SEARCH FOR HIGH FREQUENCY SIGNALS AT THE HANFORD 4K INTERFEROMETER

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It is well known that the sensitivity of the LIGO interferometers decreases at higher frequencies and indeed, for an “optimally” oriented g.w. (i.e. with the propagation vector normal to the plane of the arms) vanishes at the free spectral range (fsr) frequency \( \nu = \nu_0 = c/2L \). However “optimal” orientation is a special case and when averaged over directions of incidence and polarization of the g.w. the interferometer response remains finite at \( \nu = \nu_0 \) [1]. Furthermore the response of the Fabry-Perot cavity at the fsr is greatly enhanced [1,2]. This can be understood as parametric conversion from the carrier frequency to the next longitudinal mode of the interferometer.

Such parametric conversion has been recently demonstrated at the Hanford 4K interferometer (IFO) for single arm operation as well as with the full IFO locked in power recycled mode [3]. The data were obtained by imposing a (longitudinal) sinusoidal drive on ITMX and sweeping the frequency around \( \nu_0 \). Fig.1 shows the response of the AS_I signal for only the X-arm locked. The curve is the theoretical prediction using the mirror reflectivities and only the amplitude has been normalized for best fit.

The response with the full IFO is shown in Fig.2. We observe a broad enhancement at \( \nu = \nu_0 \) of the same width as found in Fig.1, (FWHM \( \sim300 \) Hz) and superimposed on it a narrow peak with FWHM \( \sim8 \) Hz. This latter peak reflects the, so called, “double cavity pole”. Namely resonance in both the arm cavity and the power recycling cavity.

To express the signal quantitatively we use the following notation

- \( E_2 \) carrier field incident on the beam splitter (BS).
- \( E_S \) r.f. sideband field incident at the antisymmetric (AS) port.
- \( E_A \) g.w. sideband field at AS.
- \( V_A \) photodiode voltage at AS.
- \( x_0 \) amplitude of ITMX motion.
- \( Q \) quality factor of the arm, \( Q = F(L/\lambda_c) \).
- \( F \) finesse of the arm, \( F \simeq \pi/(1 - r_1) \)
The field at the AS port for $|E_A/E_2| \ll 1$, is [4]

$$
\left| \frac{E_A}{E_2} \right| = 4 \frac{x_0}{L} Q = 4 \frac{x_0}{\lambda_c} F \tag{1}
$$

and $V_A$ can be expressed as

$$
V_A = K \left| E_2 \right|^2 \left| \frac{E_S}{E_2} \right| \left| \frac{E_A}{E_2} \right| \tag{2}
$$

Here $K$ is the conversion factor from the optical field (squared) to voltage. It is a product of the E/O shutter attenuation, of the diode conversion factor (0.54 A/W), and of the r.f. impedance (4K). Typical E/O shutter settings are 3% for single arm and “detect mode”, and 7% for “common mode”, operation. An additional attenuation factor of 4 is due to the splitting of the light from the AS port before it reaches the photodetector. We use $|E_S/E_2| \simeq 0.11$, and we take $|E_2|^2 = 20$ W, namely a recycling gain of 20 for a 1 W input. Furthermore $F_{\text{arm}} \simeq 220$ so that $Q = 8.4 \times 10^{11}$.

Data were obtained over a range of excitations $x_0$ and the results are shown in Fig. 3 where the detected voltage $V_A$ is plotted vs the equivalent strain $h = x_0/L$. To obtain $x_0$ in terms of the excitation applied to ITMX we excited the mass at 0.1 Hz and compared the result with the D.C. calibration. The value of $x_0$ at the fsr frequency $\nu_0 = 37.52$ kHz was obtained by decreasing its static value by the effect of a double pole at $\nu_P = 0.74$ Hz. Thus

$$
x_0(\nu_0) = \frac{x_{\text{DC}}}{1 + (\nu_0/\nu_P)^2} = 8 \times 10^{-16} \text{ m} \tag{3}
$$

for 1 V excitation. Namely a strain $h = 2 \times 10^{-19}$ for 1 V excitation. Using this calibration for $x_0$, the observed signal is within a factor of 2 of the prediction of Eqs.(1,2) using for $K, |E_2|^2$ and $|E_S/E_2|$ the values discussed previously.

To give an indication of the noise floor we show in Fig.4 the spectrum of the response to excitation of ITMX at a fixed frequency $\nu = \nu_0$, and for 1 V excitation. Note the appearance of the roll and bounce mode natural frequencies as sidebands to the excitation. The signal to noise is $\sim 150$ with the noise level at $V_N = 0.2 \times 10^{-6}$ V for a bandwidth $BW = 0.125$ Hz and 11 averages. Thus for a 24 hour period one can expect a noise level $V_N \simeq 6 \times 10^{-9}$ V, and therefore

$$
\frac{x_0}{L} \simeq 4 \times 10^{-23} \tag{4}
$$
should be observable with $S/N = 1$. This sensitivity is applicable for a 300 Hz (FWHM) frequency band around $\nu_0 = 37,520$ Hz.

Such high frequency is outside the band of conventional gravitational signals. However it is possible that the stochastic background spectrum extends to and beyond $\nu_0$. Of course the higher the frequency the lower the wave amplitude $h(t)$. The commonly accepted spectrum for the stochastic background has a $1/\nu$ dependence, and we write

$$\Omega_G = \frac{1}{\rho_c} \frac{d\rho_G}{d\log \nu} = \frac{1}{\rho_c} \frac{d\rho_G}{d\nu}$$

(5)

Here $\Omega_G$ is a constant, $\rho_c$ the closure density and $\rho_G$ the density in gravitational waves. The wave amplitude is then given by

$$h(t)_{\nu,\Delta\nu} = \frac{H_0}{\pi} \frac{1}{\nu} \sqrt{\frac{3}{4} \Omega_G \Delta\nu \nu}$$

(6)

where $H_0$ is the Hubble constant, which we take as $H_0/\pi \approx 0.7 \times 10^{-18}$ s$^{-1}$ and $\Delta\nu$ the measurement bandwidth.

Numerically, for $\nu = 37.5$ kHz, $\Delta\nu = 300$ Hz

$$h(t) \simeq 3 \times 10^{-24} \sqrt{\Omega_G}$$

(7)

The limits on $\Omega_G$ are at $\Omega_G < 10^{-6}$ so that the expected value is well below the sensitivity given by Eq.(4). The arguments for pursuing the measurement are two-fold. While the limits on $\Omega_G$ are for the entire universe it is possible to have a local enhancement of $\Omega_G$ within the galaxy. Furthermore if the spectrum differs from that of Eq.(5) and is peaked around $\nu_0$, then $h(t) = 3.5 \times 10^{-23} \sqrt{\Omega_G}$. To account for the random orientation and polarization of the stochastic background the limiting sensitivity given by Eq.(4) must be further increased (derated) by a factor of $\sqrt{5}$ [1,5].

The measurements at $\nu = \nu_0$ can be carried out parasitically to the normal operation of the IFO by using the AS-I monitor output directly into the SRS-785 spectrum analyzer. After averaging for periods of 1 hour the data would be transferred to a PC and written on disk for further off line analysis. Alternately the AS-I output would be mixed down to an audio frequency and the signal recorded in the DAQ stream.
References


Figure 1: Parametric conversion response to swept sine excitation around $\nu = \nu_0$ (shaking ITMX). The normalized theoretical prediction is shown by the solid curve. Single arm.

Figure 2: Parametric conversion response to swept sine at ASJ, around $\nu = \nu_0$. Fully locked interferometer.
Figure 3: Plot of the equivalent strain $h$ as a function of the observed AS$_I$ signal at frequency $\nu_0 = 32.519$ kHz.

Figure 4: Parametric conversion signal at AS$_I$ for fixed $\nu = \nu_0$. Fully locked interferometer in common mode.