# Laser Control of Correlation

Maciej Kalinski
*Department of Physics and Astronomy, University of Rochester*
*Rochester, New York 14627*

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Simple model of two fermions occupying four energy levels is presented. The analytical expression for the degree of correlation is derived and the time evolution of this parameter is calculated when the system interacts with the laser field. The case of collision is considered, when it is successfully proven that one can both decrease and increase the degree of correlation using the external field.

## I. Introduction

The correlation parameter originally introduced in [1] seems to be a promissible quantity characterizing collective behaviors of the many particle system. In the following we try to answer the question if it is possible to control the degree of the correlation using the external parameters such as a laser field. In order to do this we are dealing with the simplest nontrivial many electron system which is established by two electrons occupying four levels. The simplicity of the system makes not only possible to derive the analytic expression for the degree of correlation, but also allows us to solve the time dependent Schrödinger equation thousands of times for the different parameters of a laser field. The model may seem to be a little artificial, but we are able to recover the main character of the time dependence of K [1] even for such a physical situation like the one dimensional wave packet scattering on the the atom bounding the electron [1]. We believe that our model can describe the correlation gain in the process of the collision of two two level atoms in the presence of the laser .

## II. The model

We assume that our single electron Hilbert space can be spanned by the four orthonormal states $|1), |2), |3), |4)$. For the simplicity we assume the energy spacing between the consecutive levels is equal, which reduces the number of the parameters of the model. The full two electron state vector can be thus expand as

$$|\Psi>= c_{12}|1\,2> + c_{13}|1\,3> + c_{14}|1\,4>$$

$$+ c_{23}|2\,3> + c_{24}|2\,4> + c_{34}|3\,4>, \qquad (2.1)$$

where the two electron basis vectors are now

$$|1\,2>= \frac{1}{\sqrt{2}}[|1)\otimes|2) - |2)\otimes|1)],$$

$$|1\,3>= \frac{1}{\sqrt{2}}[|1)\otimes|3) - |3)\otimes|1)],$$

$$|1\,4>= \frac{1}{\sqrt{2}}[|1)\otimes|4) - |4)\otimes|1)],$$

$$|2\,3>= \frac{1}{\sqrt{2}}[|2)\otimes|3) - |3)\otimes|2)],$$

$$|2\,4>= \frac{1}{\sqrt{2}}[|2)\otimes|4) - |4)\otimes|2)],$$

$$|3\,4>= \frac{1}{\sqrt{2}}[|3)\otimes|4) - |4)\otimes|3)], \qquad (2.2)$$

which are also orthonormal. We also assume that our single particle basis is 'a very good' basis in the sense that all the single particle processes are already included in the diagonal parameters of the Hamiltonial $H$. This implies that the only non vanishing matrix elements of the energy operator, when the laser field is switched off, are between states which do not contain the same single particle vector, for example $<1\,3|H|2\,4>$. When the laser field is present we take only the matrix elements of the dipole operator between the neighboring levels as non vanishing and put them equal. Using all these simplifications we can write our full hamiltonian matrix as

| | $\|1\,2>$ | $\|1\,3>$ | $\|1\,4>$ | $\|2\,3>$ | $\|2\,4>$ | $\|3\,4>$ |
|---|---|---|---|---|---|---|
| $<1\,2\|$ | $\epsilon$ | $d_p$ | 0 | 0 | 0 | $v$ |
| $<1\,3\|$ | $d_p$ | $2\epsilon$ | $d_p$ | $d_p$ | $v$ | 0 |
| $<1\,4\|$ | 0 | $d_p$ | $3\epsilon$ | $v$ | $d_p$ | 0 |
| $<2\,3\|$ | 0 | $d_p$ | $v$ | $3\epsilon$ | $d_p$ | 0 |
| $<2\,4\|$ | 0 | $v$ | $d_p$ | $d_p$ | $4\epsilon$ | $d_p$ |
| $<3\,4\|$ | $v$ | 0 | 0 | 0 | $d_p$ | $5\epsilon$ |

(2.3)

so the time dependent Schrödinger equation reduces to the system of six first order differential equations for the complex functions $c_{ij}$

$$i\dot{c}_1(t) = \epsilon c_1(t) + d_p(t)c_2(t) + v(t)c_6(t),$$

$$i\dot{c}_2(t) = d_p(t)[c_1(t) + c_3(t) + c_4(t)] + 2\epsilon c_2(t) + v(t)c_5(t),$$

$$i\dot{c}_3(t) = d_p(t)[c_2(t) + c_5(t)] + 3\epsilon c_3(t) + v(t)c_4(t),$$

$$i\dot{c}_4(t) = d_p(t)[c_2(t) + c_5(t)] + v(t)c_3(t) + 3\epsilon c_4(t),$$

$$i\dot{c}_5(t) = d_p(t)[c_3(t) + c_4(t) + c_6(t)] + 4\epsilon c_5(t) + v(t)c_2(t),$$

$$i\dot{c}_6(t) = 5\epsilon c_6(t) + d_p(t)c_5(t) + v(t)c_1(t), \qquad (2.4)$$

where we denoted $c_1 \equiv c_{12}$, $c_2 \equiv c_{13}$, $c_3 \equiv c_{14}$, $c_4 \equiv c_{23}$, $c_5 \equiv c_{24}$, $c_6 \equiv c_{34}$ and $\epsilon$, $v$ and $d_p$ are adequately the spacing between the single particle energy levels, the strength of the two body interaction and the strength of the electron-laser interaction. The last two parameters can be in general dependent on the time. We see that in our representation the perpendicular to the diagonal of the Hamiltonian matrix represents the purely two particle processes.

## III. Degree of a correlation

The full two electron density matrix for our system is given by

$$\varrho = |\Psi><\Psi|. \qquad (3.1)$$

Using the expansion (2.1) we can determine the single electron reduced density matrix

$$\varrho_r = \sum_{i=1}^{4} (i|\,\varrho\,|i), \qquad (3.2)$$

by calculating the trace of $\varrho$ with respect of the single electron Hilbert space. Since the wave functions in the expansion to $|\Psi>$ in (2.1) are antysymmetric the calculation is independent of the choice of the single particle space. Then we calculate the trace of the square of the reduced density matrix $\varrho_r$, which is obviously the function of the coefficients $c_{ij}$ in the expansion (2.1). After quite cumbersome calculations we get

$$Tr\rho_r^2 = \frac{1}{2} - |(c_{12}c_{34} - c_{13}c_{24} + c_{14}c_{23}|^2, \qquad (3.3)$$

so our degree of a correlation has the analytical form and is given by

$$K(t) = \frac{1}{\frac{1}{2} - |c_{12}(t)c_{34}(t) - c_{13}(t)c_{24}(t) + c_{14}(t)c_{23}(t)|^2}. \qquad (3.4)$$

One can easily check by calculating the derivative of $K$ directly from the expression (3.4) and replacing all derivatives of $c_{ij}$ by the left side of the Schrödinger equation (2.4) that $\dot{K}(t) = 0$, when $v \equiv 0$. This means that no change of the correlation is possible, no matter how strong the laser field is, when the purely two body interaction is absent. This behavior of $K(t)$ allows us to understand the process of gaining the correlation during the collision.

## IV. Correlation control

In the following we define what we understand by the control of the correlation in the collision process within our model. First we deal with the simpler case, when the two body interaction is present all the time during the evolution.

### IV.1 Static interaction

Let us consider first the case when the two body interaction strength $v$ in the Hamiltonian matrix (2.3) is independent of time and the laser field is switched off ($d_p \equiv 0$). In this case the time evolution of the system is characterized by the eigenvalues of the Hamiltonian matrix (2.3). We put $\epsilon = 1$ for all further considerations. For the simplest initial condition, when the time evolution starts from the bare ground state (one electron occupies the state $|1)$ and the other $|2)$ or $c_1 = 1$, $c_i = 0$, $i \neq 1$ in (2.4)) the $K(t)$ can be shown to be a periodic function of time (Fig. 1). When the laser field is on (Fig. 1) the time dependence becomes pretty exotic and for the long time scale looks almost like being chaotic (Fig. 2). This can be understood when we look at the time dependence of the degree of the correlation in case of the more complicated initial condition (Fig. 1). We feel intuitively that the time dependent part of the Hamiltonian matrix due to the presence of the laser, acts similarly like the frequent setting of the initial conditions for the free field evolution. The free field time dependence of $K$ is itself complicated for the majority of the initial conditions since this parameter is a nonlinear function of $c_{ij}$, so we can expect even more complex behavior in the former case.

### IV. 2 Collision

Now we assume that the interaction strength $v$ in our Hamiltonian matrix is time dependent in a way characteristic for the collision, namely is switched on for a short time and switched off later. For the calculations we assume that the time dependence of this parameter is given by a function $v(t) = \frac{V}{\sqrt{\tau + (t-t_0)^{12}}}$ (Fig. 4). It possesses the collisional character and is smooth enough for the purpose of solving the system of differential equations. We can associate the parameter $2\tau$ with the duration of the collision and $t_0$ as the time when the collision occurs. Let us assume first that the laser interaction is not present putting $d_p \equiv 0$ in (2.3). We see (Fig. 3) that the typical time dependence of $K$ when starting from the ground state exhibits the jump. This can be explained taking into account the behavior in the case when the parameter $v$ is constant in time. Before the collision $v(t)$ is almost 0, so the $K$ parameter remains unchanged since one can show directly that $\dot{K}(t) = 0$ in this case. During the collision the two body interaction is present so $K$ can change, but after the collision is almost 0 again, so $K$ remains 'frozen'. It is interesting that within our simple model we are able to reproduce the qualitative behavior of the correlation parameter obtained for much more complicated case [1] as the one dimensional nonelastic scattering of the electron on an atom. However, as we mentioned before we feel that our model can describe in a simplest way the correlation gain in the process of the collision of two two level atoms when one excludes the ionization processes. Now we can define what we mean by the control of the correlation by the laser field within our model. Let us assume that we solved the system of equations (2.4) in the case without any external field ($d_p \equiv 0$) for a given initial condition. After the collision we will get some value of the correlation parameter $K$ which remains constant in time since the interaction between electrons is switched off. Now we can solve the Schrödinger equation with the same initial condition and the same time dependence of the interaction $v(t)$, but when the laser field is present ($d_p(t) = dsin(\omega t)$). In the later case after the collision is over the value of $K$ will also remain constant. We will say that we can control the correlation if we can both increase and decrease the value of K with respect to the zero field case after the collision, only by changing the frequency and the strength of the interaction with the laser field $d_p$. We want to underline that we compare cases when both the initial conditions and the shape of the parameter $v(t)$ are the same.

V. Results and conclusions

We solved the time dependent Schrödinger equation (2.4) plenty of times for a different strength of the collision parameter $V$, different frequencies of the laser $\omega$ and different amplitudes of the laser field $d$. In all cases we fixed the collision time $\tau$ and the time of the occurrence of the collision $t_0$. In all cases the laser field was already switched on when the collision occurred ($d_p(t) = dsin(\omega t)$). One can see that one can both decrease an increase the degree of a correlation with respect to the case when the laser field is absent (Fig. 5-6). The interesting thing is that for some values of the collision strength $V$ it is impossible to change the limiting value of K in both directions only by changing the amplitude of the laser. In some cases, for some values of the laser frequency on can only decrease or only increase K. In the regime of the parameters in investigation we see that if the frequency of the external field is too high it is impossible to change the value of the correlation parameter at all. We see that by using very strong laser field one can increase the degree of the correlation much above the gain due to the collision even if the purely two body interaction is not too strong. It suggests that the time dependent Hartree-Fock method can fail for the system in the presence of the strong laser field even if it works well for some stationary states when there is no laser field. Summarizing, we show using very simple model that it is possible both decrease and increase the degree of a correlation using the external parameters like a laser field in the system with the two body interaction. Within our model we can also show analytically that the two body interaction is necessary for any changes of the correlation parameter.

## References

[1] R. Grobe, K. Rzążewski and J. H. Eberly, submitted to Phys. Rev. Lett.