Physics 142 – November 18, 2014

- Exam Saga

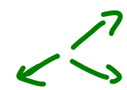
Last Time

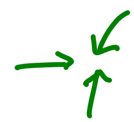
Divergence of vector field $\vec{V}$

$$\text{div}\,\vec{V} \equiv \vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

recall $\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\vec{\nabla}\cdot\vec{V} > 0$





$\vec{\nabla}\cdot\vec{V} < 0$

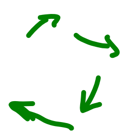
$\vec{\nabla}\cdot\vec{V} = 0$

measures Divergence or convergence of $\vec{V}$
extent to which there is a
sink or source of $\vec{V}$

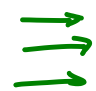
Curl of Vector field $\vec{V}$

$$\text{curl}\,\vec{V} \equiv \vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

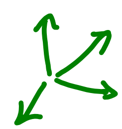
measures degree of circulation of field



$\vec{\nabla}\times\vec{V}$ not zero



$\vec{\nabla}\times\vec{V} = 0$



$\vec{\nabla}\times\vec{V} = 0$

Stokes Theorem

$$\oint \vec{V} \cdot d\vec{l} = \oint_S (\vec{\nabla} \times \vec{V}) \cdot d\vec{A}$$

Green's Theorem
Gauss' Theorem
divergence Theorem

$$\int_{vol} (\vec{\nabla} \cdot \vec{V})\, dv = \int_{surf} \vec{V} \cdot d\vec{A}$$

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow \int_V (\vec{\nabla}\cdot\vec{E})\,dv = \int_V \rho\, dv$$

$$\rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$$

$$\oint_S \vec{B}\cdot d\vec{A} = 0 \rightarrow \int_V (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

$$\rightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$$\rightarrow \int_S (\vec{\nabla}\times\vec{B})\cdot dA = \mu_0 \int_S j\cdot dA + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$$\rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{a}$$

$$\rightarrow \int_S (\vec{\nabla}\times\vec{E})\cdot dA = -\frac{\partial}{\partial t}\int B\cdot dA \rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Differential form of Maxwell's eqns

Laplacian of Scalar field, T

$$\nabla^2 T = \vec{\nabla} \cdot (\vec{\nabla} T) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

Laplacian of a Vector Field, $\vec{V}$

$$\nabla^2 \vec{V} = (\nabla^2 V_x)\hat{i} + (\nabla^2 V_y)\hat{j} + (\nabla^2 V_z)\hat{k}$$

$$\vec{\nabla} \times \vec{B} = \cancelto{0}{\mu_0 \vec{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

region with No current

TAKE curl of Both sides

$$\underbrace{\vec{\nabla} \times \vec{\nabla} \times \vec{B}}_{\text{Magic happens}} = \mu_0 \epsilon_0 \frac{\partial \, \vec{\nabla} \times \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla}\big(\cancelto{0}{\vec{\nabla} \cdot \vec{B}}\big) \underbrace{- \vec{\nabla} \cdot \big(\nabla \vec{B}\big)}_{= -\nabla^2 \vec{B}}$$

$$-\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t} = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \frac{\partial}{\partial t} \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 B_x = \mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}$$

$$\nabla^2 B_y = \mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

$$\nabla^2 B_z = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}$$

Similarly $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

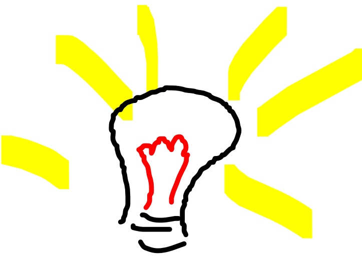
$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

1 d wave prop. on string

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 y}{\partial t^2}$$

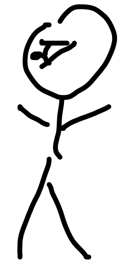
wave equations
in E, B
w/ velocity of prop.

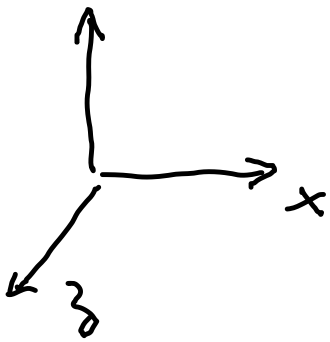
$$\sqrt{\frac{1}{\mu_0 \epsilon_0}} = c$$



Boundary conditions

Far Away





$\vec{E} = \vec{E}(x, t)$

No charge, $\rho = 0$

$\vec{j} = 0$

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 = 0$$

$$\frac{\partial E_x}{\partial x} + \cancelto{0}{\frac{\partial E_y}{\partial y}} + \cancelto{0}{\frac{\partial E_z}{\partial z}} = 0$$

E_x is constant for all x

Chose $E_x = 0$

E_y, E_z might be nonzero

$\vec{E}$ is transverse to dir. of propagation

$$\vec{E} = E(x,t)\,\hat{j}$$ direction of Polarization

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

B_x const in Time ←
$$\cancel{\frac{\partial E_z}{\partial y}}^{0} - \cancel{\frac{\partial E_y}{\partial z}}^{0} = -\frac{\partial B_x}{\partial t}$$

B_y const in Time ←
$$-\left(\cancel{\frac{\partial E_z}{\partial x}}^{0} - \cancel{\frac{\partial E_x}{\partial z}}\right) = -\frac{\partial B_y}{\partial t}$$

$$\left(\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}}^{0}\right) = -\frac{\partial B_z}{\partial t}$$

$\vec{B}$ Time dep. + $\perp$ to $\vec{E}$

E, B are Transverse and Mutually $\perp$

wave #

$$E_y(x,t) = E_{oy} \cos(kx - \omega t + \phi)$$

phase Angle

set by initial conditions

$\rightarrow 0$ for now

$\frac{2\pi}{\lambda}$

Frequency $= \frac{2\pi}{T}$

$$-\frac{\partial B_z}{\partial t} = \frac{\partial E_y}{\partial x}$$

$$B_z = -\int \frac{\partial E_y}{\partial x}\, dt$$

$$B_z = \int k E_{oy} \sin(kx - \omega t)\, dt = \frac{k}{\omega} E_{oy} \cos(kx - \omega t)$$

$$\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi/T} = \frac{T}{\lambda} = \frac{1}{c}$$

$$B_z = \frac{1}{c} E_y$$

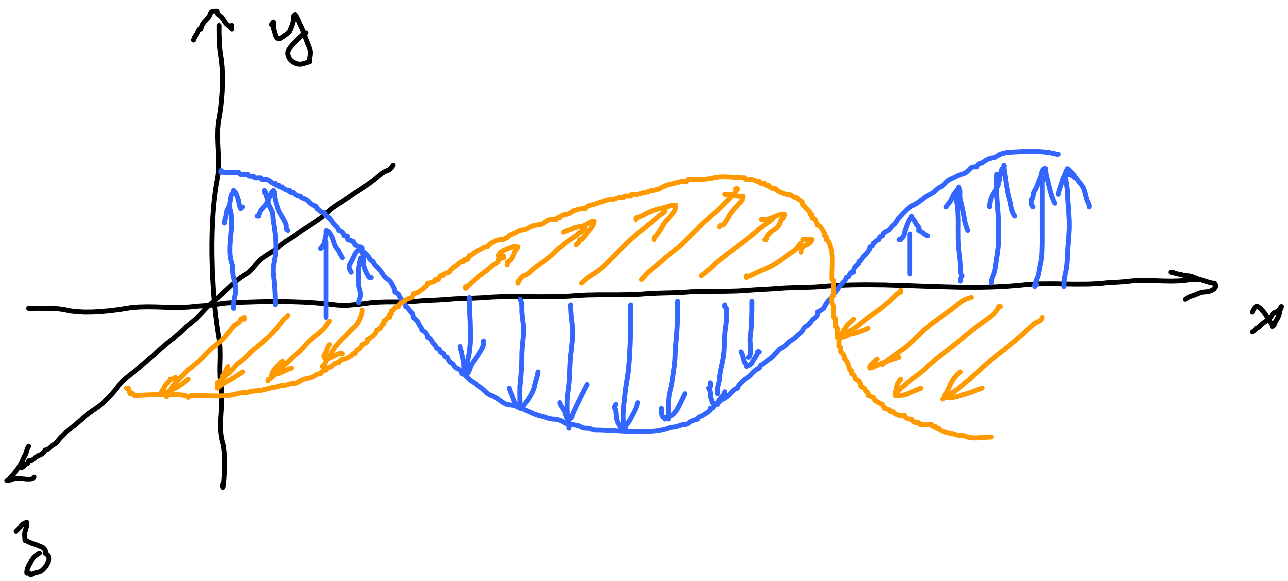
E, B are coupled

Time dependent

$|\vec{E}| = c|\vec{B}|$

in phase (Time + Space)

TRANSverse + Mutually $\perp$



$$E_y(x,t) = E_{0y}\cos(kx - \omega t)$$

$$B_z(x,t) = \frac{E_{0y}}{c}\cos(kx - \omega t)$$