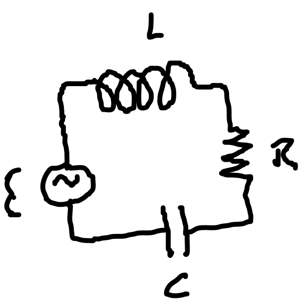
Physics 142 – November 13, 2014

- Projects . . .

Last Time

LRC circuit



$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$

general expression $I = I_{max} \sin(\omega t + \phi)$

unknown

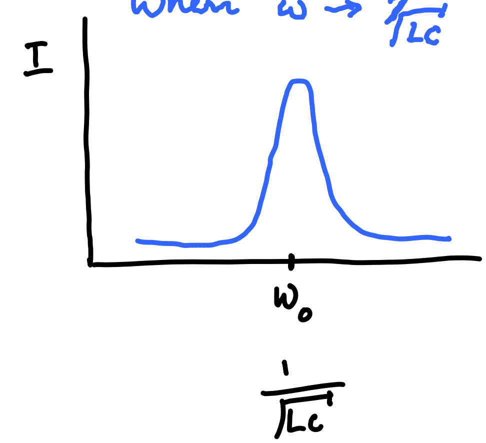
$\mathcal{E} = \Delta V_R + \Delta V_C + \Delta V_L$

$\mathcal{E}_{max} \sin \omega t = R I_{max} \sin(\omega t + \phi) - X_C I_{max} \cos(\omega t + \phi) + X_L I_{max} \cos(\omega t + \phi)$

Acts as "Resistance" of capacitor

Acts as "Resistance" of Inductor

Resonance in I when $\omega \to \frac{1}{\sqrt{LC}}$



$\tan \phi = \frac{X_L - X_C}{R}$

$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$

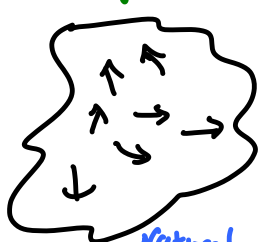
$Z \equiv \sqrt{R^2 + (X_L + X_C)^2}$

Impedance

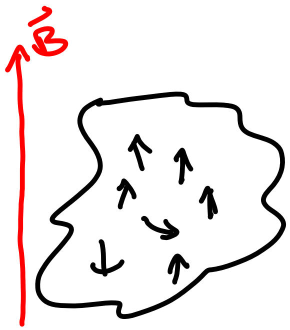
Acts as total resistance of LRC circuit

Magnetism in Materials

Paramagnetic



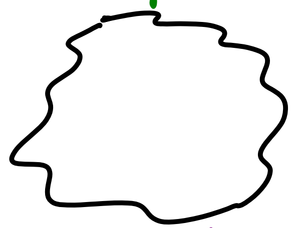
Natural Dipoles



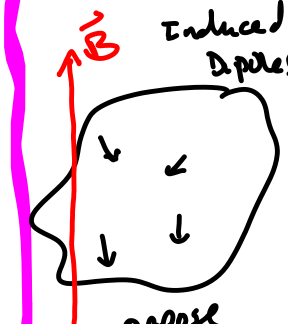
Torque from $\vec{B}$
Aligns more Dipoles
along $\vec{B}$

$\rightarrow$ $\vec{B}$ increased

Diamagnetic



No Dipole

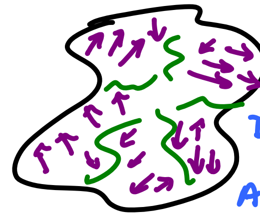


Induced Dipoles

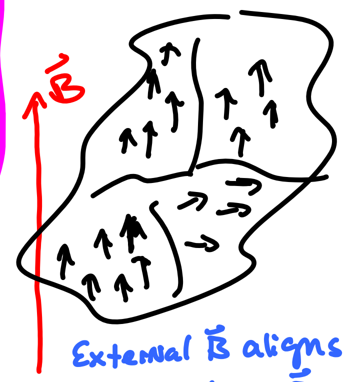
Oppose External $\vec{B}$

$\rightarrow$ $\vec{B}$ weakened

Ferromagnetic



Strong Dipoles in Domains of local Alignment



External $\vec{B}$ aligns more Domains along $\vec{B}$

$\rightarrow$ $\vec{B}$ increased

$$B = \mu_0 (1 + \chi_M) B_{free}$$

Magnetic Susceptibility

External B field

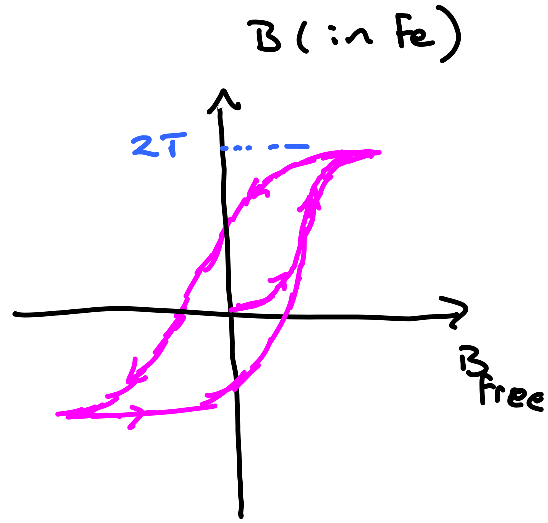
relative permeability, K_M

$\mu_0 K_M$ ~ permeability $\rightarrow \mu$

$K_M \gtrsim 1$ Paramagnetism

$\gg 1$ Ferromagnetism

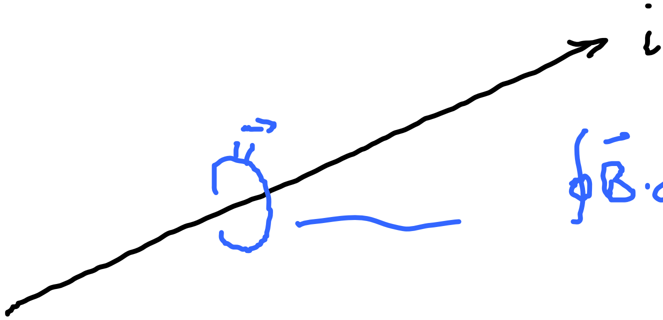
< 1 Diamagnetism



Hysteresis loop

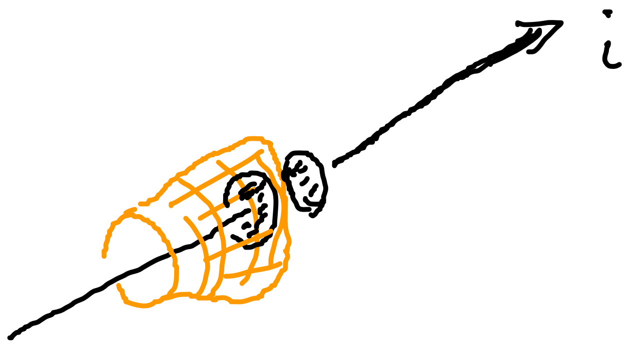
Fe "saturates" at 2T

- good thing to remember



$$\oint \vec{B}\cdot d\vec{s} = \mu_0\, i_{enclosed}$$

Ampere's law



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \underbrace{\mu_0 \epsilon_0 \frac{d}{dt}\oint_S \vec{E}\cdot d\vec{A}}$$

Maxwell's Displacement current

Integral form of Maxwell's equations

Gauss

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

No Magnetic monopole

$$\oint_S \vec{B}\cdot d\vec{A} = 0$$

Ampere

$$\oint_c \vec{B}\cdot d\vec{\ell} = \mu_0 \underbrace{\int_S \vec{j}\cdot d\vec{A}}_{I_{encl}} + \mu_0 \epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

Faraday's (law)

$$\oint_c \vec{E}\cdot d\vec{\ell} \quad -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

Transform to differential form ...

$$\vec{\nabla} \equiv \text{DEL} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{z}$$

<u>Divergence of vector field</u> (cartesian coords)

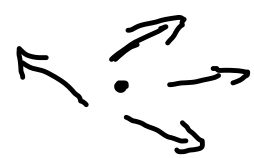
$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

scalar

div $\vec{V}$

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{j}\right)\cdot\left(V_x\hat{i} + V_y\hat{j} + V_z\hat{z}\right)$$

$$= \frac{\partial V_x}{\partial x}\hat{i}\cdot\hat{i} + \frac{\partial V_y}{\partial x}\hat{i}\cdot\hat{j} \; \cdots$$



degree to which field diverges at that point

$\rightrightarrows \quad \vec{\nabla}\cdot\vec{V} = 0$

Gauss' Theorem
Green's Theorem
divergence Theorem

$$\int_{vol} (\vec{\nabla} \cdot \vec{V})\, dv = \int_{surf} \vec{V} \cdot d\vec{A}$$

volume

Vector Field

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\int_V (\vec{\nabla} \cdot \vec{E})\, dv = \frac{\int_V \rho\, dv}{\epsilon_0}$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0}$$

$$\oint_S \vec{B} \cdot \vec{dA} = 0$$

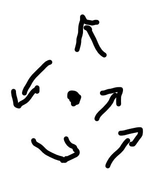
$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

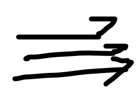
$$\oint_S \vec{B} \cdot \vec{dA} = \oint_V (\vec{\nabla} \cdot \vec{B})\, dv = 0$$

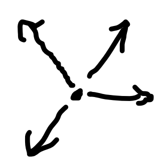
$$\vec{\nabla} \times \vec{V} \equiv \text{curl of } \vec{V} = \text{curl}\,\vec{V}$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

degree of circulation of field about a point

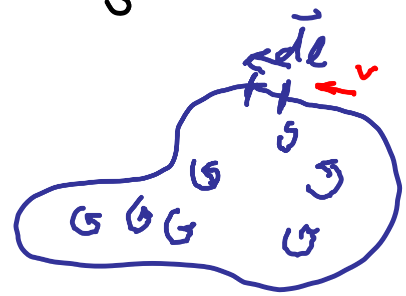
 Large curl

 0 curl

 0 curl

Stokes' theorem

$$\oint_C \vec{V}\cdot d\vec{l} = \oint_S (\vec{\nabla}\times\vec{V})\cdot d\vec{A}$$



$$\int_C \vec{E}\cdot d\vec{l} = -\frac{d\phi_M}{dt}$$

$$\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

$$\int_C \vec{B}\cdot d\vec{l} = \mu_0 \int_S \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{A}$$

$$\int_S (\vec{\nabla}\times\vec{B})\cdot d\vec{A} = \mu_0 \int_S \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{j} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\left[\begin{array}{l} \vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \\ \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\epsilon_0\dfrac{\partial\vec{E}}{\partial t} \end{array}\right]$$

differential form of Maxwell's Equations