfrac{\mathcal{E}_{max}^2}{2R}$





$$Q = C\mathcal{E} = C\mathcal{E}_{max}\sin\omega t$$

$$I = \frac{dQ}{dt} = C\omega\mathcal{E}_{max}\cos\omega t$$



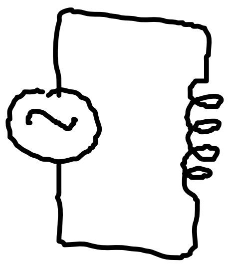
I "leads" ε by $\frac{1}{4}$ cycle

$V = IR$ $\quad I = \dfrac{\varepsilon}{R}$ $\quad I\left(\dfrac{1}{\omega C}\right) = \varepsilon_{max} \cos \omega t$

Think of this as a "resistance" in capacitive AC circuit

$\equiv$ Capacitive Reactance $\equiv X_c$

Inductive Ac circuit



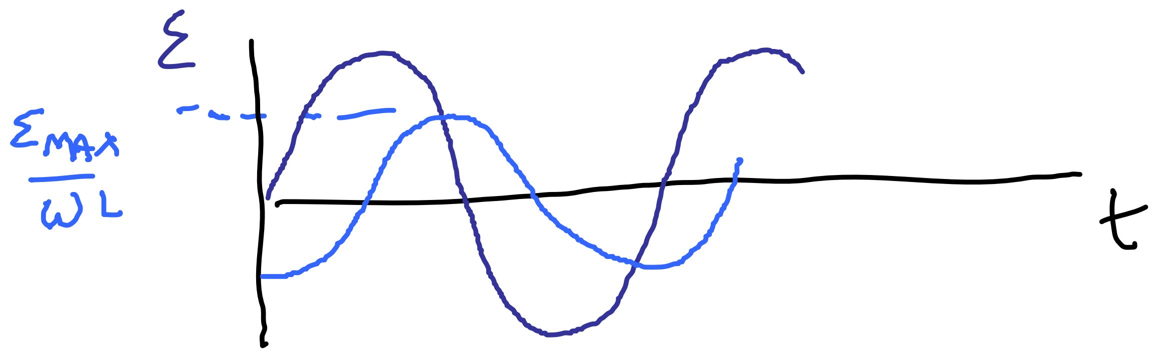
$$\mathcal{E} - L\frac{di}{dt} = 0$$

$$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$$

$$\mathcal{E}_{max} \sin \omega t = L\frac{di}{dt}$$

$$I = -\frac{\mathcal{E}_{max}}{\omega L} \cos \omega t$$

$$\omega L I = -\mathcal{E}_{max} \cos \omega t$$



ωL acts like resistance in Inductive AC circuit

$\omega L \equiv X_L \equiv$ Inductive reactance