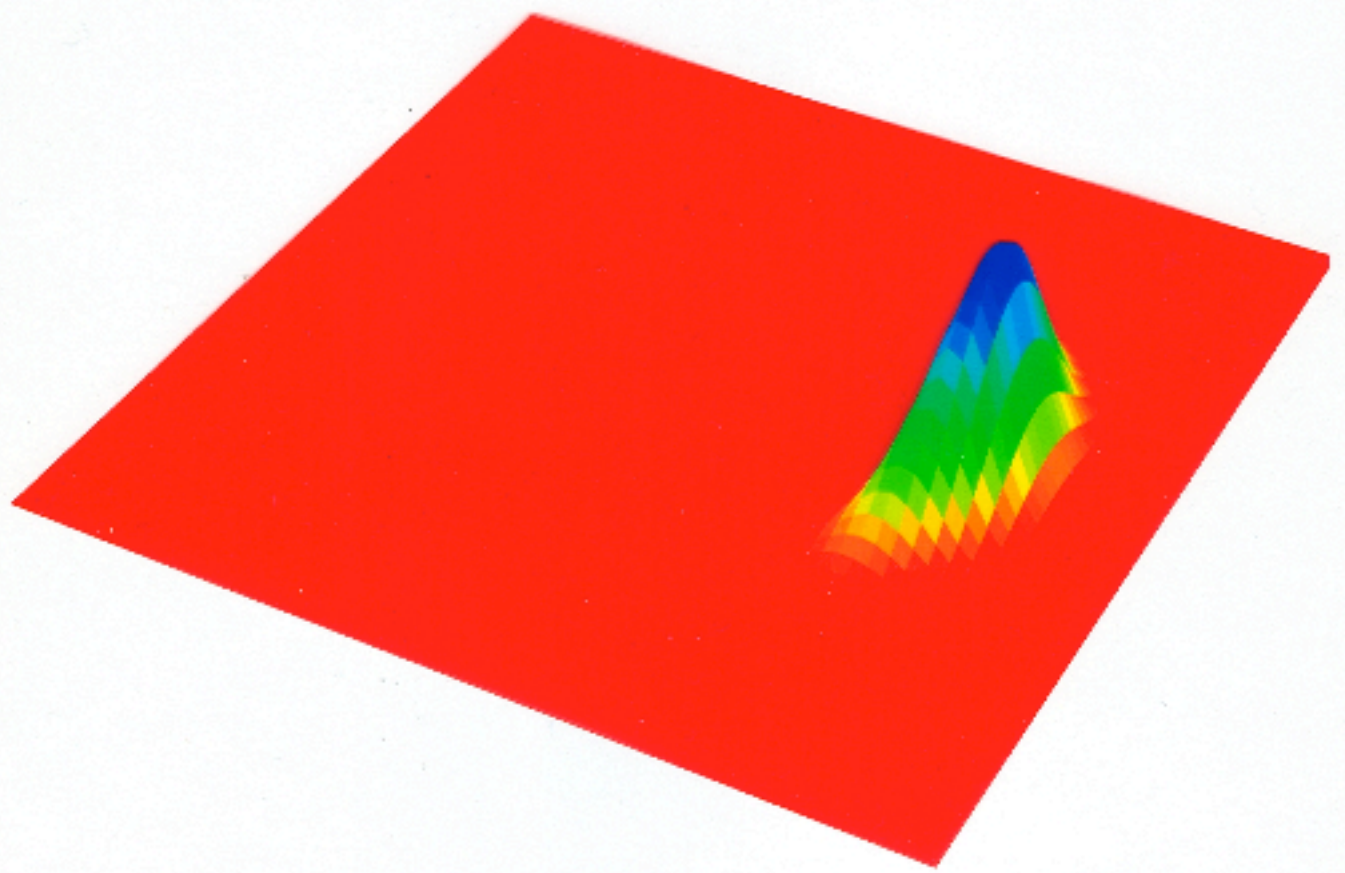


**Langmuir-Trojan Configurations in  
Spherically Harmonic Quantum Dots  
with the Coulomb Impurity in Magnetic and  
Circularly Polarized Electromagnetic Fields  
on Triangular and other Polygon Orbits**

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Utah State University**

# TROJAN WAVEPACKET



$$m_0 = 60$$

$$\omega = 1/60^3$$

$$\varepsilon = 0.041/60^4$$

# Quantum Phase Transition Theory - cranked oscillator Ex: Hopfield dielectric

$$H = \frac{p_x^2}{2} + \frac{p_y^2}{2} + \frac{(a+1)\omega^2 x^2}{2} + \frac{(b+1)\omega^2 y^2}{2} - \omega(xp_y - yp_x)$$

$$a(q) = q$$

$$b(q) = -2q$$

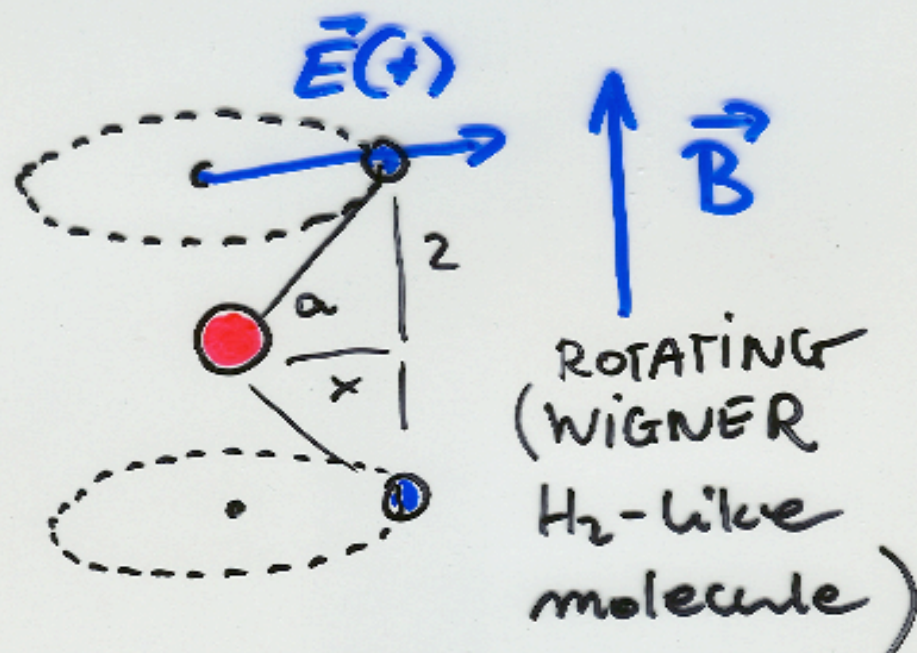
$$q = 1/\omega^2 x_0^3$$

$$\omega = \omega_c / 2$$

$$x_0 = \pm 1/\sqrt{\epsilon}$$



# LANGMUIR HELIUM IN CP AND MAGNETIC FIELDS



$$x = \frac{a\sqrt{3}}{2}$$

$$z = \frac{a}{2}$$

$$a (= a(\omega^2 - \omega_c \omega) + \left(\frac{\epsilon}{\frac{\sqrt{3}}{2}}\right)) = \frac{2}{a^2}$$

LIKE TROJAN WP WITH ANTICENTRIFUGAL  
FORCE AND  $E_c = \epsilon/\sqrt{3}$

$$\omega_c > \omega \quad E_c = \frac{\sqrt{3}}{2} \frac{3}{2^{1/2}} (\omega_c - 1)^{2/3}$$

AND TWO SOLUTIONS !

# CLASSICAL STABILITY ANALYSIS

$$\dot{p}_i = \{p_i, H\}, \quad p_i = p_{0i} + \delta p_i$$

$$\dot{q}_i = \{q_i, H\}, \quad q_i = q_{0i} + \delta q_i$$

$$\begin{bmatrix} \delta \dot{p}_i \\ \delta \dot{q}_i \end{bmatrix} = [S] \begin{bmatrix} \delta p_i \\ \delta q_i \end{bmatrix}$$

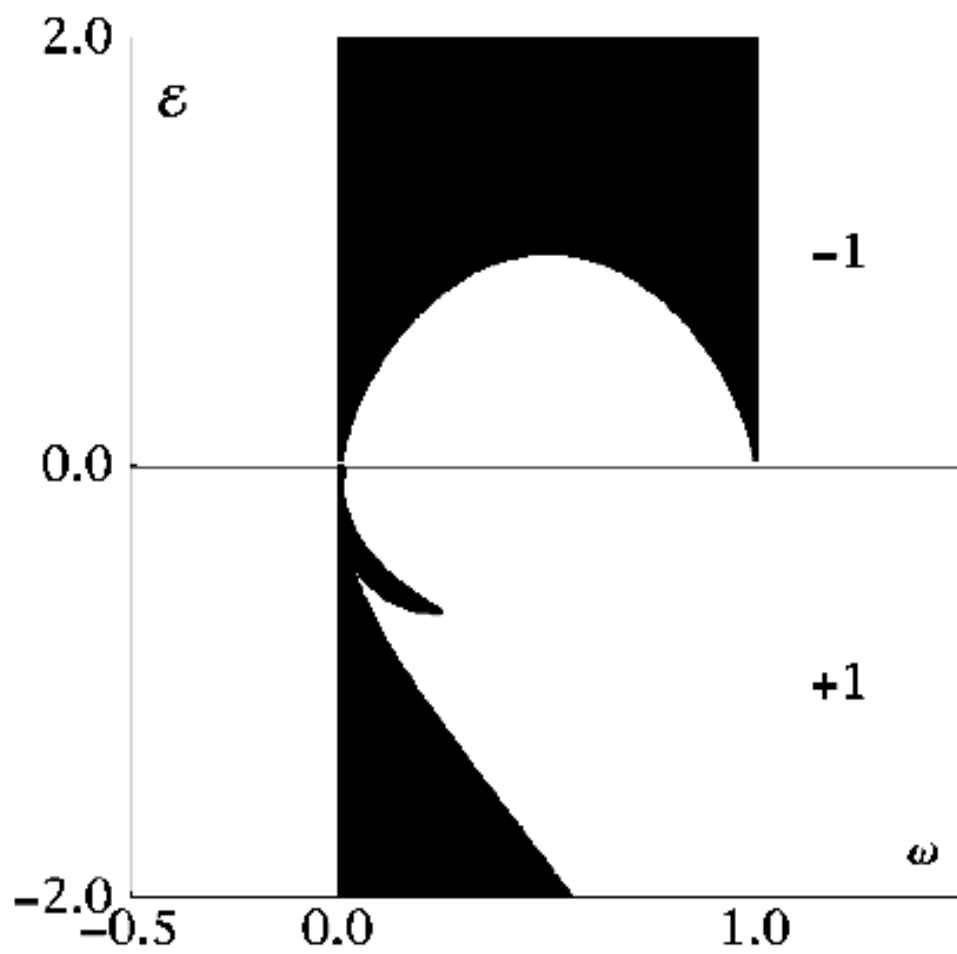
General form of stability matrix in rotating frame:

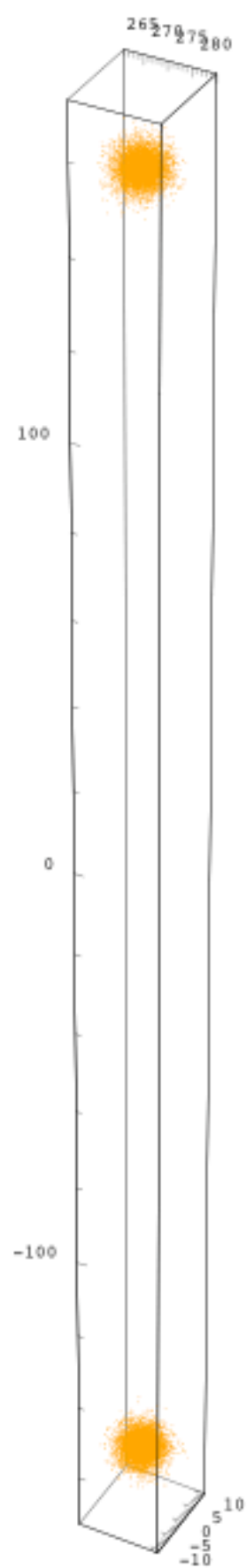
$$A = \begin{bmatrix} 0 & 1 - \frac{\omega_c}{2} & 0 \\ -1 + \frac{\omega_c}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_{ij} = - \begin{bmatrix} \frac{\partial^2 V}{\partial x_i \partial x_j} & \frac{\partial^2 V}{\partial x_i \partial y_j} & \frac{\partial^2 V}{\partial x_i \partial z_j} \\ \frac{\partial^2 V}{\partial y_i \partial x_j} & \frac{\partial^2 V}{\partial y_i \partial y_j} & \frac{\partial^2 V}{\partial y_i \partial z_j} \\ \frac{\partial^2 V}{\partial z_i \partial x_j} & \frac{\partial^2 V}{\partial z_i \partial y_j} & \frac{\partial^2 V}{\partial z_i \partial z_j} \end{bmatrix}$$

$$S_{ij} = \begin{bmatrix} A & 0 & 0 & C_{11} & \dots & C_{1N} \\ 0 & A & 0 & \dots & \dots & \dots \\ 0 & 0 & A & C_{N1} & \dots & C_{NN} \\ B & 0 & 0 & A & 0 & 0 \\ 0 & B & 0 & 0 & A & 0 \\ 0 & 0 & B & 0 & 0 & A \end{bmatrix}$$

M. Kalinski, L. Hansen, D. Farrelly, Phys. Rev. Lett. 95, 103001, (2005)







## RADIALLY OSCILLATING ELECTRON—THE BASIS OF THE CLASSICAL MODEL OF THE ATOM

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(Received 14 April 1965)

In a series of papers on the classical theory of atomic collisions recently published by the author,<sup>1</sup> it has proved necessary, both for the qualitative explanation of a number of experimental phenomena as well as for their appropriate quantitative description, to proceed with certain assumptions which are in contradiction with the prevailing classical notions regarding the structure of the atom and, of course, in contradiction with the wave approach to the atom.

First, in order to explain such phenomena as the asymptotic form of excitation and ionization formulas for high energies of the bombarding particles and the absence of a threshold for processes of inelastic collisions with heavy particles, it was necessary to assume a continuous velocity distribution of atomic electrons. Second, in order to account for the diffraction pattern associated with the crystalline structures, it was necessary to accept the existence of a strong anisotropy in the velocity distribution of atomic electrons.

Such assumptions are totally unacceptable from the point of view of electrons moving in circular or even elliptic orbits, since the range of variability of electron velocity is too narrow and the anisotropy too low. The assumption concerning the continuous velocity distribution of atomic electrons may be accounted for on the basis of classical mechanics only by the fact that the moving electron exists both beyond and in the immediate vicinity of the attracting center represented by the nucleus.

This apparent discrepancy between the conclusions derived from the classical theory of atomic collisions and Bohr's classical model of the atom, in the persistent attempt to explain atomic phenomena classically (that is, on the basis of four-dimensional space, and therefore permitting development of a mechanistic model), has led to a new analysis of Bohr's conceptions.

It appeared that Bohr's classical theory was mistaken on a point which had proved crucial

to its further development. The analogy to the planetary system has been advanced too far, and the electronic state  $n_{\text{cl}} = 0$  in which the electron would move radially has been eliminated without serious justification. The vulgarized picture of the electron falling upon the nucleus may by no means be recognized as a proof excluding such a possibility.<sup>2</sup>

The assumption that the ground-state electron is subject to oscillatory motion collisions yields a continuous velocity distribution function of the form

$$f\left(\frac{v_e}{v_e^0}\right) = \frac{4}{\pi} \frac{1}{[1 + (v_e/v_e^0)^2]^2},$$

which gives for  $v_e \gg v_e^0$  a dependence very close to that deduced from the classical theory of atomic collisions [ $f(v_e) \propto v_e^{-3}$ ]. The slight difference can be explained if the magnetic moment of the electron is taken into account (the motion of the electron is radial in the first approximation only).

In crystalline structures, oscillations of electrons can be expected to be coherent and to take place in the directions of crystal axes. This fact accounts for the "diffraction" pattern of inelastically scattered particles and the other anisotropic phenomena, such as, for instance, the strong dependence of the ranges of charged particles in a crystalline target on the orientation of its axes. The model of the atom with oscillating electrons makes it possible to understand classically a large class of phenomena.

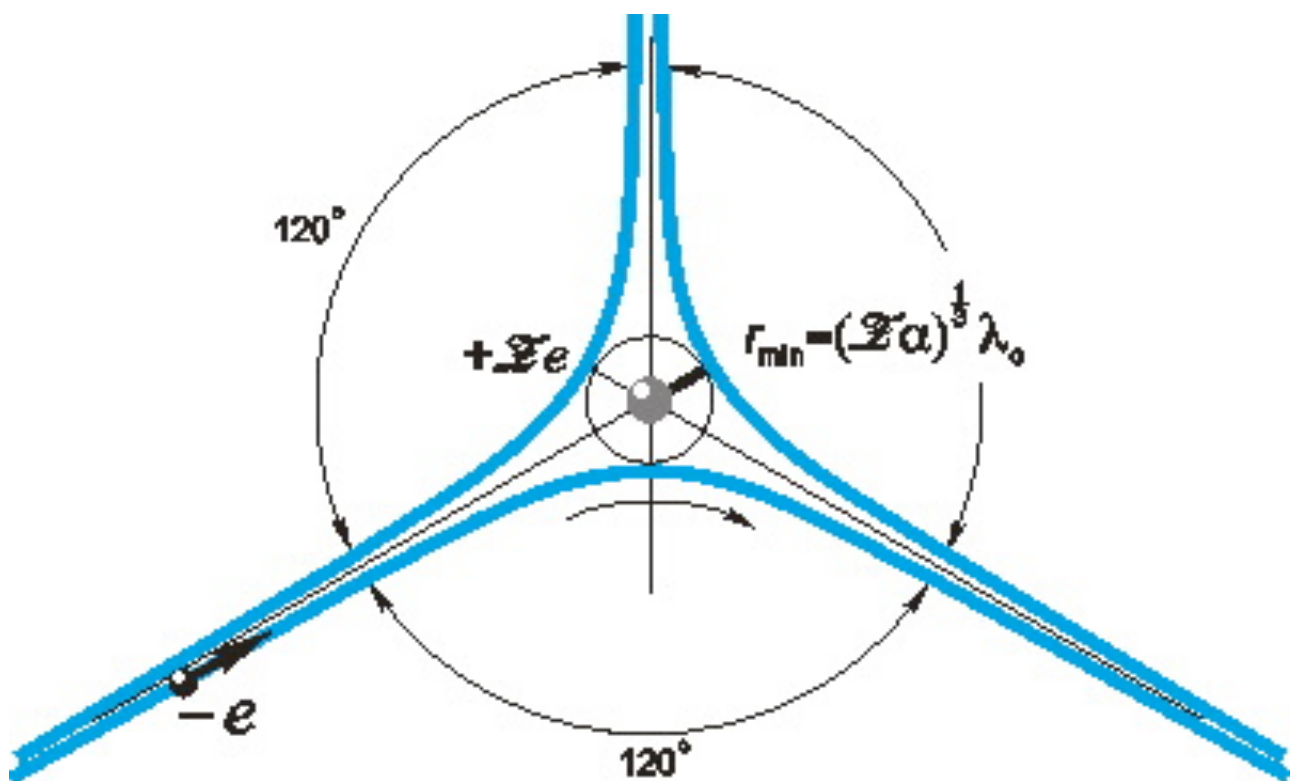
The concept of a radially oscillating electron should be regarded as the starting point for a classical understanding of the atom, and in consequence for a classical understanding of molecular forces.

<sup>1</sup>M. Gryziński, Phys. Rev. **138**, A336 (1965).

<sup>2</sup>A. Sommerfeld, *Atombau und Spektrallinien* (Friedrich Vieweg & Sohn, Braunschweig, 1951, Vol. I, p. 107).

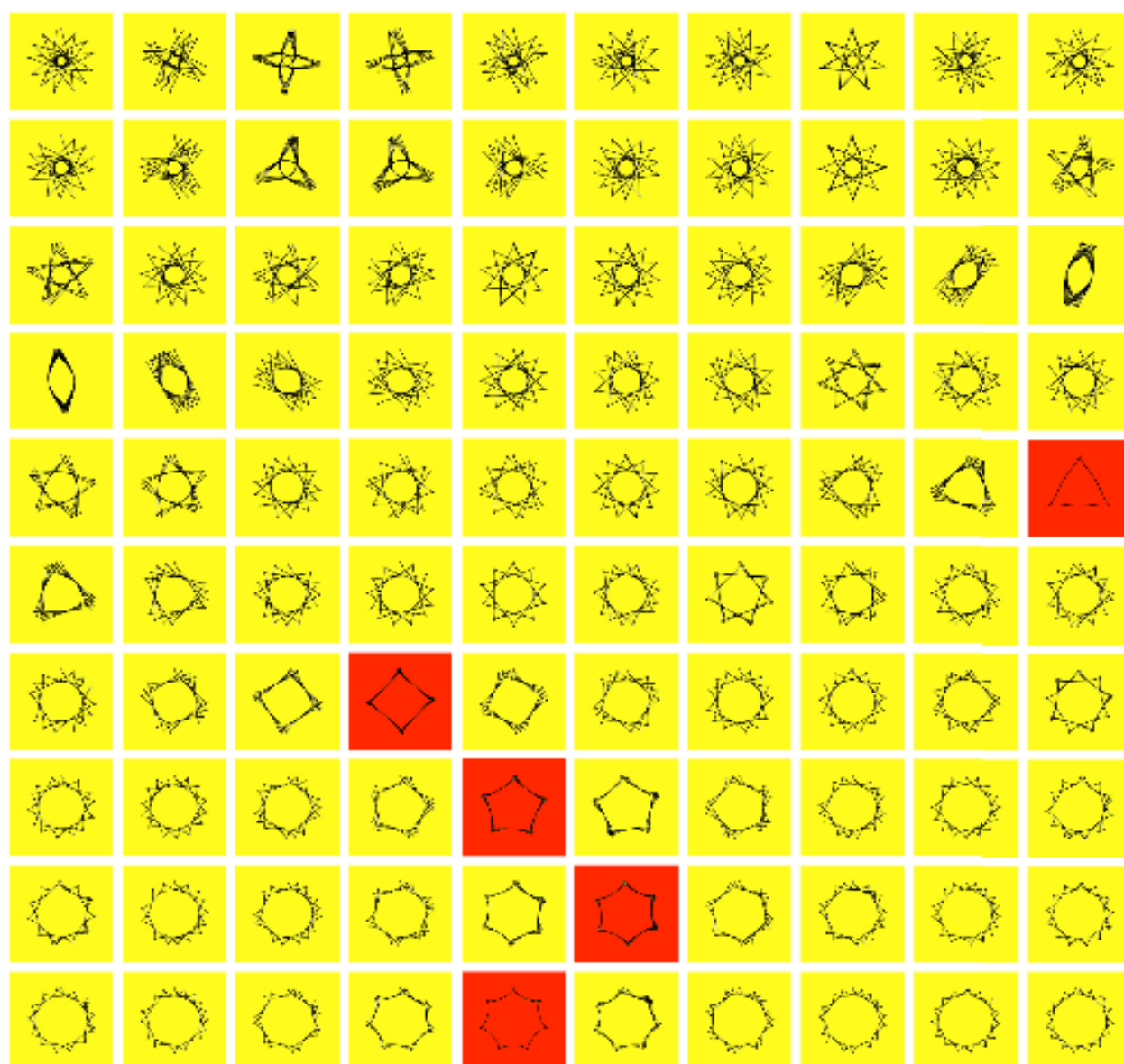


Gryzinski atom against Bohr - free Coulomb-falling classical electron is infinitesimally closely (of the order of Compton wavelength) to the nucleus deflected by the magnetic Breit-like dipole forces due to the near-nuclear instability to produce 3-symmetric trajectory



Our Triangular and Polygon trajectories: The trajectory can be piecewise elevated even to straight line when the magnetic field is strong and uniform and the Lorentz force acts against the Coulomb

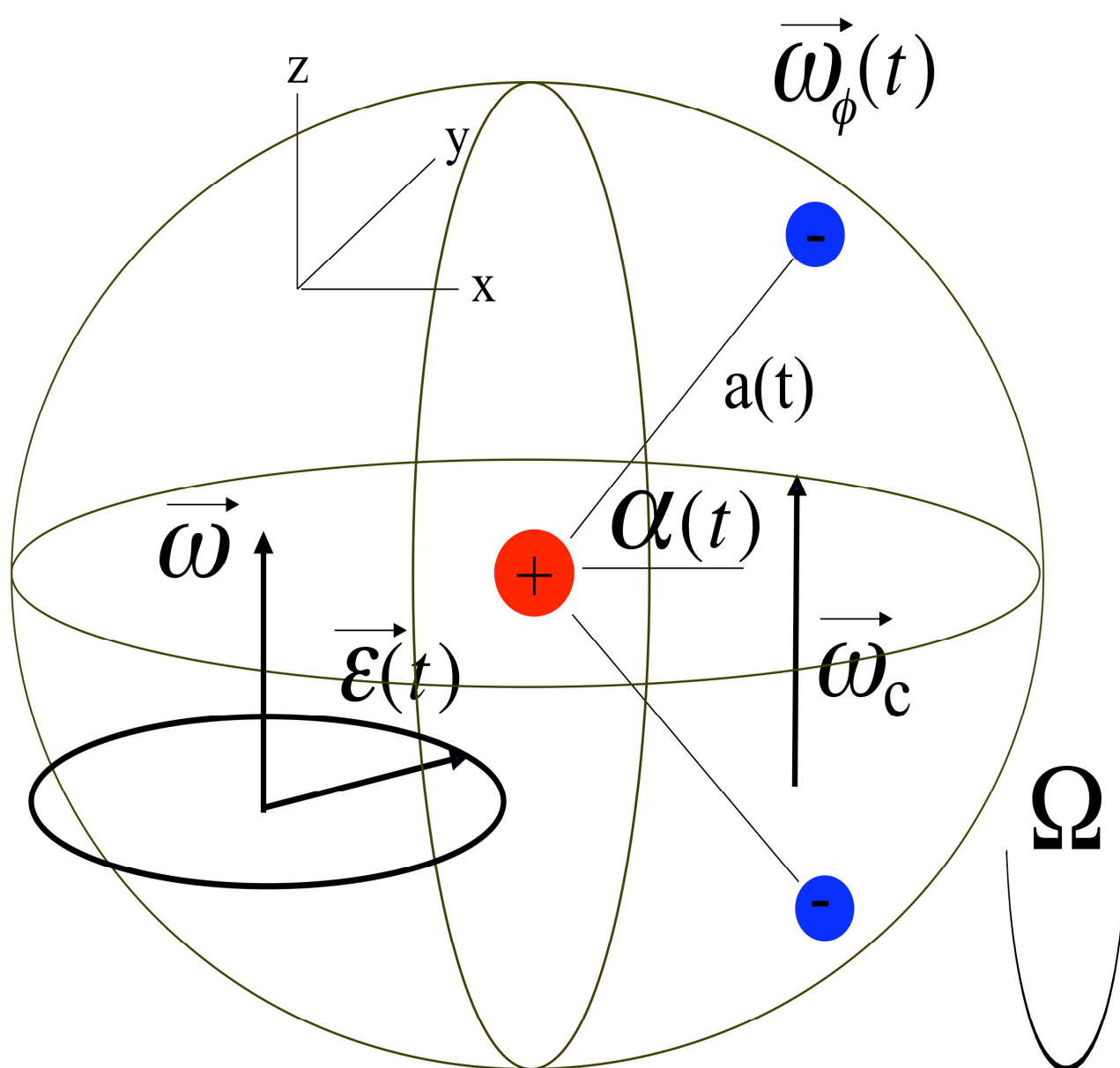
# Polygon Trajectories



$$\omega_c = 0.97 \text{ to } 2.97 \text{ step } 0.02$$

$$[x(0), y(0)] = [0, 1]$$

# Harmonic Spherical Quantum Dot Capable of Sustaining Triagle or Polygon Trajectories



## THE HAMILTONIAN

$$H = H_1 + H_2 + \frac{1}{r_{12}}$$

$$\begin{aligned} H_i = & \frac{\mathbf{p}_i^2}{2} - \frac{Z}{r_i} - \epsilon(x_i \cos \omega t + y_i \sin \omega t) \\ & - \frac{\omega_c}{2}(x_i p_{y_i} - y_i p_{x_i}) + \frac{\omega_c^2}{8}(x_i^2 + y_i^2) \\ & + \frac{\Omega^2}{2}(x_i^2 + y_i^2 + z_i^2) \end{aligned}$$

### Polygon solutions

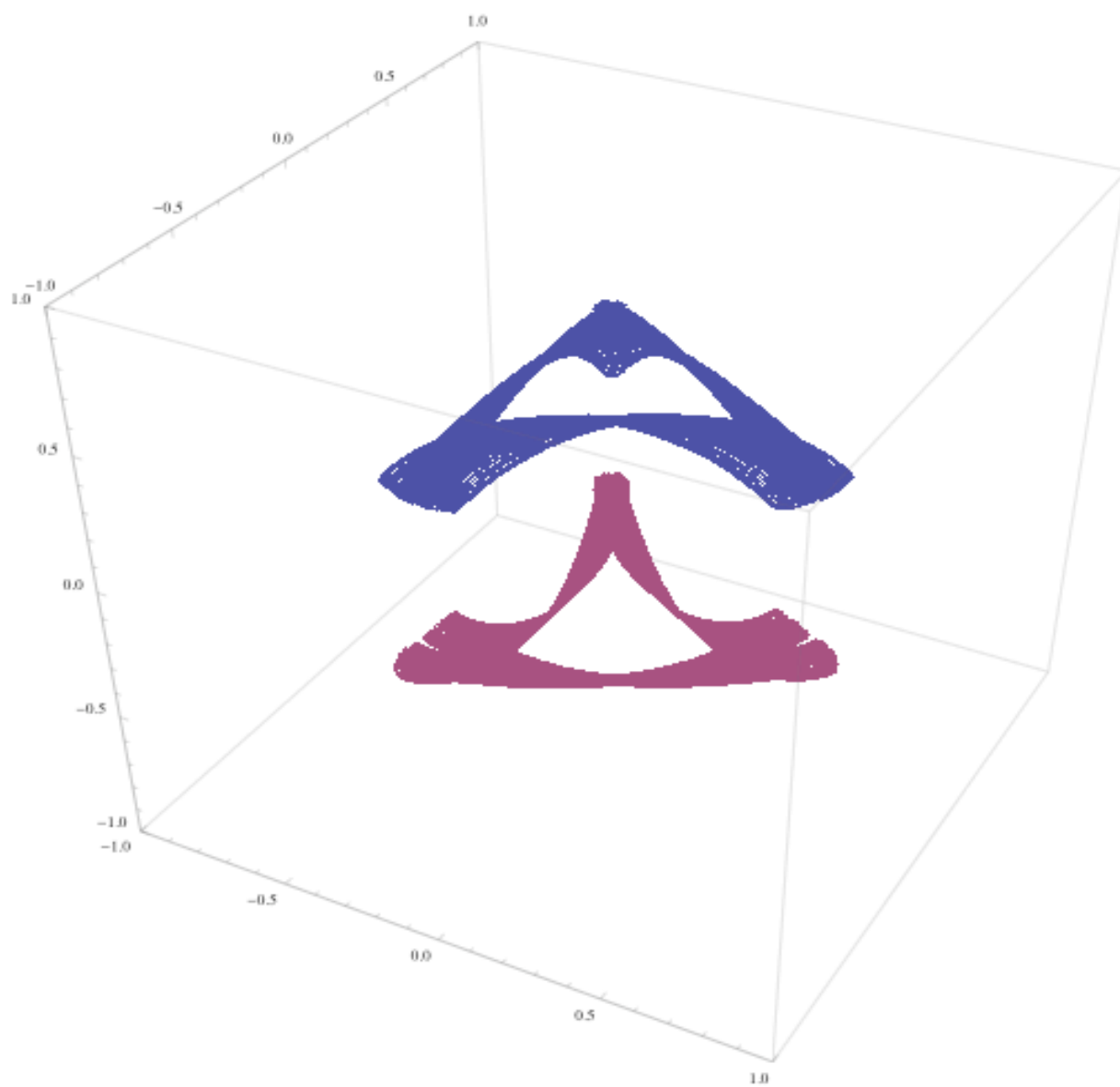
$$[x(t), y(t), z(t)], \quad [x(t), y(t), -z(t)]$$

$$x(0) = a \cos \alpha, \quad z(0) = a \sin \alpha$$

### Scaling of the solutions

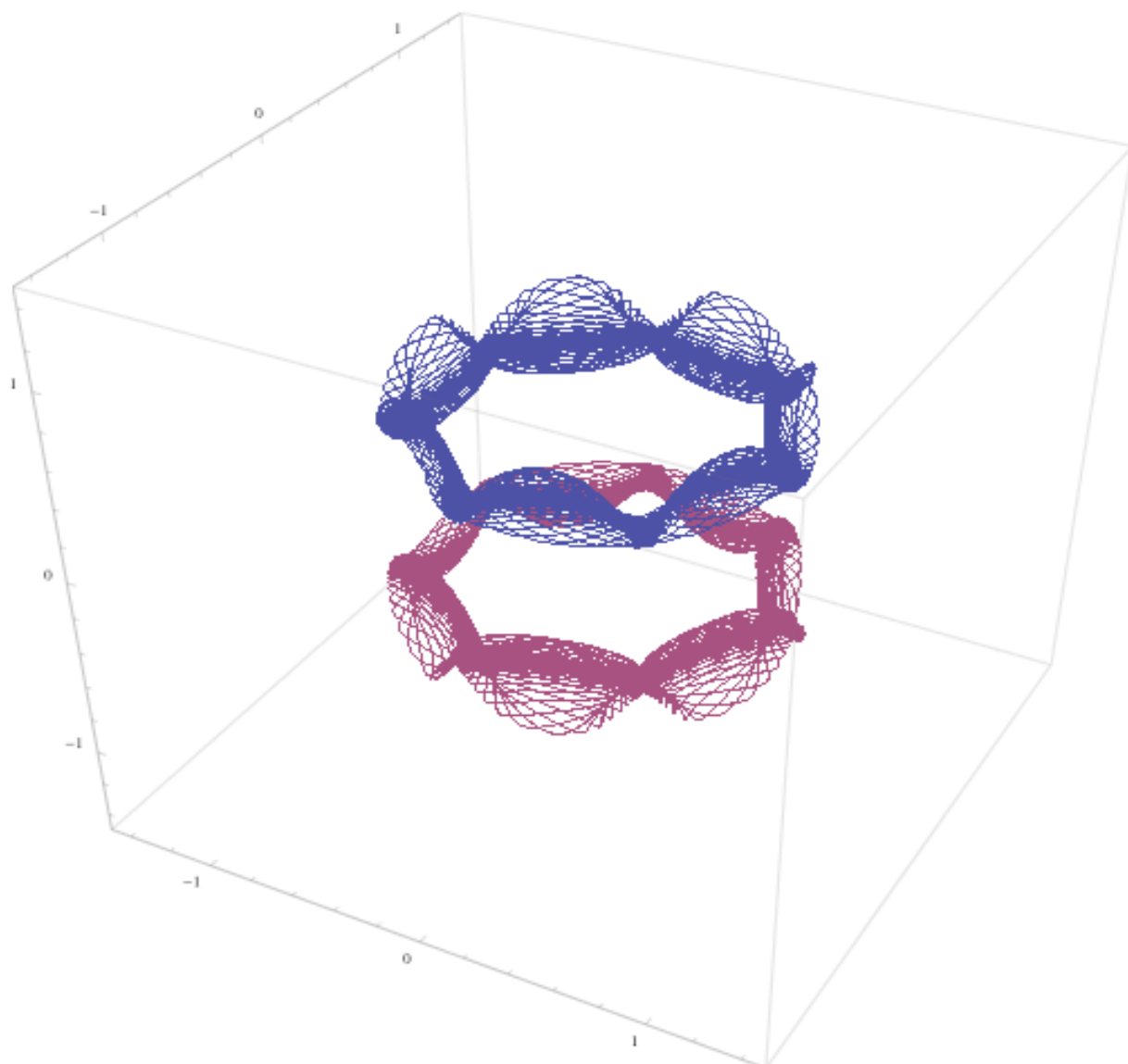
$$\mathbf{r} \rightarrow \alpha \mathbf{r}, \quad \mathbf{p} \rightarrow \mathbf{p}/\sqrt{\alpha}, \quad t \rightarrow \alpha^{3/2} t$$

$$\omega_c \rightarrow (\sqrt{\alpha}/\alpha^2)\omega_c, \quad \Omega \rightarrow \Omega/\alpha^{3/2}, \quad \epsilon \rightarrow \epsilon/\alpha^2$$

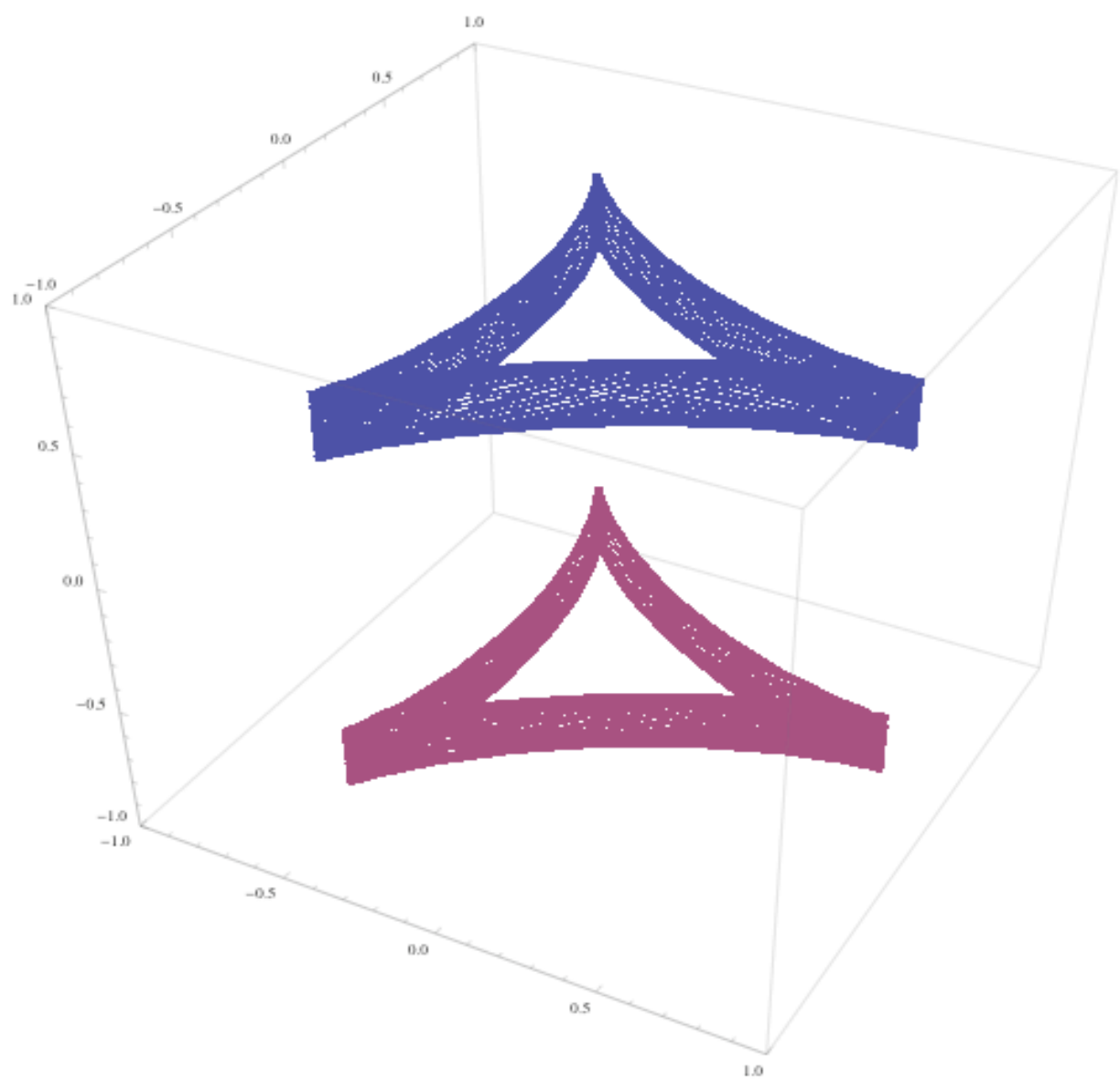


$$\mathbf{Z} = 1, \omega_c = 1.61, \epsilon = 0, \Omega = 0$$

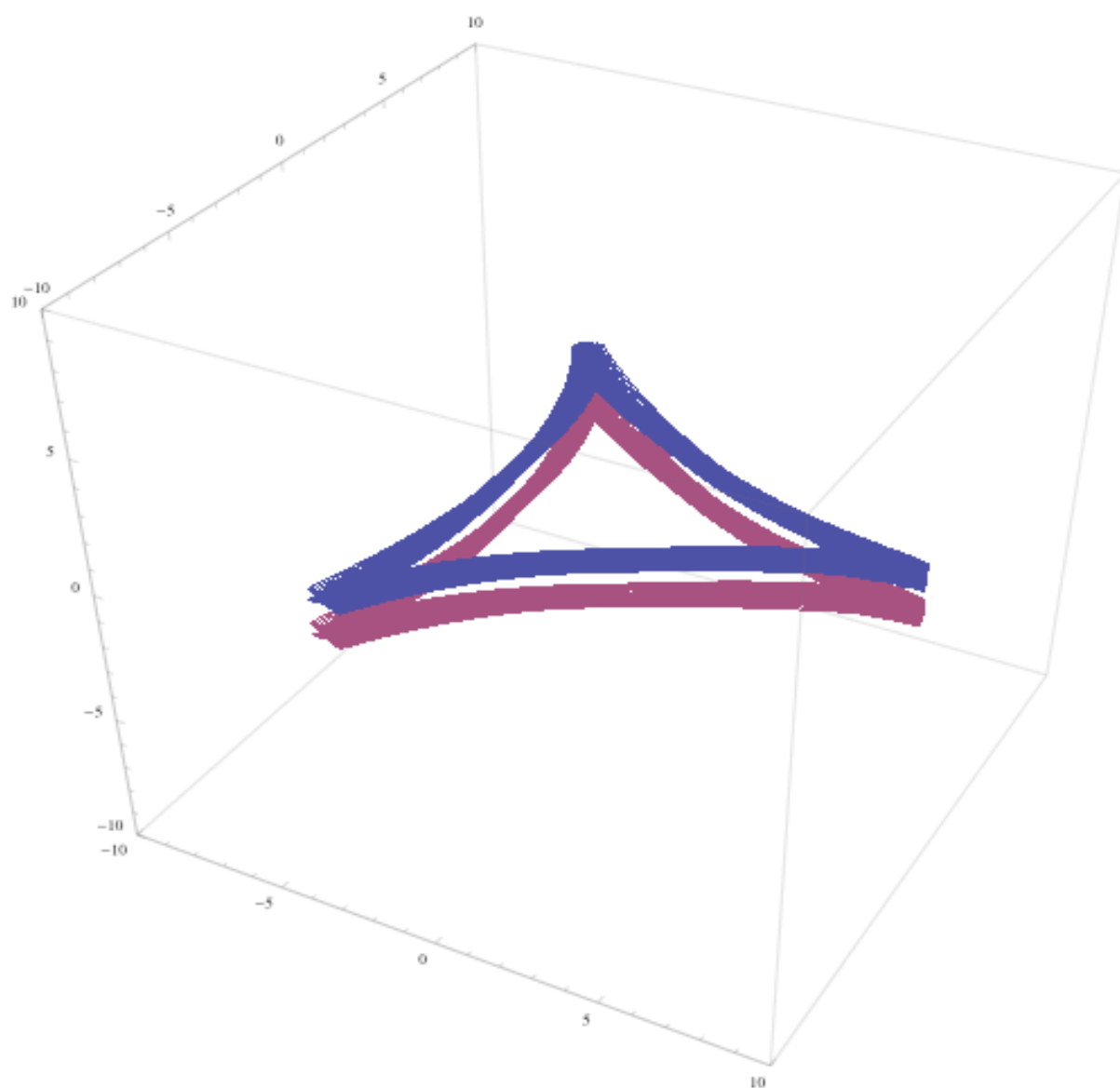




$$\mathbf{Z} = 2, \omega_c = 3.85, \epsilon = 0, \Omega = 0$$



$$\mathbf{Z} = \mathbf{0}, \omega_c = 0.71, \epsilon = 0, \Omega = 1$$



$$\mathbf{Z} = 2, \omega_c = 0.72, \epsilon = -0.01, \Omega = 1$$